

Are bottom PDFs needed at the LHC?

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Processes involving bottom quarks play a crucial role in the LHC phenomenology, from flavour physics to Higgs characterisation and as a window to new physics, appearing both as signals and irreducible background in BSM searches. These processes can be described in QCD either in a 4-flavor or 5-flavor scheme. In the former, b quarks appear only in the final state and are considered massive. In 5-flavor schemes, calculations include b quarks in the initial state. Possibly large logarithms originating from the collinear splitting of gluons into bottom pairs are resummed into the b parton distribution function (PDF). In this contribution, I describe a simple method to assess the size of the logarithms in processes initiated by bottom quarks and show how a substantial and justified agreement between calculations in the two schemes can be achieved. As a consequence both calculations can be used in different context. To conclude, an overview of the current studies aiming to generalise the current appraisal is given and some preliminary results are discussed.

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Processes that feature gluon splitting into $b\bar{b}$ pairs in the initial state are of great phenomenological interest at the LHC. Many SM analyses, such as the precise measurement of the electroweak coupling in single top production or the Higgs boson characterisation via the study of its coupling to bottom quarks, as well as searches for new physics, consider final state signatures which involve bottom quarks. The need for accurate predictions is strongly motivated.

There are two different ways to compute theoretical predictions for this class of processes, often referred to as schemes: the 4- and the 5-flavor schemes. In the former (4FS), bottom quarks are treated as massive particles which appear only in the final state. The bottom quark has a non-zero transverse momentum already at the leading order and the full kinematics of the bottom quark is accurately described in the next-to-leading order computations. Moreover the implementation of the calculation in parton shower codes is straightforward, as there is no arbitrariness in the description of massive effects. However, order by order in the matrix element, logarithms of the ratio Q^2/m_b^2 , Q^2 being of the order of the hard scale of the process, appear as a result of the collinear splitting of gluons into bottom pairs. These possibly large logarithms may spoil the convergence of the perturbative series. A way out is given by the so-called 5-flavor scheme (5FS), in which bottom PDFs are introduced. The mass of the bottom quarks is considered as a small parameter and the bottom quarks contribute to the proton wavefunction and to the running of the strong coupling constant. This way, the DGLAP evolution of the bottom PDFs from threshold to the hard scale of the process automatically resums the initial state collinear logarithms to all orders in perturbative QCD. Calculations in the 5FS always start with at least a power of α_s less than the corresponding 4FS calculations and the bottom quarks have zero transverse momentum at leading order. Some improved versions of the 5-flavor scheme have been proposed in which the effects of the mass of the bottom quarks are included by replacing higher order contributions with no b quarks in the initial state by corresponding contributions with $m_b \neq 0$ [1]. In principle the two schemes can be combined and consistently matched via an approach based on Ref. [2], in which the massive calculation in the 4FS is supplemented by the resummation of the initial state collinear logarithms and double counting is properly taken into account. However matched resummed calculations are available only for a limited number of processes at colliders and they are more difficult to be implemented in the description of exclusive observables. Therefore, in practice, total cross section predictions in the two schemes are often combined via some pragmatical matching prescription, not based on a thorough field-theoretic analysis, such as the one proposed in [3], currently adopted by the LHC Higgs working group.

Clearly, if all perturbative orders were included, the 4-flavor and the improved 5-flavor schemes would yield identical results. However the terms in the perturbative expansion are organised differently and at any finite order in perturbative QCD results are different. In the past the discrepancy between the predictions formulated in the two schemes appeared to be extremely large, a glaring example being the factor of 10 difference in the case of the leading order b -initiated Higgs production. Several studies have been performed to investigate its origin, e.g. [4, 5]. In all cases of study, increasing the perturbative order of the predictions did obviously help in reducing the discrepancy, but this was not enough to reconcile the results obtained in the two schemes unless they were compared by using a factorisation scale smaller than the typical hard scale of the process. Also, contrary to one's naïve expectations that the collinear logarithms would have more space to develop at large energies, the discrepancy between predictions obtained the two schemes was found

to be larger at the Tevatron than at the LHC.

Despite the effort invested in investigating this issue, we still were not able to answer a number of crucial questions: according to which criterion can one establish which is the best way for describing bottom-initiated processes at colliders? What is the typical size of the effects of the resummation of initial-state collinear logs of the type $\log \frac{\mathcal{Q}^2}{m_b^2}$ with respect to an approximation where only logs at a finite order in perturbation theory are kept? Also, what is the typical size of logarithms themselves in phenomenologically relevant processes at the LHC? Finally, what justifies the use of a smaller factorisation scale when comparing the 5FS predictions to the 4FS ones?

