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Leptoproduction of vector meson from the small x to the valence region

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We show that the longitudinal helicity amplitude of the hard vector meson leptoproduction within the so-called modified perturbative approach (MPA) in a kinematic where the incoming and outgoing partons are kept on their mass-shells, leads to results that can be connected with the amplitude obtained within k_T-factorization. This correspondence sheds light on the differences in the way of treating the nucleon interactions with the hard-subprocess partons within these two schemes. We compare predictions from both factorization schemes with H1 data and discuss the role of the quark contribution which is neglected within the k_T-factorization scheme. We also briefly discuss the parton phase space where saturation effects could play an important role within the collinear factorization scheme. Finally we give an estimate of the vector meson width a_V for the case of the ρ meson, in order to show the role of the Sudakov form factor and the vector meson ansatz on the predictions.

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1. Introduction

The hard exclusive vector meson leptoproduction in the small$-x$ regime,

$$\gamma^*(q,\lambda_\gamma)N(p) \to V(p_V,\lambda_V)N'(p'),$$

where λ_γ and λ_V denote respectively the polarizations of the virtual photon and the vector meson, can be described within different theoretical schemes inspired by the perturbative QCD calculations : the collinear factorization approach and the k_T-factorization approach. These two schemes are based on different kinematical assumptions. The collinear factorization is valid in the Bjorken limit when the virtuality of the photon Q and the energy in the center of mass of the system $\gamma^* N$ denoted W, are assumed to be asymptotically large $(-q^2 = Q^2, W) \to \infty$, but when the Bjorken variable $x \approx Q^2/W^2$ is finite. A proof of factorization was given long time ago in ref. [1] for the leading twist amplitude where both the virtual photon and the vector meson are longitudinally polarized. The k_T-factorization approach [2], as well as the dipole model [3], is based on the eikonal limit when $W^2 \gg Q^2 \gg \Lambda^2_{QCD}$. In this study, we compare the results derived within k_T-factorization and collinear factorization schemes for the leading helicity amplitude. We then discuss the numerical results and the contribution of the quark exchange in $t-$channel.

2. k_T-factorization aproach

Within the k_T-factorization framework, the helicity amplitudes $\mathscr{M}_{V,\{\lambda_\rho,\lambda_\gamma\}}$ of the diffractive vector meson leptoproduction are factorized using the eikonal approximation and reads as the convolution of the $\gamma^*(q,\lambda_\gamma) \to V(\lambda_V)$ impact factor, denoted $\Phi^{\gamma^*_{\lambda_\gamma}\to V_{\lambda_V}}(\underline{k})$ and the unintegrated gluon density $\mathscr{F}(x,\underline{k})$[1],

$$\mathscr{M}_{V,\{\lambda_\rho,\lambda_\gamma\}} = is\int \frac{d^2\underline{k}}{(\underline{k}^2)^2}\,\Phi^{\gamma^*_{\lambda_\gamma}\to\rho_{\lambda_\rho}}(\underline{k})\,\mathscr{F}(x,\underline{k})\,. \tag{2.1}$$

The impact factor is defined as

$$\Phi^{\gamma^*_{\lambda_\gamma}\to\rho_{\lambda_\rho}} = \frac{1}{2s}\int\frac{d\kappa}{2\pi} i\mathscr{M}(\gamma^*(\lambda_\gamma,q)+g(k_1)\to\rho(\lambda_\rho,p_1)+g(k_2)),$$

with $\kappa = (q+k_1)^2$. For large enough values of Q ($Q^2 \gg \Lambda^2_{QCD}$), it is possible to calculate the impact factor $\Phi^{\gamma^*\to V}$ within the light-cone collinear factorization scheme. The result obtained for the helicity amplitude $\mathscr{M}_{V,\{0,0\}}$ in ref. [4], reads

$$\begin{aligned}\mathscr{I}m\,\mathscr{M}_{V,\{0,0\}} = &\int d\tau\int d^2\underline{r}\sum_f C_V^f\,\frac{1}{2}\sqrt{\frac{\pi}{N_c}}f_V\,\varphi_1(\tau,\mu^2)\sum_{h,\bar{h}}\delta_{h,-\bar{h}}\Psi^{\gamma^*,f}_{h,\bar{h}}(\tau,\underline{r})\\ &\times\mathscr{I}m\,\mathscr{N}^{q\bar{q}}(x,\underline{r})\end{aligned} \tag{2.2}$$

where $\Psi^{\gamma^*,f}_{h,\bar{h}}(\tau,\underline{r})$ is the virtual photon wavefunction and where $\mathscr{N}^{q\bar{q}}(x,\underline{r})$ is the dipole/target scattering amplitude. The vector meson leading twist distribution amplitude (DA) is denoted $\varphi_1(\tau,\mu^2)$

[1] We denote $\underline{k}$ the euclidean transverse vector such that $\underline{k}^2 = -k_\perp^2$.

