PROCEEDINGS OF SCIENCE

Helicity amplitudes for high-energy scattering processes

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We present a prescription to construct manifestly gauge invariant tree-level helicity amplitudes with one or two off-shell initial-state partons, and arbitrary particles in the final state. Such amplitudes are needed in calculations within high-energy factorization schemes, in which the initial-state partons have non-vanishing transverse momentum components. The prescription allows for efficient calculations that are easy to automate, and leads to results that are equivalent to those obtained with the well-known effective action approach.

11th International Symposium on Radiative Corrections (Applications of Quantum Field Theory to Phenomenology) (RADCOR 2013),
22-27 September 2013
Lumley Castle Hotel, Durham, UK

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PoS(RADCOR 2013)036

1. Introduction

In collinear factorization, only fractions of the, light-like, momenta of the colliding hadrons in hadron collisions are transported into the hard process, and conventionally denoted by x. Extensions allowing for the initial-state partons to have non-vanishing transverse momentum components go under the name of *Transversal Momentum Dependent* (TMD) factorization (see [1] and references therein). Since the introduction of extra momentum degrees of freedom implies the introduction of an extra energy scale, one may consider the kinematical region in which this scale is much smaller than the total collision energy. This *high-energy factorization* [2,3] becomes relevant when the transverse components are sizable compared to the longitudinal components carried into the partonic process, *i.e.* for low values of x.

While high-energy factorization allows for kinematical effects in the hard matrix elements at the lowest perturbative order that only appear at higher order in collinear factorization, it also raises the question of how to define the matrix elements in a gauge invariant manner. One approach to achieve this is by making use of effective actions that have been constructed to this end [4, 5]. Explicit expressions for matrix elements with off-shell initial-state partons have been derived using these [6, 7]. Here, we present an alternative approach, which allows for the efficient numerical calculation of helicity amplitudes with off-shell initial-state partons and arbitrary particles in the final state. In particular, the approach only requires the introduction of a few extra Feynman rules besides the usual ones for the calculation of on-shell amplitudes, and can therefor readily employ well-known efficient numerical methods that largely work algorithmically and avoid the need for the derivation of expressions [8–12].

2. Prescription

For any space-like momentum k, a light-like momentum ℓ_k and a transverse momentum $k_\perp$ with $\ell_k \cdot k_\perp = 0$ and $k_\perp^2 = k^2$ can be found such that

$$k^\mu = \ell_k^\mu + k_\perp^\mu \, . \tag{2.1}$$

For example in the frame in which k is along the z-axis, one has

$$k^\mu = E(1,0,0,\tfrac{1}{z}) \quad , \quad |z| < 1 \, , \tag{2.2}$$

and one can take, with $y = \sqrt{1-z^2}$ and for any angle ϕ,

$$\ell_k^\mu = E(1, y\sin\phi, y\cos\phi, z) \quad , \quad k_\perp^\mu = E(0, -y\sin\phi, -y\cos\phi, \tfrac{1}{z} - z) \, . \tag{2.3}$$

The space-like momenta entering the hard scattering process within high-energy factorization are restricted such that the light-like components are exactly proportional to the momenta ℓ_1, ℓ_2 of the colliding hadrons, and are given by

$$k_1^\mu = x_1 \ell_1^\mu + k_{1\perp}^\mu \quad , \quad k_2^\mu = x_2 \ell_2^\mu + k_{2\perp}^\mu \, . \tag{2.4}$$

The transverse momenta are transverse to both light-like momenta: $k_{1,2\perp} \cdot \ell_{1,2} = 0$. The strategy to arrive at gauge invariant amplitudes with off-shell initial-state momenta above is to embed the

