Detailed study of the K_{e4} decay mode properties

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The largest data samples of semi-leptonic charged kaon decays in K_{e4}^{+-} ($K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu_e$) and K_{e4}^{00} ($K^{\pm} \to \pi^0\pi^0 e^{\pm}\nu_e$) modes have been collected by the NA48/2 experiment at CERN SPS. We report improved measurements of branching fractions and detailed form factors studies. The achieved remarkable precisions of the results bring new inputs to low energy QCD description and crucial tests of predictions from Chiral Perturbation Theory.

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1. Introduction

Kaon decays are perfect for studying low energy strong interactions. Semi-leptonic four-body decays are of particular interest because of the small number of hadrons in the final state which are related to the investigation of $\pi\pi$ interaction. The global analysis of $\pi\pi$, πK and K_{e4} data allow the determination of Low Energy Constants of Chiral Perturbation Theory (ChPT) at Leading and Next to Leading Orders [1, 2] and subsequent predictions of form factors and branching fractions.

A brief introduction to the NA48/2 experiment, concise descriptions of the event selections and obtained measurements of K_{e4} branching ratios and form factors are reported.

2. The NA48/2 experiment

The beam line of the NA48/2 experiment was designed to produce two simultaneous K^+ and K^- beams by 400 GeV/c primary protons delivered by the CERN SPS and impinging on a 400 mm long beryllium target. The opposite charged particles are selected with a momentum range of (60 ± 3) GeV/c by two systems of dipole magnets with zero total deflection, focusing quadrupoles, muon sweepers and collimators. The positive and negative kaon beams travel further through a 114 m vacuum decay volume. Their axes coincide within 1 mm. A detailed description of the NA48 experimental set-up can be found in [3]. The momenta of the charged decay products are measured by a magnetic spectrometer located in a tank filled with helium at nearly atmospheric pressure. It consists of four octagonal drift chambers (DCH) and a large aperture dipole magnet. The momentum resolution achieved in the spectrometer is $\sigma(p)/p = (1.02 \oplus 0.044p)\%$ (p in GeV/c). A fast signal for the trigger is provided by a scintillator hodoscope (HOD) situated just after the spectrometer. It measures the time of arrival of charged particles with an offline resolution of ~ 150 ps. Downstream of the HOD a quasi-homogeneous liquid Krypton calorimeter (LKr) is placed. It is used to measure the energy deposits of photons and electrons. The calorimeter is 27 X_0 deep and its energy resolution is $\sigma(E)/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\%$ (E in GeV). The muon veto counter (MUV) is the last component of the NA48/2 detector set-up. It is composed of three planes of alternating horizontal and vertical scintillator strips separated by 80 cm thick iron walls. Its time resolution is below 1 ns and about a per mil inefficiency for muons with momenta higher than 10 GeV/c.

3. K_{e4} Braching ratio measurements

The branching ratio (BR) of each K_{e4} mode is measured relative to a normalization decay using the following formula:

$$BR(K_{e4}^{+-}) = \frac{N_s - N_b}{N_n} \cdot \frac{A_n \varepsilon_n}{A_s \varepsilon_s} \cdot BR(n),$$

where N_s, N_b, N_n are the numbers of signal, background and normalization events, A_s and ε_s (A_n and ε_n) are the geometrical acceptance and trigger efficiency for the signal (normalization) sample, respectively. The normalization branching ratio $BR(n)$ is taken from world average [4].

NA48/2 has measured the BR of the charged and neutral K_{e4} modes by using for reference channels $K^\pm \to \pi^+\pi^-\pi^\pm$ ($K_{3\pi}^{+-}$) and $K^\pm \to \pi^0\pi^0\pi^\pm$ ($K_{3\pi}^{00}$), respectively. The signal and the normalization samples are collected concurrently using the same trigger logic and a common event selection is considered as much as possible in order to cancel partially systematic effects.

Detailed GEANT-3-based Monte Carlo simulations are used to compute the acceptances of the signal and normalization channels in both analyses. Each simulation includes full detector and material description, DCH alignment, DCH and LKr local inefficiencies and kaon beam properties throughout the running period.