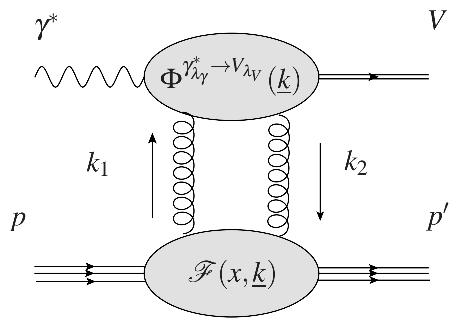

Figure 1: Impact factor representation of the helicity amplitudes.

with $\mu^2 \sim Q^2$ the renormalization scale. In the forward limit, the dipole scattering amplitude is related by the optical theorem to the dipole cross-section,

$$\mathscr{I}m\,\mathscr{N}^{q\bar{q}}(x,\underline{r}) = s\,\hat{\sigma}(x,\underline{r})\,. \tag{2.3}$$

For our purpose, we will use the saturation model of Golec-Biernat and Wüsthoff (GBW) [5] to model the dipole cross-section. In this model, saturation effects become important for $|\underline{r}| > 2R_0(x)$ with $R_0(x)$ the saturation radius. Note that the skewness effects are neglected in this treatment but some of the dipole models [6] takes them into account.

3. Collinear factorization approach and modified perturbative approach

In this study, we use the modified perturbative approach (MPA) in order to evaluate the amplitude $\mathscr{M}_{V,\{0,0\}}$. The approach we follow is very similar in spirit to the model of Goloskokov and Kroll (GK) [7]. The main difference is that we keep the external hard subprocess partons on their mass-shells. In this kinematic, the virtual photon wavefunction factorizes in the result and then it allows to identify the dipole scattering amplitude in terms of generalized parton distributions (GPDs).

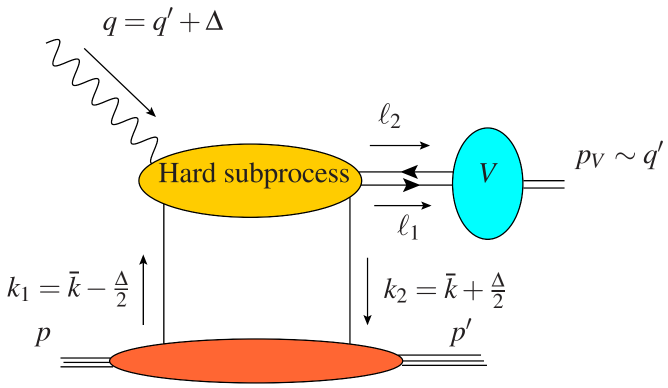

Figure 2: Diagram with the parton kinematic.

Within MPA, as illustrated in fig. 2, the amplitude reads like the convolution of the hard subprocess, the vector meson wavefunction $\Psi_V(\tau,\underline{\ell})$ and the GPDs of the partons exchanged in the

$t-$channel with the nucleon. The skewness is denoted ξ and $\xi \sim x/2$ in the small$-x$ regime. The result within the MPA for the amplitude $\mathscr{M}_{V,\{0,0\}}$ in terms of the gluon GPD $H^g(x,\xi,t)$ and the quark GPDs $H^f(x,\xi,t)$ reads [8],

$$\mathscr{M}_{V,\{0,0\}} = \int_0^1 d\tau \int d^2\underline{r} \sum_f C_V^f \hat{\Psi}_V(\tau,-\underline{r}) \sum_{h\bar{h}} \delta_{h,-\bar{h}} \hat{\Psi}_{h,\bar{h}}^{\gamma^*,f}(\tau,\underline{r}) \exp\left(-S(\tau,\underline{r},Q^2)\right)$$
$$\times \left(\frac{\alpha_s\sqrt{2\pi}}{N_c} \frac{1}{\tau\bar{\tau}} \int_0^1 dx \frac{H^g(x,\xi,t) + C_F\, x \left(H^f(x,\xi,t) - H^f(-x,\xi,t)\right)}{(x-\xi+i\varepsilon)(x+\xi-i\varepsilon)} \right), \tag{3.1}$$