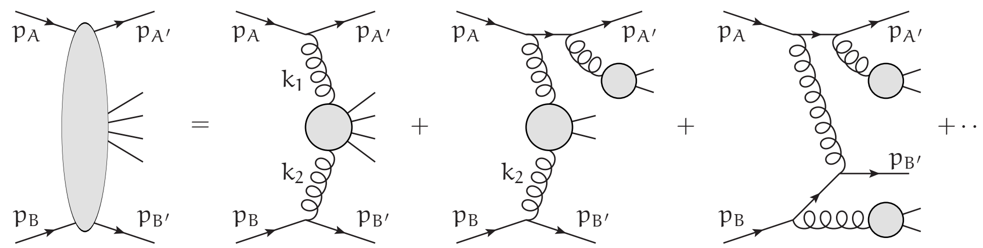

Figure 1: The embedding of $g^* g^* \to X$ into $q_A(p_A)\, q_B(p_B) \to q_A(p_{A'})\, q_B(p_{B'})\, X$.

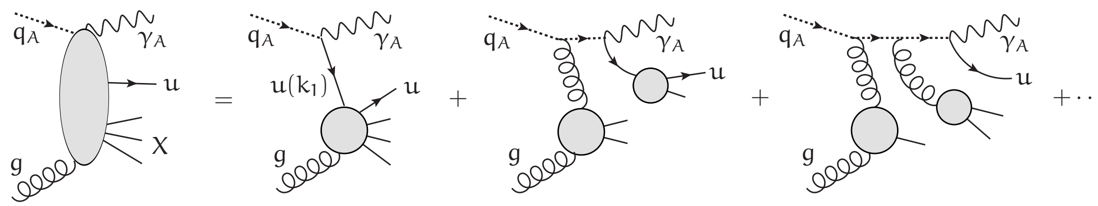

Figure 2: The embedding of $u^* g \to X$ into $q_A(p_A)\, g \to \gamma_A(p_{A'})\, u X$.

process under consideration into a larger on-shell process. In case the off-shell parton is a gluon, the embedding is obtained by replacing the gluon by an extra auxiliary quark, and adding a quark of the same type in the final state. This idea is depicted in Fig. 1 for the case of two off-shell initial-state gluons. Since all external particles in the embedding are on-shell, the amplitude is manifestly gauge invariant. The only requirement is to take into account the contribution of all Feynman graphs, also the ones that do not contain gluon lines with momenta k_1, k_2 that can be identified with the desired off-shell initial-state gluons. In case the off-shell parton is a quark, the embedding requires the introduction of an auxiliary photon that interacts only via vertices involving the desired off-shell quark, say a u-quark, and a companying auxiliary quark. Fig. 2 depicts the situation for a single off-shell initial-state u-quark.

While the embeddings ensure gauge invariance, it is not *a priori* clear that they allow for the desired kinematics of Eq. (2.4). For the embeddings, it implies that

$$p_A^\mu - p_{A'}^\mu = x_1 \ell_1^\mu + k_{1\perp}^\mu \quad , \quad p_B^\mu - p_{B'}^\mu = x_2 \ell_2^\mu + k_{2\perp}^\mu \ . \tag{2.5}$$

Clearly, the momenta of the initial-state auxiliary quarks cannot be equal to ℓ_1 and ℓ_2 respectively. In [13, 14], a compromise is proposed ensuring on-shellness of all external particles and ensuring Eq. (2.5) at the same time. Furthermore, it immediately implies the natural values for the spinors and polarization vectors of the auxiliary quarks and photons in terms of ℓ_1 and ℓ_2. The external momenta of the auxiliary particles are fixed up to parameter denoted by Λ, which determines the fraction of ℓ_1 in p_A and $p_{A'}$ and the fraction of ℓ_2 in p_B and $p_{B'}$. The compromise lies in the fact that these momenta are not real and have imaginary components. Observing, however, that gauge invariance and Eq. (2.5) hold for any value of Λ, also for $\Lambda \to \infty$, and that the imaginary momentum components become negligible in that limit, the desired physical amplitude is obtained as the coefficient of the highest power in Λ, namely Λ^2.