3.1 $K^\pm \to \pi^\pm\pi^0 e^\pm \nu_e$ **decay**

The K_{e4}^{+-} and $K_{3\pi}^{+-}$ candidates are reconstructed from three charged tracks which form one common vertex in the decay volume and satisfy all requirements of DCH, HOD, LKr and MUV geometrical acceptances. The ratio of energy deposition in the LKr calorimeter and the momentum measured by the spectrometer (E/p) is used for pion/electron identification. The track is identified as an electron if its momentum is greater than 2.75 GeV/c and it has an associated cluster with $0.9 < E/p < 1.1$. The criteria for pion identification require momentum larger than 5 GeV/c and E/p smaller than 0.8. The electric charge of the two pions should be 0. Additional suppression of pions mis-identified as electrons is achieved by using a dedicated discriminant variable defined as a linear combination of quantities related to shower properties in the electromagnetic calorimeter (such as E/p, radial shower width and energy weighted track-cluster distance at the LKr front face). Rejection of the significant $K_{3\pi}^{+-}$ background source is obtained by requiring the invariant mass ($M_{3\pi}$) of the three tracks in $\pi^+\pi^-\pi^\pm$ hypothesis and the transverse momentum relative to the beam axis (p_t) to be outside of an half-ellipse centred on the PDG value of the kaon mass [4] and zero p_t, with semi-axes 20 MeV/c^2 and 35 MeV/c, respectively. The squared missing mass is required to be larger than 0.04 $(GeV/c)^2$ in order to remove $K^\pm \to \pi^\pm\pi^0$ events with a subsequent $\pi^0 \to e^+e^-\gamma$ decay. The normalization $K_{3\pi}^{+-}$ sample is selected by applying all possible common criteria with the main channel and additionally demanding the $M_{3\pi}$ and the p_t to be inside a smaller ellipse with semi-axes 12 MeV/c^2 and 25 MeV/c respectively.

A total sample of 1.1×10^6 K_{e4}^{+-} candidate-events (712 288 K^+ and 396 653 K^-) and prescaled 18.8×10^6 $K_{3\pi}$ candidates (12.09×10^6 K^+ and 6.73×10^6 K^-) are selected from data recorded during 2003-2004.

The main remaining background is the $K^\pm \to \pi^+\pi^-\pi^\pm$ channel followed by the decay in flight of a single pion $\pi^\pm \to e^\pm \nu$ or mis-identification of a pion as an electron. Another background source is $K^\pm \to \pi^0(\pi^0)\pi^\pm$ with subsequent Dalitz decay of $\pi^0 \to e^+e^-\gamma$, where the $\pi^\pm$ fakes the electron response and photon(s) is (are) undetected. The total background contamination in the signal region is of order 1%.

The final result is a weighted average of 16 values obtained in eight independent data subsamples and both kaon charges. The obtained K_{e4}^{+-} branching ratio by NA48/2 [5] is $BR(K_{e4}^{+-}) = (4.257 \pm 0.004_{stat.} \pm 0.016_{syst.} \pm 0.031_{ext.}) \times 10^{-5}$. The precision of the result is 3 times better than the available PDG value [4].

3.2 $K^\pm \to \pi^0\pi^0 e^\pm \nu_e$ **decay**

The neutral K_{e4}^{00} and $K_{3\pi}^{00}$ candidates are selected by requiring one charged track and at least four

photons, forming two π^0's, pointing to the same vertex. The track is reconstructed by the magnetic spectrometer and the photons are identified as electromagnetic showers, developed in the LKr, without associated track and with energy above 3 GeV. For each track candidate with momentum larger than 5 GeV/c, the charged vertex longitudinal position (Z_c) is defined as the closest distance of approach to the beam line smaller than 5 cm. On the other hand, the reconstructed longitudinal positions of both $\pi^0 \to \gamma\gamma$ decay candidates (Z_1 and Z_2) are required to be within 500 cm. The neutral vertex, $Z_n = (Z_1 + Z_2)/2$, is determined as the average longitudinal positions of both π^0's. Track candidate for which Z_c and Z_n are compatible within ± 800 cm is retained further in the analysis.

The track is identified as an electron if it has an associated cluster with E/p between 0.9 and 1.1 or as a charged pion if its momentum is above 5 GeV/c (it may have an associated cluster or not). Powerful suppression of pions mis-identified as electrons is obtained by using the above mentioned discriminant variable as in $K_{e4}^{\pm}$ decay analysis.