where $\bar{\tau} = 1-\tau$, C_V^f is the vector meson wavefunction coefficient for the quark of flavor f and

$$\hat{\Psi}_V(\tau,\underline{r}) = \pi\sqrt{\frac{2}{N_c}} f_V \varphi_1(\tau,\mu^2) \exp(-\tau\bar{\tau}\frac{r^2}{4a_V^2}). \tag{3.2}$$

The factor "$\exp\left(-S(\tau,\underline{r},Q^2)\right)$" is the Sudakov form factor that is known up to next to leading log accuracy [9] and which resums the emission of soft gluons from the quark antiquark pair. Note that the leading twist result can be straightforwardly obtained from eq. (3.1) by replacing $\hat{\Psi}_V(\tau,-\underline{r})$ by $\hat{\Psi}_V(\tau,0)$ and taking the limit $S(\tau,\underline{r},Q^2) \to 0$. We can then identify the imaginary part of the gluon contribution of the leading twist result with the k_T-factorization result eq. (2.2) and get that

$$\mathscr{I}m\,\mathscr{N}^{q\bar{q}}(x,\underline{r}) \leftrightarrow -s\frac{\pi^2\alpha_s}{N_c} r_0^2 H^g(\xi,\xi,t\sim 0) \tag{3.3}$$

with $r_0^2 = 4/(\tau\bar{\tau}Q^2)$. Note that in the leading log approximation, the gluon GPD can be related to the gluon PDF: $H^g(\xi,\xi,t\sim 0) \sim xg(x)$. Then, up to a relative sign, the result of eq. (3.3) in the forward case is similar to the result of ref. [10] for small dipole sizes $r = |\underline{r}|$ derived in the forward case $\xi = 0$,

$$\hat{\sigma}(x,\underline{r}) = \frac{\pi^2\alpha_s}{N_c} r^2 xg(x), \tag{3.4}$$

where r^2 is replaced by r_0^2. Saturation effects are expected to be important for $|\underline{r}| > 2R_0(x)$ and consequently, one can define the intervals $[0,\tau_0(x,Q^2)]$ for τ or $\bar{\tau}$, with $\tau_0(x,Q^2) \sim (R_0^2(x)Q^2)^{-1}$ in the limit $Q \gg R_0^{-1}(x)$, where $r_0^2 > 4R_0^2(x)$ and in which one should implement such effects. These intervals correspond to aligned jet configurations when τ or $\bar{\tau} \to 0$, which are suppressed by the overlap of the wavefunctions and by the Sudakov form factor within MPA. Note also that the sign difference between the results (3.1) and (2.2) is to take with precaution as it is clear from eq. (2.3) that $\mathscr{I}m\,\mathscr{N}^{q\bar{q}}(x,\underline{r}) > 0$ in the forward case.

4. Comparison of the results

In fig. 4, we compare the leading twist result and the result obtained within k_T-factorization using the GBW saturation model, with the data of the H1 collaboration for the longitudinally polarized $\rho-$meson cross-section. We show also the leading twist results obtained in the collinear factorization scheme, using the GPD model of GK [7] and the asymptotic shape of the DA. We put in α_s the renormalization scale $\mu^2 = Q^2 + m_V^2$ with m_V the mass of the vector meson. The GPD model is based on the double distribution ansatz [11] and the GPD evolution in Q^2 is approximated

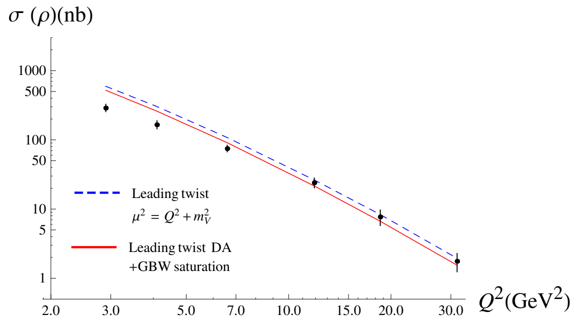

Figure 3: Imaginary part of the $t-$channel gluon exchange contribution $\mathscr{I}m\mathscr{M}^g_{V,\{0,0\}}$ versus Q^2 vs the data of H1 [12] for $W = 75$ GeV. Leading twist result (blue dashed) and k_T-factorization result (red) using the GBW saturation model for the dipole scattering amplitude.

by the DGLAP evolution of the PDFs. The forward limit of the double distribution of the GPDs $H_i(x, \xi, t)$ is fitted on the CTEQ6M PDF set.