Eventually, the following prescription is derived in [13, 14] to calculate tree-level helicity amplitudes with off-shell initial-state partons:

1. Consider the embedding of the process, in which initial state partons are replaced, and final-state partons are added as follows:

initial state	off-shell parton type	new initial state	extra final state
1	gluon	A-quark	A-quark
1	quark	A-quark	A-photon
1	anti-quark	A-anti-quark	A-photon
2	gluon	B-quark	B-quark
2	quark	B-quark	B-photon
2	anti-quark	B-anti-quark	B-photon

   The A-photon only interacts with A-quarks and quarks of the type of the original initial state, and the B-photon only interacts with B-quarks and quarks of the type of the original initial state. The auxiliary quarks further interact with gluons. All of these vertices are the usual ones.

2. Momentum flow is as if the new initial-state partons carry the momentum of the off-shell partons, and the extra final-state particles carry vanishing momentum.

3. A-quark propagators are interpreted as $i\not\ell_1/(2\ell_1\cdot p)$ and are diagonal in color space,
   B-quark propagators are interpreted as $i\not\ell_2/(2\ell_2\cdot p)$ and are also diagonal in color space.

4. To the external auxiliary particles of the A-line, spinors and polarization vectors are assigned as follows:

initial auxiliary	initial spinor	final spinor	final polarization vector
quark $-$	$\|\ell_1]$	$\langle\ell_1\|$	$\langle\ell_1\|\gamma^\mu\|\ell_2]/\big(\sqrt{2}[\ell_1\|\ell_2]\big)$
quark $+$	$\|\ell_1\rangle$	$[\ell_1\|$	$\langle\ell_2\|\gamma^\mu\|\ell_1]/\big(\sqrt{2}\langle\ell_2\|\ell_1\rangle\big)$
anti-quark $+$	$[\ell_1\|$		$\langle\ell_1\|\gamma^\mu\|\ell_2]/\big(\sqrt{2}[\ell_1\|\ell_2]\big)$
anti-quark $-$	$\langle\ell_1\|$		$\langle\ell_2\|\gamma^\mu\|\ell_1]/\big(\sqrt{2}\langle\ell_2\|\ell_1\rangle\big)$

   For the B-line, the role of ℓ_1 and ℓ_2 are interchanged.

5. Multiply the amplitude with $x_1\sqrt{-k_1^2/2}/g_S$ if off-shell parton 1 is a gluon,
   and with $\sqrt{-x_1k_1^2/2}$ if it is a (anti-)quark.
   Multiply the amplitude with $x_2\sqrt{-k_2^2/2}/g_S$ if off-shell parton 2 is a gluon,
   and with $\sqrt{-x_2k_2^2/2}$ if it is a (anti-)quark.

For the rest, normal Feynman rules apply.

Regarding the momentum flow we remark that momentum components proportional to k_1 and k_2 do not contribute in the eikonal propagators, and there is a freedom in the choice of the momenta flowing through the eikonal lines.

Regarding the helicities of the initial-state quarks in the embedding, the simplest way to formulate the rule for the squared amplitude is to *sum over all helicities*. For some helicity configurations the resulting amplitudes will vanish or their squares will be identical, but within a purely numerical Monte Carlo set-up, these kind of issues are usually easier dealt with using a numerical method. It does mean, however, that the squared amplitudes receive an extra factor $1/2$ for each off-shell initial-state gluon.

In order to get the right power of the coupling constant, the vertices with auxiliary photons can simply be taken with unit coupling. Gluons coupled to eikonal lines must carry a coupling constant however, which is the reason for the overall factors $1/g_S$ in rule 5. The rest of those factors are necessary to get the correct collinear limit of the amplitude.

Stated as such, finally, the rules admit auxiliary photon propagators. These, however, must be omitted.

3. Results

In [13,14] it has been shown that the prescription above leads to matrix elements equivalent to those obtained with the effective action approach of [4,5]. In case of only one off-shell initial state gluon, the method is also equivalent to [15].

