Forward and low-\(x\) physics

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In this introductory talk, recent progress in forward and low-\(x\) physics was discussed. Emphasis was put on the search for BFKL effects, saturation and multi-parton interactions.
1. Introduction

A popular approach to describe the hard scattering of hadrons in quantum chromodynamics is based on fixed-order perturbation theory and collinear factorization. As such, the calculation of the cross section is factorized in parts dominated by weak and strong coupling dynamics. Schematically:

$$\sigma_{AB} = f_A^i(x_1, \mu^2) \otimes \hat{\sigma}(i + j \rightarrow X) \otimes f_B^j(x_2, \mu^2),$$

(1.1)

with $\hat{\sigma}$ the hard scattering matrix element describing the scattering of parton $i$ and $j$ into a final state $X$, which can be calculated at fixed order, and $f_A^i$, $f_B^j$ the parton density functions (PDFs) for hadrons $A$ and $B$, depending on the longitudinal momentum fraction $x$ of the hadron carried by the parton and on the resolution scale $\mu$. The evolution of the PDFs is given by the DGLAP equations, meaning that $f(x, \mu^2)$ is fully determined by $f(x_0 > x, \mu_0^2 < \mu^2)$. In this collinear approach, the PDFs do not depend on the parton transverse momentum, $k_T$, so that, at leading order, the state $X$ must be collinear with the incoming hadrons, and, as it is leading twist, a single parton is picked from the hadron. This approach is valid for hard momentum scales and for hadrons consisting of a dilute set of partons. It has been found to work extremely well for the description of inclusive cross sections.

The forward production of high-mass particles or large-$p_T$ jets may be the result of the collision between a low and a high $x$ parton, or of the collision between partons with similar (low) $x$, but where additional QCD radiation occurs between the hard scattering system and the beam remnant. A useful pocket formula is $x_{1,2} = M \sqrt{s} \exp(\pm y)$, which relates the rapidity $y$ of the forward system with scale $M$ to the momentum fraction $x$ of the incoming partons in a collision with centre-of-mass energy $\sqrt{s}$. From this equation, it can be seen that “forward physics” is largely equivalent to “low-$x$ physics”. At the LHC ($\sqrt{s} = 14$ TeV), values of $x \approx 10^{-6}$ are reached for $M = 10$ GeV and $y = 6$, substantially extending the reach of the HERA collider.

The collinear picture described above makes use of parton showers governed by the DGLAP equations. These equations cover contributions which are leading in $\log Q^2$ and lead to parton emissions that are strongly ordered in $k_T$. While they are valid for medium to large $x$ and large $Q^2$, the picture is expected to break down at low $x$. Some transverse momentum may be injected in the hard scattering system by (perturbative) parton showers, even in the collinear approach, but this may not be adequate at low $x$ and moderate $\mu^2$. An alternative approach, based on $k_T$-dependent parton densities is described in several contributions to these proceedings [1–3]. The BFKL equations offer an alternative for the QCD evolution and re-sum terms in $\log(1/x)$ to all orders in $\alpha_S$. The parton emissions exhibit a random walk in transverse momentum, resulting in a diffusion of $k_T$ towards small $x$. Finally, the BFKL equations naturally incorporate unintegrated PDFs.

The HERA experiments have explored the low-$x$ structure of the proton and have shown that the proton becomes increasingly densely packed towards small $x$. This must eventually violate unitarity bounds and therefore it is clear that at some point non-linear evolution of parton densities (induced by parton recombinations) must set in and therefore parton densities must saturate. This may happen at a large scale $Q^2$ so that the coupling is still weak and may thus lead to a parton level understanding of the dense limit of QCD. The saturation scale, defined by a packing factor of order
unity, is given by
\[ \text{density per unit transverse area} \sim 1 \Rightarrow x g(x, Q^2_s) \sim 1 \Rightarrow Q^2_s \sim \frac{Q^2_0}{x} \lambda. \] (1.2)

Saturation effects are discussed in these proceedings in [4–6]. One of the discussion points in low \( x \) physics is related to the interplay between re-summations (BFKL) and non-linear evolution effects.

