## Next-to-eikonal corrections in the CGC

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We present a new method to systematically calculate the corrections to eikonal approximation in the background field formalism. These corrections are power suppressed corrections due the finite width of the target or the finite energy of the projectile. We use this method to study the single inclusive gluon production and polarized gluon production in pA collisions at next-to-eikonal accuracy.
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## 1. Introduction

High energy scattering processes are treated usually in eikonal approximation. For the scattering of a dilute projectile on dense target, the high density of the target makes it possible to perform a semi-classical approximation that amounts to replace the target by an intense classical background field. The Color Glass Condensate (CGC, see [1] and references therein) is the effective theory that is used to study such scatterings within the eikonal and semi-classical approximations. In the CGC, one can calculate the observables for such processes in a weak coupling expansion and resum highenergy leading and next-to-leading logarithms. Corrections suppressed by inverse powers of the energy of the collision are systematically neglected within the eikonal limit. We study such power suppressed corrections to the CGC, namely next-to-eikonal contributions due to finite length of the target [2]. For the production of a gluon with momentum $k=\left(k^{+}, k^{-}, \mathbf{k}\right)$ off a target of light-cone thickness $L^{+}$, the eikonal expansion amounts to assume that $\sqrt{k^{+} / L^{+}}$is larger than any available transverse momentum scale, like the transverse momentum of the produced gluon, $\mathbf{k}$, or the saturation scale of the target. Next-to-eikonal corrections are then suppressed as $L^{+} / k^{+}$compared to the strict eikonal terms.

## 2. Eikonal expansion of the retarded gluon propagator in a background field

One of the main building blocks for dense-dilute scattering processes at high energy is the retarded gluon propagator in a classical background field. A highly boosted left-moving target can be described by a classical gluon background field $\mathscr{A}_{a}^{-}\left(x^{+}, \mathbf{x}\right)$. Only the ( - ) component of this field is enhanced by a Lorentz gamma factor, so that the other components are negligible in comparison. Moreover, due to time dilation, the $x^{-}$dependence of the field can be neglected. In the presence of such a background field, it is natural to work in the light-cone gauge $A_{a}^{+}=0$. Linearizing the Yang-Mills equations around the background field $\mathscr{A}_{a}^{-}\left(x^{+}, \mathbf{x}\right)$, one finds the equation satisfied by the gluon propagators in that background

$$
\begin{align*}
& {\left[g_{\mu \nu}\left(\delta_{a b} \square_{x}-2 i g\left(\mathscr{A}^{-}\left(x^{+}, \mathbf{x}\right) \cdot T\right)_{a b} \partial_{x^{-}}\right)-\delta_{a b} \partial_{x^{\mu}} \partial_{x^{v}}\right] G^{v \rho}(x, y)_{b c}} \\
& \quad=i g_{\mu^{\rho}} \delta_{a c} \delta^{(4)}(x-y), \quad \text { for } \mu \neq+. \tag{2.1}
\end{align*}
$$

For our purposes, we only need the retarded propagator. Since the background field $\mathscr{A}_{a}^{-}\left(x^{+}, \mathbf{x}\right)$ is independent of $x^{-}$, it is convenient to introduce the one dimensional Fourier transform of the retarded gluon propagator

$$
\begin{equation*}
G_{R}^{(\mu v)}(x, y)_{a b}=\int \frac{\mathrm{d} p^{+}}{2 \pi} e^{-i p^{+}\left(x^{-}-y^{-}\right)} \frac{1}{2\left(p^{+}+i \varepsilon\right)} \mathscr{G}_{p^{+}}^{(\mu \nu)}(\underline{x} ; \underline{y})_{a b} . \tag{2.2}
\end{equation*}
$$

In principle one can write all the components of the retarded gluon propagator but we only need the ( $i-$ ) component for the calculations of the observables that we are interested in. The (i-) component of the retarded gluon propagator is given in terms of the background scalar propagator as

$$
\begin{equation*}
\mathscr{G}_{k^{+}}^{i-}(\underline{x} ; \underline{y})^{a b}=\frac{i}{k^{+}+i \varepsilon} \partial_{y^{1}} \mathscr{G}_{k^{+}}^{a b}(\underline{x} ; \underline{y}) . \tag{2.3}
\end{equation*}
$$