For one off-shell initial-state parton, some analytic results have been obtained. For example for the process $\emptyset \to g^*(p_1+k_T)\,g(p_2)\,g(p_3)\,g(p_4)$ with $p_1\cdot k_T = 0$, the color-ordered helicity amplitudes are given by

$$\mathcal{A}(2^-,3^-,4^-) = 0 \qquad \mathcal{A}(2^+,3^+,4^+) = 0 \tag{3.1}$$

$$\mathcal{A}(2^-,3^-,4^+) = \frac{[3|\not{k}_T|1\rangle}{|k_T|[31]}\,\frac{[41]^4}{[12][23][34][41]} \qquad \mathcal{A}(2^+,3^+,4^-) = \frac{\langle 1|\not{k}_T|3]}{|k_T|\langle 13\rangle}\,\frac{\langle 41\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \tag{3.2}$$

$$\mathcal{A}(2^+,3^-,4^-) = \frac{[3|\not{k}_T|1\rangle}{|k_T|[31]}\,\frac{[12]^4}{[12][23][34][41]} \qquad \mathcal{A}(2^-,3^+,4^+) = \frac{\langle 1|\not{k}_T|3]}{|k_T|\langle 13\rangle}\,\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \tag{3.3}$$

$$\mathcal{A}(2^-,3^+,4^-) = \frac{[3|\not{k}_T|1\rangle}{|k_T|[31]}\,\frac{[31]^4}{[12][23][34][41]} \qquad \mathcal{A}(2^+,3^-,4^+) = \frac{\langle 1|\not{k}_T|3]}{|k_T|\langle 13\rangle}\,\frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle} \tag{3.4}$$

Since $\big|[3|\not{k}_T|1\rangle\big| = \big|\langle 1|\not{k}_T|3]\big| = \big||k_T|[31]\big| = \big||k_T|\langle 13\rangle\big|$, we see that up to phase factors, the expressions for the amplitudes in terms of the momenta $p_{1,2,3,4}$ are very similar to the those for the process in which all gluons are on-shell, despite the fact that these momenta here do not satisfy momentum conservation.

The same can be seen for the process $\emptyset \to g(p_1)\,g(p_2)\,q(p_q)\,\bar{q}(p_{\bar{q}}+k_T)$, now with $p_{\bar{q}}\cdot k_T = 0$. The color-ordered helicity amplitudes are given by

$$\mathcal{A}(1^+,2^-,q^+,\bar{q}^+) = -\frac{[\bar{q}|\not{k}_T|1\rangle}{|k_T|\langle\bar{q}1\rangle}\,\frac{\langle\bar{q}1\rangle^3\langle q1\rangle}{\langle q1\rangle\langle 12\rangle\langle 2\bar{q}\rangle\langle\bar{q}q\rangle} \tag{3.5}$$

$$\mathcal{A}(1^-,2^+,q^+,\bar{q}^+) = -\frac{[\bar{q}|\not{k}_T|2\rangle}{|k_T|\langle\bar{q}2\rangle}\,\frac{\langle\bar{q}2\rangle^3\langle q2\rangle}{\langle q1\rangle\langle 12\rangle\langle 2\bar{q}\rangle\langle\bar{q}q\rangle} \tag{3.6}$$

$$\mathcal{A}(1^+,2^-,q^-,\bar{q}^-) = \frac{\langle\bar{q}|\not{k}_T|1]}{|k_T|[\bar{q}1]}\,\frac{[\bar{q}1]^3[q1]}{[q1][12][2\bar{q}][\bar{q}q]} \tag{3.7}$$

$$\mathcal{A}(1^-,2^+,q^-,\bar{q}^-) = \frac{\langle\bar{q}|\not{k}_T|2]}{|k_T|[\bar{q}2]}\,\frac{[\bar{q}2]^3[q2]}{[q1][12][2\bar{q}][\bar{q}q]} \tag{3.8}$$

$$\mathcal{A}(1^+,2^+,q^-,\bar{q}^-) = -|k_T|\,\frac{\langle\bar{q}q\rangle^3}{\langle q1\rangle\langle 12\rangle\langle 2\bar{q}\rangle\langle\bar{q}q\rangle} \tag{3.9}$$

$$\mathcal{A}(1^-,2^-,q^+,\bar{q}^+) = |k_T|\,\frac{[\bar{q}q]^3}{[q1][12][2\bar{q}][\bar{q}q]} \tag{3.10}$$