Another complication at low scales is the occurrence of multiple parton interactions (MPI). Because the partonic cross section
\[ \frac{d \hat{\sigma}}{d p_T^2} = \frac{8 \pi \alpha_s^2(p_T^2)}{9 p_T^2}, \] (1.3)
diverges for \( p_T \to 0 \), some regularization has to be applied to avoid that it exceeds the total inelastic cross section. This can be understood since at very small \( p_T \), the exchanged gluon can no longer resolve the individual colour charges of the parton. Therefore the effective coupling will decrease and the cross section is suppressed. In PYTHIA, a two-fold solution is applied. The cross section itself is regularized by introducing a cut-off parameter \( p_{T,0} \) in
\[ \frac{d \hat{\sigma}}{d p_T^2} = \frac{8 \pi \alpha_s^2(p_T^2 + p_{T,0}^2)}{9(p_T^2 + p_{T,0}^2)^2}, \] (1.4)
which itself is also energy dependent: \( p_{T,0}(\sqrt{s}) = p_{T,0}(\sqrt{s}_0) \cdot \left( \frac{\sqrt{s}}{\sqrt{s}_0} \right)^{\epsilon} \). In addition, one interaction between protons may consist of multiple parton interactions. The number of parton interactions is given by
\[ \langle n \rangle(p_{T,\text{min}}) = \frac{\sigma_{\text{in}}(p_{T,\text{min}})}{\sigma_{\text{tot}}}, \] (1.5)
so that more MPI activity is predicted for smaller values of \( p_{T,0} \).

In the remainder of this talk, several existing results were discussed related to the topics mentioned above: the search for BFKL effects, saturation physics and multiple parton interactions. Here, these results are briefly summarized and reference are given to the original papers.

2. Search for BFKL effects

In [7], the CMS Collaboration published results on the inclusive to exclusive dijet ratio. Dijets with a transverse momentum \( p_T > 35 \text{ GeV/c} \) are considered. An exclusive (inclusive) dijet sample is obtained by requiring exactly (at least) one pair of jets in the event. In the inclusive case, each pair-wise combination of jets enters in the sample. A Mueller-Navelet dijet sample is defined by considering only the most forward/backward jet pair in the event. The ratios \( R^{\text{incl}} = \frac{\sigma_{\text{incl}}}{\sigma_{\text{tot}}} \) and \( R^{\text{MN}} = \frac{\sigma_{\text{MN}}}{\sigma_{\text{tot}}} \) are calculated. The influence of the parton distribution function is reduced in these ratios, while they are expected to be particularly sensitive to parton radiation patterns. BFKL evolution predicts a strong increase of the ratios with increasing rapidity separation between the jets (\( \Delta y \)). The CMS Collaboration observes ratios that increase moderately with \( \Delta y \), as is expected from the increased phase space for parton radiation. PYTHIA agrees with the data, while HERWIG overestimates the measured ratios at medium and large rapidity separations. BFKL-motivated models, such as CASCADE and HEJ, strongly overestimate the data. This can be explained by the
fact that a large dijet mass in the exclusive sample implies high $x$, while no valence contributions
is present in these models.

The ATLAS collaboration looked at the fraction of events with reduced activity inbetween
jets as function of the rapidity separation [8]. They selected the most forward/backward jet and
applied a veto on the 3rd jet with $p_T$ above $(p_{T,1} + p_{T,2})/2$. The fraction of events passing the
veto condition is more or less equivalent to the inverse of the inclusive over exclusive cross section
ratio obtained by CMS. Again, this gap fraction is well described by DGLAP-based models such
as POWHEG+PYTHIA or HERWIG, while HEJ undershoots the data. No sign for BFKL effects has
thus been found in these results.

Another sign for BFKL dynamics could be found in the de-correlations in azimuthal angle
of dijets with increasing jet separation. This requires measuring the fourier coefficients in the
expansion of the $\Delta \phi$ distribution, given by $\langle \cos(n(\pi - \Delta \phi)) \rangle$. For back-to-back jets, the average
cosines would all be equal to unity. However, as BFKL dynamics predicts an increasing number of
partons with increasing rapidity interval between the jets, one expect the average cosines to become
smaller than unity. The average cosines relect properties of the BFKL evolution equations that are
absent in DGLAP and one furthermore expects that the ratios of average cosines further suppresses
DGLAP contributions. The CMS collaboration has obtained results with events with at least two
jets with $p_T > 35\text{GeV}$ and $|y| < 4.7$ [9]. The Mueller-Navelet jet pair is defined to be the jet pair
with the largest rapidity separation. An increasing decorrelation with rapidity separation is indeed
observed. However, DGLAP-based models, especially HERWIG, give a reasonable description of
the data, while the BFKL-inspired CASCADE model predicts too strong decorrelations. For more
information, the reader is referred to [10] and [11].