Note that the notation $(\underline{x}) \equiv\left(x^{+}, \mathbf{x}\right)$ is introduced for simplicity. The background scalar propagator, $\mathscr{G}_{k^{+}}^{a b}(\underline{x} ; \underline{y})$ satisfies the scalar Green's equation:

$$
\begin{equation*}
\left[\delta^{a b}\left(i \partial_{x^{+}}+\frac{\partial_{\mathbf{x}}^{2}}{2\left(k^{+}+i \varepsilon\right)}\right)+g\left(\mathscr{A}^{-}(\underline{x}) \cdot T\right)^{a b} \mathscr{G}_{k^{+}}^{b c}(\underline{x} ; \underline{y})=i \delta^{a c} \delta^{(3)}(\underline{x}-\underline{y}),\right. \tag{2.4}
\end{equation*}
$$

whose solution can be formally written as a path integral

$$
\begin{equation*}
\mathscr{G}_{k^{+}}^{a b}(\underline{x} ; \underline{y})=\boldsymbol{\theta}\left(x^{+}-y^{+}\right) \int_{\mathbf{z}\left(y^{+}\right)=\mathbf{y}}^{\mathbf{z}\left(x^{+}\right)=\mathbf{x}} \mathscr{D} \mathbf{z}\left(z^{+}\right) \exp \left[\frac{i k^{+}}{2} \int_{y^{+}}^{x^{+}} d z^{+} \dot{\mathbf{z}}^{2}\left(z^{+}\right)\right] \mathscr{U}^{a b}\left(x^{+}, y^{+},\left[\mathbf{z}\left(z^{+}\right)\right]\right) \tag{2.5}
\end{equation*}
$$

with the Wilson line

$$
\begin{equation*}
\mathscr{U}^{a b}\left(x^{+}, y^{+},\left[\mathbf{z}\left(z^{+}\right)\right]\right)=\mathscr{P}_{+} \exp \left\{i g \int_{y^{+}}^{x^{+}} d z^{+} T \cdot \mathscr{A}^{-}\left(z^{+}, \mathbf{z}\left(z^{+}\right)\right)\right\}^{a b} \tag{2.6}
\end{equation*}
$$

following the Brownian trajectory $\mathbf{z}\left(z^{+}\right)$. In the last expression $\mathscr{P}_{+}$indicates the ordering of color generators $T^{a}$ along $x^{+}$. The path integral is actually defined through discretization as

$$
\begin{align*}
\mathscr{G}_{k^{+}}^{a b}(\underline{x} ; \underline{y})=\lim _{N \rightarrow+\infty} \theta\left(x^{+}-y^{+}\right) & \int\left(\prod_{n=1}^{N-1} \mathrm{~d}^{2} \mathbf{z}_{n}\right)\left(\frac{-i\left(k^{+}+i \varepsilon\right) N}{2 \pi\left(x^{+}-y^{+}\right)}\right)^{N} \\
& \times \exp \left[\frac{i\left(k^{+}+i \varepsilon\right) N}{2\left(x^{+}-y^{+}\right)} \sum_{n=0}^{N-1}\left(\mathbf{z}_{n+1}-\mathbf{z}_{n}\right)^{2}\right] \mathscr{U}^{a b}\left(x^{+}, y^{+},\left\{\mathbf{z}_{n}\right\}\right), \tag{2.7}
\end{align*}
$$

with $N$ being the number of discretization steps, $\mathbf{z}_{0}=\mathbf{y}$ and $\mathbf{z}_{N}=\mathbf{x}$. Here, $\mathscr{U}^{a b}\left(x^{+}, y^{+},\left\{\mathbf{z}_{n}\right\}\right)$ is the discretized Wilson line in the adjoint representation, defined as

$$
\begin{equation*}
\mathscr{U}^{a b}\left(x^{+}, y^{+},\left\{\mathbf{z}_{n}\right\}\right)=\mathscr{P}_{+}\left\{\prod_{n=0}^{N-1} \exp \left[i g \frac{\left(x^{+}-y^{+}\right)}{N}\left(\mathscr{A}^{-}\left(z_{n}^{+}, \mathbf{z}_{n}\right) \cdot T\right)\right]\right\}^{a b} \tag{2.8}
\end{equation*}
$$

where

$$
\begin{equation*}
z_{n}^{+}=y^{+}+\frac{n}{N}\left(x^{+}-y^{+}\right) \tag{2.9}
\end{equation*}
$$