In fact, here we see that the expression in terms of $p_{1,2,q,\bar{q}}$ for the sum over the first four helicity comfigurations of the squared amplitudes is identical to the one for the process in which all quarks are on-shell, again despite the fact that those momenta here do not satisfy momentum conservation. The last two helicity configurations vanish for the on-shell process, and are proportional to $|k_T|$ here.

The prescription has been implemented into a numerical program, and in [16] a phenomenological study has been performed requiring matrix elements with an off-shell gluon for the production of three jets in the context of saturation effects in p-p and p-pB collisions. A numerical program based on the approach of [15] has also been used in this study, and the results have been cross-checked and confirmed. The study considers the kinematical situation in which one of the partons entering the hard process carries a large fraction x of the initial hadron momentum, while the other parton carries a small fraction. Then a hybrid factorization approach is applied, in which large-x parton is treated within collinear factorization, and the low-x parton within high-energy factorization. Then, different choices for the unintegrated PDFs from [17] for the low-x parton are compared. One particular studied observable is the angular decorrelation ϕ_{13} when all jets are in the forward region. It is the absolute value of the azimuthal angle between the hardest and the softest jet. Fig. 3 shows the differential cross section for three choices of the unintegrated PDF, namely the non-linear PDFs for the proton and lead from [17], and the proton PDF with linear evolution [18]. We observe significant differences between the three scenarios (nonlinear proton, nonlinear Pb and linear proton). The right plot shows the nuclear modification factor, *i.e.* the ratio of the distributions of “proton nonlinear”/“Pb nonlinear”. The significant deviation from unity indicates that the observable is sensitive to nonlinear effects.

4. Summary

We presented a prescription to calculate tree-level helicity amplitudes with off-shell initial-state partons and arbitrary particles in the final state, which can be used in calculations within high-energy factorization. The amplitudes are manifestly gauge invariant, and the prescription admits efficient numerical approaches. It has be implemented into a numerical Monte Carlo program with which studies of the production of tree jets in the forward region have been performed. More studies with multiple-particle final states are expected to follow.

Acknowledgments

This work was partially supported by NCBiR grant LIDER/02/35/L-2/10/NCBiR/2011.

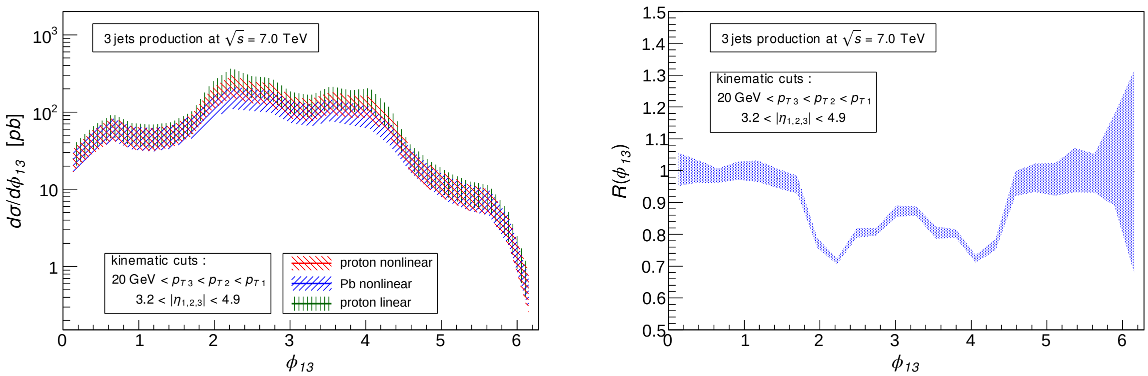

Figure 3: On the left, the differential cross section in the difference of the azimuthal angles between the hardest and the softest jet. On the right the nuclear modification factor: the ratio of the distributions of "proton nonlinear"/"Pb nonlinear". All bands represent the theoretical uncertainty due to scale variation, and statistical errors.