Why does DGLAP do such a good job at low $x$? In a recent talk [12], G. Gustafson argued that
it is already known for a long time that NLO corrections to BFKL tame the growth of the parton
density towards low $x$. In addition, with the introduction of NLO corrections, also the running of
$\alpha_s$ became relevant in BFKL calculations and a BFKL chain with running coupling factors favours
an initial piece with limited $k_T$, followed by a $k_T$-ordered rise to high virtuality. Therefore, at small
$x$ and large $Q^2$, the result can be well described by a DLGAP chain, starting with a tuned input at
low $k_T$. This seems to be confirmed by a recent calculation [13], with a NLL BFKL kernel and
NLO impact factors which describes data on the azimuthal decorrelations nicely.

A couple of questions might summarize the current state of the search for BFKL effects. As we
don’t find any clear sign for BFKL in experimental data, one might first of all ask whether the right
observables are being looked at. Is it a coincidence that state-of-the-art BFKL calculations resemble
DGLAP predictions? Probably not, since, to infinite order, both should give the same answers.
Moreover, in detailed (MC) calculations both approaches are being modified through different
effects. DGLAP-based models need extra $k_T$ generated by parton showers, angular ordering of
emissions, multi-parton interactions, etc. On the other hand, BFKL calculations need NLO/NLL
corrections. Finally, is the BFKL vs. DGLAP debate being pushed towards more and more extreme
corners of phase space, now that higher order, multi-leg matrix element calculations are available?

3. Saturation

As is well known, the DIS cross section levels off when decreasing $Q^2$ towards the photopro-
duction limit. This is expected from saturation, as e.g. described in dipole models. At HERA, this happened however at very small values of $Q^2$ and could therefore by dismissed as non-perturbative physics. Many studies of the structure function $F_2$ based on saturation-inspired models exists and these are being applied to new observables, also at the LHC. For an update, the reader is refered to [14–16].

An example of this is a new measurement proposed in [17] and performed by the CMS Collaboration [18]. Here, the leading jet $p_T$ spectrum for $|\eta| < 2.5$ is integrated above $p_{T,\text{min}}$:

$$\sigma(p_{T,\text{min}}) = \int_{p_{T,\text{min}}} d \sigma d p_T^2 dy \frac{d^2 \sigma}{d p_T^2 dy}$$  \hspace{1cm} (3.1)

This integrated cross section should, by definition, approach $\sigma_{\text{inel}}$ for $p_{T,\text{min}} \to 0$. By using the leading jet in the event, one is less sensitive to MPIs. The integrated cross section should however be sensitive to the regularization of $\sigma_{\text{int}}$ and the saturation of parton densities.

The CMS Collaboration performed this analysis with leading charged particles. The cross section is integrated from a minimal $p_T$ of the leading track and scaled to $\sigma_{\text{inel}}$. The result is generally well described by PYTHIA, but there is a large sensitivity to the tunes that are utilized. One observes a turn-over over the cross section at a $p_{T,\text{min}}$ of a few GeV. It would be interesting to see how this evolves as function of rapidity, as forward leading tracks would access lower $x$ values. Comparisons to calculations including saturation would also be very interesting. More results on this were presented in [19].

4. Multiple parton interactions

The Underlying Event (UE) is defined to consist of all final state particles except those originating from the hard scattering (even though a strict separation cannot be made in the quantum-mechanical sense). Multiple parton interactions (MPI) are well established in the description of the UE. The most convincing argument in favour of MPIs is the high multiplicity observed in hadronic collisions. This is indeed very difficult to explain without MPI. Understanding the UE is crucial for precision measurements of the Standard Model and for the search for new physics, but its dynamics is not well understood. Phenomenological models typically involve parameters that have to be tuned to observed data.