In the large $k^{+}$limit, one can basically expand the Brownian trajectory followed by the Wilson line around a classical path :

$$
\begin{equation*}
\mathbf{z}_{n}=\mathbf{z}_{n}^{\mathrm{cl}}+\mathbf{u}_{n} \tag{2.10}
\end{equation*}
$$

where $\mathbf{z}_{n}^{\mathrm{cl}}=\mathbf{y}+\frac{n}{N}(\mathbf{x}-\mathbf{y})$. The retarded background propagator, $\mathscr{G}_{k^{+}}^{a b}(\underline{x} ; \underline{y})$, is proportional to $\exp \left[\frac{\left(x^{+}-y^{+}\right)}{N} i g T \cdot \mathscr{A}^{-}\left(z_{n}^{+}, \mathbf{z}_{n}^{\mathrm{cl}}+\mathbf{u}_{n}\right)\right]$. Thus, in the large $k^{+}$limit, one should Taylor expand the gauge link, $\mathscr{A}^{-}$, around $\mathbf{u}_{n}=0$ at each discretization step and take the product of the Taylor expansion of all links and collect the terms according to the power of $\mathbf{u}_{n}$. After this expansion, one should re-expand the result around the fixed initial transverse position since in the limit $\mathbf{z}^{\mathrm{cl}}-\mathbf{y}$ is small at each step. All in all, the background propagator reads

$$
\begin{align*}
& \int d^{2} \mathbf{x} e^{-i \mathbf{k} \cdot \mathbf{x}} \mathscr{G}_{k^{+}}^{a b}(\underline{x} ; \underline{y})=\theta\left(x^{+}-y^{+}\right) e^{-i \mathbf{k} \cdot \mathbf{y}} e^{-i k^{-}\left(x^{+}-y^{+}\right)}\left\{\mathscr{U}\left(x^{+}, y^{+}, \mathbf{y}\right)\right. \\
&\left.+\frac{\left(x^{+}-y^{+}\right)}{k^{+}} \mathbf{k}^{i} \mathscr{U}_{(1)}^{i}\left(x^{+}, y^{+}, \mathbf{y}\right)+i \frac{\left(x^{+}-y^{+}\right)}{2 k^{+}} \mathscr{U}_{(2)}\left(x^{+}, y^{+}, \mathbf{y}\right)\right\}^{a b} \tag{2.11}
\end{align*}
$$

where the explicit forms of the first corrections to strict eikonal limit are

$$
\begin{align*}
\mathscr{U}_{(1)}^{i, a b}\left(x^{+}, y^{+}, \mathbf{y}\right)=\int_{y^{+}}^{x^{+}} d z^{+}\left(\frac{z^{+}-y^{+}}{x^{+}-y^{+}}\right) & \left\{\mathscr{U}\left(x^{+}, z^{+}, \mathbf{y}\right)\right. \\
\times & {\left.\left[i g T \cdot \partial_{\mathbf{y}^{i}} A^{-}\left(z^{+}, \mathbf{y}\right)\right] \mathscr{U}\left(z^{+}, y^{+}, \mathbf{y}\right)\right\}^{a b} } \tag{2.12}
\end{align*}
$$