References

[1] P. Mulders and T. Rogers, *Gauge Links, TMD-Factorization, and TMD-Factorization Breaking*, 1102.4569.

[2] L. Gribov, E. Levin, and M. Ryskin, *Semihard Processes in QCD*, *Phys.Rept.* **100** (1983) 1–150.

[3] S. Catani, M. Ciafaloni, and F. Hautmann, *High-energy factorization and small x heavy flavor production*, *Nucl.Phys.* **B366** (1991) 135–188.

[4] L. Lipatov, *Gauge invariant effective action for high-energy processes in QCD*, *Nucl.Phys.* **B452** (1995) 369–400, [hep-ph/9502308].

[5] L. Lipatov and M. Vyazovsky, *QuasimultiRegge processes with a quark exchange in the t channel*, *Nucl.Phys.* **B597** (2001) 399–409, [hep-ph/0009340].

[6] E. Antonov, L. Lipatov, E. Kuraev, and I. Cherednikov, *Feynman rules for effective Regge action*, *Nucl.Phys.* **B721** (2005) 111–135, [hep-ph/0411185].

[7] M. Nefedov, V. Saleev, and A. V. Shipilova, *Dijet azimuthal decorrelations at the LHC in the parton Reggeization approach*, *Phys.Rev.* **D87** (2013) 094030, [1304.3549].

[8] M. L. Mangano, M. Moretti, F. Piccinini, R. Pittau, and A. D. Polosa, *ALPGEN, a generator for hard multiparton processes in hadronic collisions*, *JHEP* **0307** (2003) 001, [hep-ph/0206293].

[9] A. Kanaki and C. G. Papadopoulos, *HELAC: A Package to compute electroweak helicity amplitudes*, *Comput.Phys.Commun.* **132** (2000) 306–315, [hep-ph/0002082].

[10] M. Moretti, T. Ohl, and J. Reuter, *O'Mega: An Optimizing matrix element generator*, hep-ph/0102195.

[11] T. Gleisberg and S. Hoeche, *Comix, a new matrix element generator*, *JHEP* **0812** (2008) 039, [0808.3674].

[12] R. Kleiss and G. van den Oord, *CAMORRA: a C++ library for recursive computation of particle scattering amplitudes*, *Comput.Phys.Commun.* **182** (2011) 435–447, [1006.5614].

[13] A. van Hameren, P. Kotko, and K. Kutak, *Helicity amplitudes for high-energy scattering*, *JHEP* **1301** (2013) 078, [1211.0961].

[14] A. van Hameren, K. Kutak, and T. Salwa, *Scattering amplitudes with off-shell quarks*, *Phys.Lett.* **B727** (2013) 226–233, [1308.2861].

[15] A. van Hameren, P. Kotko, and K. Kutak, *Multi-gluon helicity amplitudes with one off-shell leg within high energy factorization*, *JHEP* **1212** (2012) 029, [1207.3332].

[16] A. van Hameren, P. Kotko, and K. Kutak, *Three jet production and gluon saturation effects in p-p and p-Pb collisions within high-energy factorization*, *Phys.Rev.* **D88** (2013) 094001, [1308.0452].

[17] K. Kutak and S. Sapeta, *Gluon saturation in dijet production in p-Pb collisions at Large Hadron Collider*, *Phys.Rev.* **D86** (2012) 094043, [1205.5035].

[18] J. Kwiecinski, A. D. Martin, and A. Stasto, *A Unified BFKL and GLAP description of F2 data*, *Phys.Rev.* **D56** (1997) 3991–4006, [hep-ph/9703445].

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