A quantitative analysis of the UE is possible by studying final state activity as a function of the hard scale in the event. As such, one can divide the azimuthal phase space to separate the UE from the hard scatter. A so-called “toward” and “away” region capture the hard scatter, while the “transmax” and “transmin” capture MPI+PS and just MPI, respectively. One typically looks at particle densities, energies, etc. in the transverse region as function of the hard scatter $p_T$ scale obtained from the leading jets, Drell-Yan pairs, etc. This enables tuning of Monte Carlo models [20, 21].

The term “Double Parton Scattering” (DPS) is used when more than one MPI is a hard parton scattering. The cross section for a generic DPS process can be written as

$$\sigma_{AB} = \frac{m \sigma_A \sigma_B}{2 \sigma_{\text{eff}}}$$  \hspace{1cm} (4.1)
where $\sigma_{\text{eff}} \sim \sigma_{\text{inel}}$. An enhanced DPS cross section (i.e. correlated production of $A$ and $B$) is indicated by $\sigma_{\text{eff}} < \sigma_{\text{inel}}$. The effective cross section should be process and energy independent. However, many effects are neglected in the above, simplified formula. Does factorization hold for a double-parton density? Is there correlation in momentum fraction, spin, colour or flavour? How to take into account a perturbative splitting of a single parton that could lead to DPS?

Because of the large parton density at small $x$ and the high rate of processes such as dijet production, double hard parton scattering must occur at the LHC. Several final states can be investigated, e.g. $W$ bosons + jets. Single parton scattering (SPS) is an irreducible background for such processes. However, difference in the kinematics of DPS and SPS can be exploited to extract the DPS fraction. A number of different observables can be used, such as:

- the azimuthal separation $\Delta \phi$ between the jets: DPS yields more back-to-back jets;
- the $p_T$ balance between the jets: DPS yields more balanced jets
- the azimuthal angle between the $W$ boson and the jet pair: DPS yields a random (flat) distribution.

The CMS Collaboration has looked at these variables in [22]. A event sample was obtained with a $W$ boson and at least 2 jets with $p_T > 20$ GeV and $|\eta| < 2$ in the final state. PYTHIA8 does not describe the data and would require a large fraction of DPS. However, MADGRAPH interfaced to PYTHIA reproduces the data well and needs MPI to describe the data. The effective cross section has been extracted by fitting SPS and DPS templates to data. However, in this approach it is crucial to get a good definition of the SPS background. The determination of the DPS fraction thus relies on the de-correlation between final-state systems and one should note that similar effects were predicted in the search for BFKL. Care has thus to be taken to disentangle both effects. New results are presented in [23, 24].

A new, interesting approach to extract the effective cross section was recently proposed by P. Gunnellini. The idea consists of retuning MC models (e.g. MADGRAPH + PYTHIA) to DPS observables obtained from data. This typically leads to somewhat smaller values for $\sigma_{\text{eff}}$ than in tunes to the UE [25], therefore indicating some tension between tunes of the soft UE and the hard DPS.

Finally, an old idea that merits some further study is to make the link between MPI and the rapidity gap survival probability in hard diffraction. The suggestion is that if a diffractive interaction occurs, the rapidity gap may be destroyed by additional partonic interactions. The numbers used in various models roughly agree: 10-20% of events have more than 1 MPI, which would lead to a survival factor of about 1 out of 10. However, this approach awaits a detailed simulation and comparison to data. One caveat may be that the survival of the proton is a soft (i.e. long time scale) process and therefore the question may be asked how this can depend on the (semi-) hard (i.e. short time scale) MPIs. Maybe the idea needs to be rephrased as a reduced survival probability in case of multiple partonic interactions.

5. Summary

Forward and low $x$ processes are the area where the conventional (collinear) QCD description
of hardonic scattering is challenged. Many (related) effects are expected and/or observed: alternative QCD shower dynamics, saturation of parton densities, multiple parton interactions and hard double parton scattering. The interpretation of the measurements is however often difficult and real deviations from the standard description are sometimes surprisingly difficult to find. Still, forward and low $x$ QCD is a vibrant field with many experimental and theoretical ideas and discussion, as exemplified by the many contributions related to the subject presented at this conference.

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