and

$$
\begin{gather*}
\mathscr{U}_{(2)}^{a b}\left(x^{+}, y^{+}, \mathbf{y}\right)=\int_{y^{+}}^{x^{+}} d z^{+}\left(\frac{z^{+}-y^{+}}{x^{+}-y^{+}}\right)\left\{\mathscr{U}\left(x^{+}, z^{+}, \mathbf{y}\right)\left[i g T \cdot \partial_{\mathbf{y}}^{2} A^{-}\left(z^{+}, \mathbf{y}\right)\right] \mathscr{U}\left(z^{+}, y^{+}, \mathbf{y}\right)\right\}^{a b} \\
+2 \int_{y^{+}}^{x^{+}} d z^{+} \int_{y^{+}}^{z^{+}} d w^{+}\left(\frac{w^{+}-y^{+}}{x^{+}-y^{+}}\right)\left\{\mathscr{U}\left(x^{+}, z^{+}, \mathbf{y}\right)\left[i g T \cdot \partial_{\mathbf{y}^{i}} A^{-}\left(z^{+}, \mathbf{y}\right)\right]\right. \\
\left.\times \mathscr{U}\left(z^{+}, w^{+}, \mathbf{y}\right)\left[i g T \cdot \partial_{\mathbf{y}^{i}} A^{-}\left(w^{+}, \mathbf{y}\right)\right] \mathscr{U}\left(w^{+}, y^{+}, \mathbf{y}\right)\right\}^{a b} \tag{2.13}
\end{gather*}
$$

## 3. Single inclusive gluon production in pA collisions beyond the eikonal approximation

In the CGC formalism, a highly boosted left-moving nucleus is usually described by a classical gluon shockwave $\mathscr{A}_{a}^{\mu}(x)=\delta^{\mu-} \delta\left(x^{+}\right) \mathscr{A}_{a}^{-}(\mathbf{x})$ in the light-cone gauge $A_{a}^{+}=0$. That field has indeed a vanishing longitudinal width and no $x^{-}$dependence in the limit of infinite boost.

Consider instead a background field

$$
\begin{equation*}
\mathscr{A}_{a}^{\mu}(x)=\delta^{\mu-} \mathscr{A}_{a}^{-}\left(x^{+}, \mathbf{x}\right) \tag{3.1}
\end{equation*}
$$

with a finite support along the $x^{+}$direction, from $x^{+}=0$ to $x^{+}=L^{+}$. In the case of a large nucleus, this should be the dominant finite-boost correction with respect to the usual gluon shockwave. On the other hand, a highly boosted right-moving proton, considered as dilute, is described by a classical color current

$$
\begin{equation*}
j_{a}^{\mu}(x)=\delta^{\mu-} j_{a}^{+}(x) \tag{3.2}
\end{equation*}
$$

with zero width along $x^{-}: j_{a}^{+}(x) \propto \delta\left(x^{-}\right)$. Let us consider a proton-nucleus collision with a particular impact parameter $\mathbf{B}$ and choose the center of the nucleus as the reference point for the transverse plane, so that a generic point $\mathbf{x}$ in the transverse plane is at a distance $|\mathbf{x}-\mathbf{B}|$ from the center of the proton and at a distance $|\mathbf{x}|$ from the center of the nucleus. Then, the color current $j_{a}^{+}(x)$ can be written as $j_{a}^{+}(x)=\delta\left(x^{-}\right) \mathscr{U}^{a b}\left(x^{+},-\infty, \mathbf{x}\right) \rho^{b}(\mathbf{x}-\mathbf{B})$ where $\rho^{b}$ is the transverse density of color charges inside the proton before it reaches the nucleus, and $\mathscr{U}^{a b}\left(x^{+},-\infty, \mathbf{x}\right)$ is the Wilson line implementing the color precession of these color charges in the background field $\mathscr{A}_{a}^{-}\left(x^{+}, \mathbf{x}\right)$ of the nucleus. One can define the Fourier transform of the color charge density

$$
\begin{equation*}
\rho^{a}(\mathbf{y}-\mathbf{B})=\int \frac{d^{2} \mathbf{q}}{(2 \pi)^{2}} e^{i \mathbf{q} \cdot(\mathbf{y}-\mathbf{B})} \tilde{\rho}(\mathbf{q}) \tag{3.3}
\end{equation*}
$$

and the gluon-nucleus reduced amplitude, $\overline{\mathscr{M}}_{\lambda}^{a b}(\underline{k}, \mathbf{q})$, as