In Reference [6] a reappraisal was formulated, which answered to the above questions by keeping into account two simple facts, one of dynamical and one of kinematical origin. The first one concerns the evolution of the bottom PDFs. In Fig. 1, taken from [6], the ratio of $\tilde{b}^{(2)}$, i.e. of the approximated *b* distribution that one obtains when truncating the perturbative expansion of the bottom PDF evolution at $\mathcal{O}(\alpha_s^2)$, and the full *b* PDF obtained by solving the DGLAP evolution at next-to-leading order, is displayed. One can observe that the effects of the resummation of the $\log \frac{\mu_f^2}{m_b^2}$'s is quite small and relevant mainly at large Bjorken *x*. So, in general, keeping only the explicit logs appearing at NLO is a good approximation and it stops being good only when the mass of the produced particle is very large as compared to the centre of mass energy. This observation accounts for previously noticed behaviours, such as the more sizable differences between predictions in the two schemes for single top and $bb \rightarrow H$ at the Tevatron than at the LHC [7, 8]. Recently

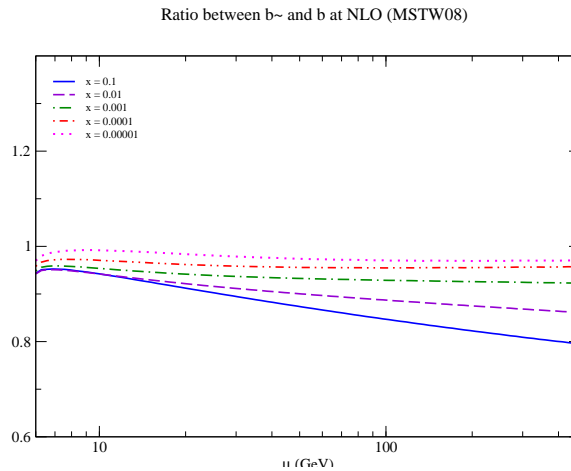


Figure 1: Ratio \tilde{b}/b for several values of *x* as a function of the scale μ . The 4F-FFNS and GM-VFNS are associated to the \tilde{b} and *b* PDF computations respectively at NLO order for the MSTW2008 parton set.

a similar study was performed to assess the size of the logarithms resummed in top PDFs at future 100 TeV colliders with very similar findings [9]. To conclude, unless the typical Bjorken *x* probed by the process is large, the effects of initial-state collinear logs is always modest, and, even though total cross sections computed in 5-flavor schemes may indeed display a smaller uncertainty, such logarithms do not spoil the convergence of perturbation theory in 4-flavor scheme calculations.

Furthermore, we showed that the effective scale \mathcal{Q} which enters the initial-state collinear logarithms, while proportional to hardest scale(s) in the process, turns out to be modified by universal

phase space factors. The latter are valid at all orders and are independent of the details of the splitting. Considering the $m_b^2 \rightarrow 0$ limit of the lowest-order 4FS cross-sections, which present a collinear singularity due to the gluon splitting into $b\bar{b}$ pair in the initial state, we have shown that the logarithmically-enhanced contributions to the cross-sections are proportional to

$$L = \log \frac{\mathcal{Q}^2(z)}{m_b^2}, \quad (1)$$

where $\mathcal{Q}^2(z)$ is a universal factor, a function of the mass of the produced particles M^2 and the virtuality of the exchanged bosons in the t -channel Q^2 as well as their ratio z with the partonic centre-of-mass energy \hat{s} :

$$\mathcal{Q}^2(z) = (M^2 + Q^2) \frac{(1-z)^2}{z} \frac{1}{1 - \frac{zQ^2}{M^2 + Q^2}} \quad \text{with} \quad z = \frac{M^2 + Q^2}{\hat{s} + Q^2}, \quad (2)$$

The collinear logarithm reduces to $L_{\text{DIS}} = \log \left[\frac{Q^2}{m_b^2} \frac{1-z}{z} \right]$ in the case of DIS $b\bar{b}$ production ($M^2 \rightarrow 0$) and to $L_{\text{DY}} = \log \left[\frac{M^2}{m_b^2} \frac{(1-z)^2}{z} \right]$ in the case of Drell-Yan production ($Q^2 \rightarrow 0$). The explicit scale \mathcal{Q}^2 is different from the one the we would naively expect $M^2 + Q^2$, i.e., of the hard scale to the collinear regulator, to develop in the integrated cross section.