The separation between the signal candidates from the abundant $K_{3\pi}^{00}$ decays is obtained by means of an elliptic requirement in the plane of missing mass ($M_{3\pi} - M_K$) and missing transverse momentum (p_t) assuming the $\pi^0\pi^0\pi^{\pm}$ decay hypothesis. $M_{3\pi}$ is the reconstructed three pion invariant mass and M_K is the nominal kaon mass [4]. K_{e4}^{00} events are selected outside an ellipse centered on the nominal $K^{\pm}$ mass and 5 MeV/c p_t, with semi-axes of 15 MeV/c^2 and 30 MeV/c, respectively. $K_{3\pi}^{00}$ candidates are required to be inside an ellipse with the same center but with semi-axes of 10 MeV/c^2 and 20 MeV/c respectively.

The invariant mass of the electron-neutrino system is required to be between 0 and 0.25 GeV/c^2.

A total sample of 65 210 K_{e4}^{00} events and almost 94 million $K_{3\pi}^{00}$ are selected and used for the branching ratio measurement.

The background from fake-electron tracks is evaluated to be 0.65% of the signal events and a contribution of 0.12% comes from the $K_{3\pi}^{00}$ channel with subsequent $\pi^{\pm} \to e^{\pm}\nu$ decay. The background of accidental coincidence of another kaon decay with an additional track or photon and resulting in the same final state topology as the K_{e4}^{00} signal is estimated as 0.23%. The total background to signal contamination is 1%.

The value of the branching ratio is measured by NA48/2 [6] as the combination of ten statistically independent samples: $BR(K_{e4}^{00}) = (2.552 \pm 0.010_{stat.} \pm 0.010_{syst.} \pm 0.032_{ext.}) \times 10^{-5}$, where the error is dominated by the external uncertainty from the branching fraction of the normalization decay mode. It is 13 times more precise measurement than the PDG corresponding value [4]. The systematic uncertainty includes contributions from the control of background estimate, radiative effects, trigger efficiency and acceptance.

4. Form factor measurement

The decay amplitude of the K_{e4}^{+-} is a product of the weak leptonic and (V-A) hadronic currents. The latter is described in terms of three axial-vector (F,G,R) and one vector (H) complex form factors. The R form factor is not taken into account in the final expression of the K_{e4} matrix element because it is always multiplied by the squared electron mass and therefore can be neglected. The remaining form factors may be developed in a partial wave expansion with respect to

$\cos\theta_\pi$. We limit the expansions to S- and P-waves and consider a unique phase (δ_p) for all P-waves contributions (in absence of CP-violating weak phases):

$$F = F_s e^{i\delta_{fs}} + F_p e^{i\delta_{fp}}\cos\theta_\pi, \;\; G = G_p e^{i\delta_{gp}}\cos\theta_\pi, \;\; H = H_p e^{i\delta_{hp}}\cos\theta_\pi$$

Therefore the decay amplitude depends on the real form factor magnitudes F_s, F_p, G_p, H_p, a single phase shift $\delta = \delta_s - \delta_p$ and five Cabibbo-Maksymowicz variables [7] which fully describe the K_{e4} kinematics (the squared invariant masses of dipion (S_π) and dilepton (S_e) systems, the angle θ_π (θ_e) of the $\pi^\pm$ ($e^\pm$) in the dipion (dilepton) rest frame with respect to the the flight direction of dipion (dilepton) in the $K^\pm$ center-of-mass system, and the angle ϕ between dipion and dilepton planes).

The K_{e4} decay form factors can be expressed as a series expansion of dimensionless invariants $q^2 = [(S_\pi/4m_\pi^2) - 1]$ and $(S_e/4m_\pi^2)$. All form factor values are obtained relative to a common value f_s which represents the S-wave axial vector form factor, F_s, at $q^2 = 0$ and $S_e = 0$:

$$F_s/f_s = 1 + (f_s'/f_s)q^2 + (f_s''/f_s)q^4 + (f_e'/f_s)S_e/4m_\pi^2, \tag{4.1}$$

$$F_p/f_s = f_p/f_s, \;\; G_p/f_s = g_p/f_s + (g_p'/f_s)q^2, \;\; H_p/f_s = h_p/f_s. \tag{4.2}$$

The K_{e4}^{+-} hadronic form factors and the S-wave $\pi\pi$ scattering lengths with $I = 0$ and $I = 2$, a_0^0 and a_2^0, have been extensively studied by NA48/2 and all results have been published in [8].