$$
\begin{equation*}
\mathscr{M}_{\lambda}^{a}(\underline{k}, \mathbf{B})=\int \frac{d^{2} \mathbf{q}}{(2 \pi)^{2}} e^{-i \mathbf{q} \cdot \mathbf{B}} \overline{\mathscr{M}}_{\lambda}^{a b}(\underline{k}, \mathbf{q}) \tilde{\rho}^{b}(\mathbf{q}) \tag{3.4}
\end{equation*}
$$

where the gluon production amplitude is $\mathscr{M}_{\lambda}^{a}(\underline{k}, \mathbf{B})$ is given as

$$
\begin{equation*}
\mathscr{M}_{\lambda}^{a}(\underline{k}, \mathbf{B})=\varepsilon^{i *}\left(2 k^{+}\right) \lim _{x^{+} \rightarrow \infty} \int d^{2} \mathbf{x} \int d x^{-} e^{i k \cdot x} \int d^{4} y G_{R}^{i-}(x, y)_{a b} j_{b}^{+}(y) \tag{3.5}
\end{equation*}
$$

The cross section for single inclusive gluon production can be written as

$$
\begin{equation*}
k^{+} \frac{d \sigma}{d k^{+} d^{2} \mathbf{k}}=\int \frac{d^{2} \mathbf{q}}{(2 \pi)^{2}} \varphi_{p}(\mathbf{q}) \frac{\mathbf{q}^{2}}{4} \frac{1}{N_{c}^{2}-1} \sum_{\lambda \text { phys. }}\left\langle\overline{\mathscr{M}}_{\lambda}^{a b}(\underline{k}, \mathbf{q})^{\dagger} \overline{\mathscr{M}}_{\lambda}^{a b}(\underline{k}, \mathbf{q})\right\rangle_{A} . \tag{3.6}
\end{equation*}
$$

where $\varphi_{p}(\mathbf{q})$ is the unintegrated gluon distribution. By using eqn. 2.3 in the definition of the gluon production amplitude and using the expanded expression for the background gluon propagator one write the reduced gluon-nucleus amplitude at next-to-eikonal accuracy as

$$
\begin{gather*}
\overline{\mathscr{M}}_{\lambda}^{a b}(\underline{k}, \mathbf{q})=i \varepsilon^{i *} \int d^{2} \mathbf{y} e^{-i(\mathbf{k}-\mathbf{q}) \cdot \mathbf{y}}\left\{2\left[\frac{\mathbf{k}^{i}}{\mathbf{k}^{2}}-\frac{\mathbf{q}^{i}}{\mathbf{q}^{2}}\right] \mathscr{U}\left(L^{+}, 0 ; \mathbf{y}\right)+\frac{L^{+}}{k^{+}}\left[\delta^{i j}-2 \frac{\mathbf{q}^{i}}{\mathbf{q}^{2}} \mathbf{k}^{j}\right] \mathscr{U}_{(1)}^{j}\left(L^{+}, 0 ; \mathbf{y}\right)\right. \\
-  \tag{3.7}\\
\left.-\frac{L^{+}}{k^{+}} \frac{\mathbf{q}^{i}}{\mathbf{q}^{2}} \mathscr{U}_{(2)}\left(L^{+}, 0 ; \mathbf{y}\right)\right\}^{a b}
\end{gather*}
$$