With the GK model for GPDs, the cross-section increases by about 60% in the kinematic shown in fig. 4 when one takes into account all the contributions, i.e. real and imaginary parts of the quark and the gluon contributions. Hence, the quark contributions are not negligeable even for the small$-x$ values we have considered. It is mostly due to the interference terms between the quark and the gluon contributions. Note that the leading twist result is above the data. It can be decreased by fitting the vector meson wavefunction width a_V within MPA (see fig. 4). The effect of the Sudakov form factor decreases the values of the cross-section for small values of Q^2 ($Q^2 \lesssim 10$ GeV^2) .

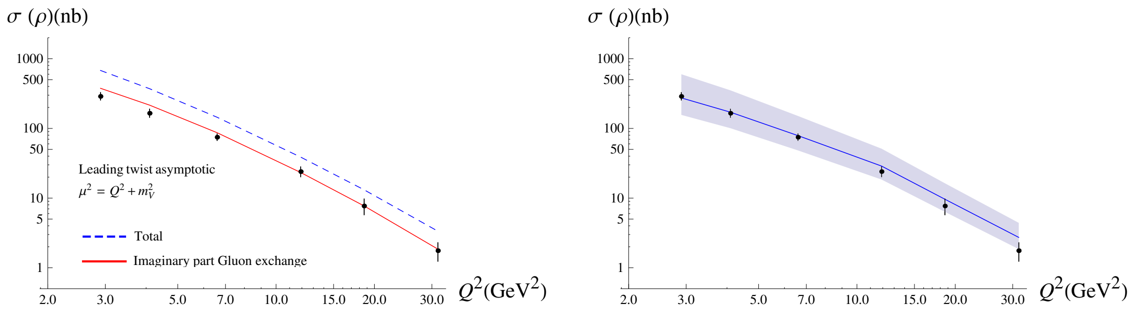

Figure 4: Longitudinally polarized $\rho-$meson cross-section versus Q^2 for $W = 75$ GeV. Left : total leading twist contribution (blue dashed) and imaginary part of the gluon contribution (red plain). Right : predictions with $a_V = 0.5$ GeV^{-1} and taking into account the Sudakov form factor. The gray area corresponds to the uncertainty on the renormalization scale by changing $\mu^2 = Q^2 + m_V^2$ in $4\mu^2$ and $\mu^2/4$. The predictions are compared with the H1 collaboration data [12].

5. Conclusion

In this study, we proposed a modification of the kinematics to calculate the hard sub-process associated to the leading helicity ampltiude $\mathscr{M}_{V,\{0,0\}}$. Keeping the partons on the mass-shell allows to compare results from the k_T-factorization approach with the MPA results. As a matter of fact, we find that the imaginary part of the dipole scattering amplitude which is positive in dipole models, corresponds to a negative quantity in MPA which is expressed in terms of the gluon GPD $H(\xi,\xi,t)$. Up to this relative sign, we recognize in the forward limit the well known formula of color transparency for the small dipole size if we interpret $r_0^2 = 4/(\tau\bar{\tau}Q^2)$ as the relevant dipole size. The dipole size r_0 reaches the saturation region ($r_0 > 2R_0(x)$) for aligned jet configurations when $\tau \to 0$ or 1. The numerical results shows that the imaginary part of the gluon contributions are similar for both approaches but taking into account the real parts and the quark contributions, the result at leading twist overestimates the longitudinal cross-section. The wavefunction ansatz as well as the inclusion of the Sudakov form factor allows to decrease enough the results to fit data for meson width $a_V \sim 0.5$ GeV^{-1} but it is not clear if the discrepancy with data is due to the lack of saturation effects within MPA or the lack of knowledge of the vector meson wavefunction.

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