Interestingly, the universal phase space factor tends to reduce the size of the logarithms for processes taking place at hadron colliders, while it enhances them in the case of DIS. In particular the suppression observed at the LHC is stronger than at the Tevatron and it gets stronger the heavier is the mass of the produced particle. By weighting the logarithm on the events according to a simple analytic formula, one can derive the factorisation scale at which to perform comparisons between calculations in the two schemes. For the processes that have been analysed, single top production and vector boson associate production, the scale turned out to be similar to the scales used in previous phenomenological analyses, about $m_t/4$ for single top production and $m_W/3$ for Wb associated production. However its origin is now properly physically-motivated. As a result, a consistent and quantitative explanation is provided of the many examples where a substantial agreement between total cross sections obtained at NLO (and beyond) in the two schemes can be found within the expected uncertainties. A recent study of the heavy Charged Higgs production cross section in the two schemes, which has been preliminary presented in [10] and which is soon going to be published, reinforces the findings of [6]. The choice of the factorisation scale in the 5-flavour scheme calculation driven by Eq. (2) significantly improves the agreement between predictions in the two schemes, and leads to a reliable NLO QCD prediction for heavy charged Higgs boson production which can be used in the searches at the second run of the LHC.

To answer the question that opens this contribution: do we need b -PDFs at the LHC? The main outcome of our study is that 4- and 5-flavor schemes provide complementary information. It is therefore strongly motivated having calculations at higher orders available in both schemes for any given process. (Improved) 5-flavor schemes, for example, can typically provide quite accurate predictions for total rates and being simpler, in some cases allow the calculations to be performed at NNLO, such as those already available for $bb \rightarrow H, Z$. On the other hand, being often the effects of resummation very mild, 4-flavor calculations can be also employed. They can be useful to achieve accurate fully exclusive predictions, such as those obtained from Monte Carlo programs at

NLO accuracy. Promoting a 4-flavor calculation at NLO to an event generator is nowadays a fully automatic procedure and kinematic effects due to the *b* quark mass can be taken into account from the start leading for example to a more accurate description of the kinematics of the spectator *b* quarks in all phase space.

The analysis presented in this contribution naturally leads to a number of relevant follow-up studies. Up to now, we focussed on processes featuring a single bottom in the initial state. Processes that can be described by two *b* quarks in the initial state, such as $pp \rightarrow Hbb$ and $pp \rightarrow Zbb$ entail the first simple extension of the approach, which we are currently working on [11]. The analytical expression of the leading order 4FS cross section for both processes in the collinear limit is proportional to the product of two logarithms in the form of L_{DY} , defined in terms of two partonic variables z_1 and z_2 which depend on the partonic cross section, the mass of the produced particle, m_Z or m_H , and the invariant mass of the bottom pair. The distribution of the factors that multiply the hard scales of the process in the logarithms are displayed in Fig. 2. The suppression of scale of the

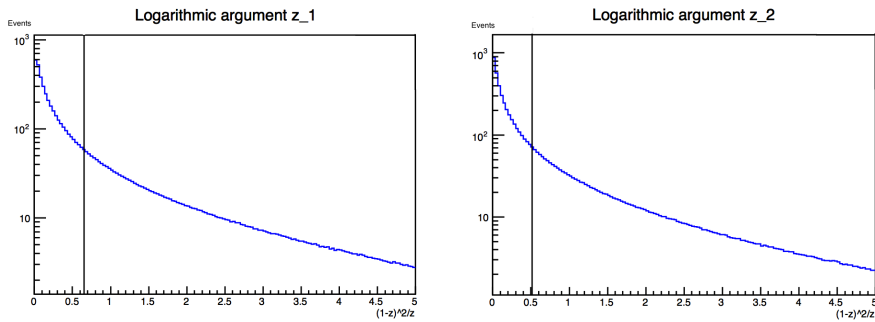


Figure 2: Associate *H* and bottom pair production at the LHC 7 TeV. $m_H = 126$. The distribution of the events (in pb/bin) as a function of $\mathcal{Q}_{DY}^2(z)/m_H^2 = (1 - z_1)^2/z_1$ (left) and $\mathcal{Q}_{DY}^2(z)/m_H^2 = (1 - z_2)^2/z_2$ (right) are displayed. 68% of the events lie on the left of the vertical line.

logarithms with respect to the hard scale of the process due to the kinematical phase space factor is apparent. The findings are similar to the ones presented in this contribution and are generalised to the production of heavier particles at future colliders.

To conclude, two major lines of research that we are pursuing are: the generalisation of the analysis from the level of inclusive cross sections to the level of differential distributions and the study of processes featuring heavy quarks in the final state. In the latter case fragmentation functions play the role of parton distribution functions and the final state collinear logarithms are resummed by the time-like DGLAP evolution equations. A thorough understanding of differential distributions, of the matching of fixed-order calculations involving bottom quarks with parton showers and an assessment of the effect of the resummation of final state collinear logarithms would complete the picture and provide a clear overview over a broad class of phenomenologically relevant processes.

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