The square of the reduced amplitude reads

$$
\begin{align*}
& \frac{1}{N_{c}^{2}-1} \sum_{\lambda \text { phys. }}\left\langle\overline{\mathscr{M}}_{\lambda}^{a b}(\underline{k}, \mathbf{q})^{\dagger} \overline{\mathscr{M}}_{\lambda}^{a b}(\underline{k}, \mathbf{q})\right\rangle_{A}=\frac{1}{\mathbf{k}^{2} \mathbf{q}^{2}} \int d^{2} \mathbf{b} \int d^{2} \mathbf{r} e^{-i(\mathbf{k}-\mathbf{q}) \cdot \mathbf{r}} \\
& \times\left\{4(\mathbf{k}-\mathbf{q})^{2} S_{A}(\mathbf{r}, \mathbf{b})+2 \frac{L^{+}}{k^{+}}\left[(\mathbf{k}-\mathbf{q})^{2} \mathbf{k}^{j}+\mathbf{k}^{2}\left(\mathbf{k}^{j}-\mathbf{q}^{j}\right)\right]\left[\mathscr{O}_{(1)}^{j}(\mathbf{r}, \mathbf{b})+\mathscr{O}_{(1)}^{j}(-\mathbf{r}, \mathbf{b})\right]\right. \\
& \left.\quad+2 i \frac{L^{+}}{k^{+}} \mathbf{k} \cdot(\mathbf{k}-\mathbf{q})\left[\mathscr{O}_{(2)}(\mathbf{r}, \mathbf{b})-\mathscr{O}_{(2)}(-\mathbf{r}, \mathbf{b})\right]\right\}, \tag{3.8}
\end{align*}
$$

where the operators $S_{A}(\mathbf{r}, \mathbf{b}), \mathscr{O}_{(1)}^{j}(\mathbf{r}, \mathbf{b})$ and $\mathscr{O}_{(2)}(\mathbf{r}, \mathbf{b})$ are defined as

$$
\begin{align*}
S_{A}(\mathbf{r}, \mathbf{b}) & =\frac{1}{N_{c}^{2}-1}\left\langle\operatorname{tr}\left[\mathscr{U}^{\dagger}\left(L^{+}, 0 ; \mathbf{b}-\frac{\mathbf{r}}{2}\right) \mathscr{U}\left(L^{+}, 0 ; \mathbf{b}+\frac{\mathbf{r}}{2}\right)\right]\right\rangle_{A},  \tag{3.9}\\
\mathscr{O}_{(1)}^{j}(\mathbf{r}, \mathbf{b}) & =\frac{1}{N_{c}^{2}-1}\left\langle\operatorname{tr}\left[\mathscr{U}^{\dagger}\left(L^{+}, 0 ; \mathbf{b}-\frac{\mathbf{r}}{2}\right) \mathscr{U}_{(1)}^{j}\left(L^{+}, 0 ; \mathbf{b}+\frac{\mathbf{r}}{2}\right)\right]\right\rangle_{A},  \tag{3.10}\\
\mathscr{O}_{(2)}(\mathbf{r}, \mathbf{b}) & =\frac{1}{N_{c}^{2}-1}\left\langle\operatorname{tr}\left[\mathscr{U}^{\dagger}\left(L^{+}, 0 ; \mathbf{b}-\frac{\mathbf{r}}{2}\right) \mathscr{U}_{(2)}\left(L^{+}, 0 ; \mathbf{b}+\frac{\mathbf{r}}{2}\right)\right]\right\rangle_{A} . \tag{3.11}
\end{align*}
$$

The combinations $\left[\mathscr{O}_{(1)}^{j}(\mathbf{r}, \mathbf{b})+\mathscr{O}_{(1)}^{j}(-\mathbf{r}, \mathbf{b})\right]$ and $\left[\mathscr{O}_{(2)}(\mathbf{r}, \mathbf{b})-\mathscr{O}_{(2)}(-\mathbf{r}, \mathbf{b})\right]$ that are appearing in the cross section vanish upon integration over $\mathbf{b}$ due to rotational symmetry. Thus, the next-toeikonal contributions vanish for single inclusive gluon cross section and we recover the well-known $k_{t}$ factorisation formula [3], [4].

## 4. The cross section for the polarized gluon production in unpolarized $\mathbf{p A}$ collisions

Next-to-eikonal corrections to single inclusive gluon cross-section vanish due to the symmetry properties of the next-to-eikonal operators upon integration over the impact parameter. In practice, one can also study the next-to-eikonal corrections to the cross-section for the polarized gluon production in unpolarized pA collisions.

There are typically two mechanisms leading to polarized hadron production in unpolarized pA collisions. One possibility is that a polarized quark or gluon is produced, which then fragments in a standard way to a polarized hadron. The other possibility is that the polarization is induced during the fragmentation process. That second possibility has been studied in the CGC framework in the case of transversely polarized hyperon production [5]. By contrast, the focus on the present section is on the first type of mechanism. For simplicity, let us restrict ourselves to the study of the asymmetry in the light-front helicity of the produced gluon, assuming that this asymmetry is preserved at the hadron level by fragmentation.

The calculation of that asymmetry is calculated by taking the difference between the $\lambda=+1$ and $\lambda=-1$ contributions of the helicity $\lambda= \pm 1$ of the produced gluon, i.e.

$$
\begin{equation*}
k^{+} \frac{d \sigma^{+}}{d k^{+} d^{2} \mathbf{k}}-k^{+} \frac{d \sigma^{-}}{d k^{+} d^{2} \mathbf{k}}=\int \frac{d^{2} \mathbf{q}}{(2 \pi)^{2}} \varphi_{p}(\mathbf{q}) \frac{\mathbf{q}^{2}}{4} \frac{1}{N_{c}^{2}-1} \sum_{\lambda \text { phys. }} \lambda\left\langle\overline{\mathscr{M}}_{\lambda}^{a b}(\underline{k}, \mathbf{q})^{\dagger} \overline{\mathscr{M}}_{\lambda}^{a b}(\underline{k}, \mathbf{q})\right\rangle_{A} . \tag{4.1}
\end{equation*}
$$

The difference between the two light-front helicity states can be calculated by using the following identity

$$
\begin{equation*}
\sum_{\lambda \text { phys. }} \lambda \varepsilon_{\lambda}^{i} \varepsilon_{\lambda}^{* j}=-i \varepsilon^{i j} \tag{4.2}
\end{equation*}
$$

with $\varepsilon^{i j}$ being an antisymmetric matrix. Then the cross-section for the polarized gluon production reads

$$
\begin{align*}
& k^{+} \frac{d \sigma^{+}}{d k^{+} d^{2} \mathbf{k}}-k^{+} \frac{d \sigma^{-}}{d k^{+} d^{2} \mathbf{k}}=\frac{L^{+}}{k^{+}} \int \frac{d^{2} \mathbf{q}}{(2 \pi)^{2}} \varphi_{p}(\mathbf{q}) \mathbf{q}^{2} \int d^{2} \mathbf{b} \int d^{2} \mathbf{r} e^{-i(\mathbf{k}-\mathbf{q}) \cdot \mathbf{r}} \\
& \quad \times\left\{-i\left[\left(\frac{\mathbf{k}^{i}}{\mathbf{k}^{2}}-\frac{\mathbf{q}^{i}}{\mathbf{q}^{2}}\right) \varepsilon^{i j}-2 \frac{\left(\varepsilon^{i l} \mathbf{k}^{i} \mathbf{q}^{l}\right)}{\mathbf{k}^{2} \mathbf{q}^{2}} \mathbf{k}^{j}\right] \mathscr{O}_{(1)}^{j}(\mathbf{r}, \mathbf{b})-\frac{\left(\varepsilon^{i j} \mathbf{k}^{i} \mathbf{q}^{j}\right)}{\mathbf{k}^{2} \mathbf{q}^{2}} \mathscr{O}_{(2)}(\mathbf{r}, \mathbf{b})\right\} \tag{4.3}
\end{align*}
$$

Thus, for the polarized gluon production cross-section, the strict eikonal term vanishes and next-to-eikonal terms become the leading contribution.

## 5. Conclusions

We presented a systematic eikonal expansion of the retarded gluon propagator in a background field which is one of the most crucial building blocks of the high energy dense-dilute scattering processes and also medium induced gluon radiation. We apply this method to single inclusive gluon cross section in order to study the corrections to the CGC beyond the eikonal limit. The strict eikonal term provides the usual $k_{t}$ factorisation formula, whereas the first next-to-eikonal corrections vanish for this particular observable. On the other hand, we also applied the same method to calculate the cross section for the polarized gluon production in unpolarized pA collisions. For this
observable, we have seen that the strict eikonal terms vanish leaving the next-to-eikonal corrections as the leading contribution.

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