In the neutral K_{e4}^{00} mode, the presence of two identical particles in the final state, reduces the independent kinematic variables to three: S_π, S_e and θ_e. Moreover, only one form factor (F_s) enters the decay amplitude expression because the dipion system can have only isospin $I = 0$ and orbital momentum $l = 0$. The allowed kinematic range of q^2 spans both positive and negative values. The ratio of data and simulation (including a constant form factor) distributions projected on the q^2 axis is shown on Fig.1. A quadratic behaviour is observed for the positive q^2 values with the same parameterization (4.1) of the F_s/f_s as in the analysis of $K_{e4}^\pm$, and a deficit of events is observed below the $2m_{\pi^+}$ threshold ($q^2 = 0$). The following fitting function $F_s/f_s = 1 + d\sqrt{|q^2/(1+q^2)|} + (f_e'/f_s)S_e/4m_\pi^2$ is taken for $q^2 < 0$. This cusp-like shape at the $2m_\pi^+$ threshold can be explained by the final state charge exchange scattering ($\pi^+\pi^- \to \pi^0\pi^0$) as in the $K_{3\pi}^{00}$ mode [9].

The results are in agreement with NA48/2 K_{e4}^{+-} analysis: $f_s'/f_s = 0.149 \pm 0.033_{stat} \pm 0.014_{syst}$, $f_s''/f_s = -0.070 \pm 0.039_{stat} \pm 0.013_{syst}$, $f_e'/f_s = 0.113 \pm 0.022_{stat} \pm 0.07_{syst}$, $d = -0.256 \pm 0.049_{stat} \pm 0.016_{syst}$. More details can be found in [6].

5. Summary

More than one million K_{e4}^{+-} decay candidates with less than one percent background has been collected by the NA48/2 experiment at CERN SPS in 2003-2004. The rich statistics gives a great opportunity to perform a detailed study of decay properties. The BR(K_{e4}^{+-}), inclusive of $K_{e4\gamma}^{+-}$ mode, is obtained with total relative error of 0.8%. This measurement complements the investigation of S- and P-wave hadronic form factors [5].

The K_{e4}^{00} Branching ratio, inclusive of radiative decays, has been measured by NA48/2 from a sample of 65210 events with 1% background contamination. The first measurement of the hadronic form factor has been obtained including its variation in the (S_π, S_e)-plane and providing an evidence for final state charge exchange scattering (K_{e4}^{+-}) in the decay mode below the $2m_{\pi^+}$ threshold [6].
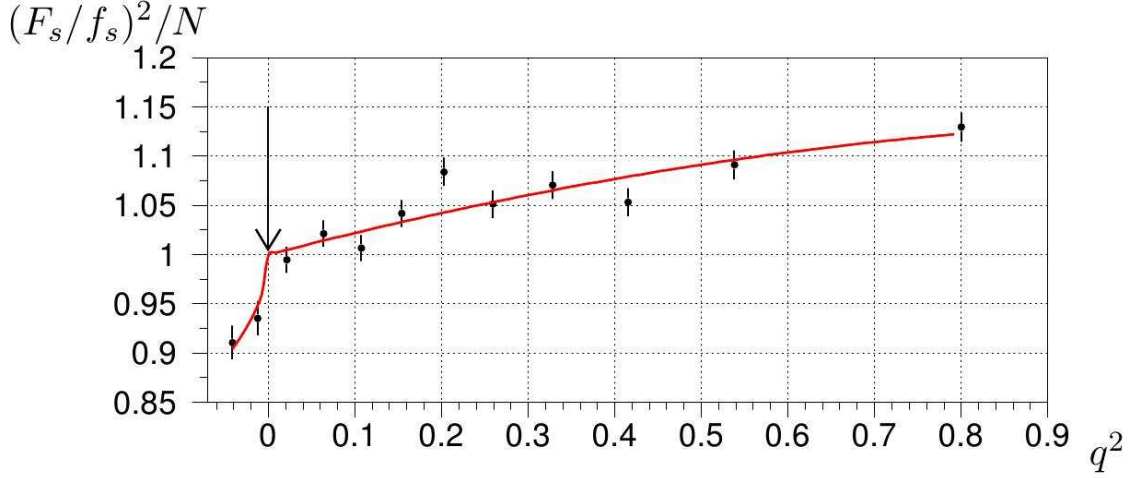
Figure 1: Data and simulation (including a constant form factor) q^2 distributions in equal population bins. Each symbol is plotted at the barycenter position of the data events in the bin. The line gives the empirical description using the best fit-parameter values: a 2-degree polynomial above $q^2 = 0$ and a cusp-like function below. N is a normalization factor.

The analysis of the $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ performed by NA48/2 has enlightened a cusp in the $M_{\pi^0\pi^0}$ distribution which may be explained by re-scattering effects in the $\pi\pi$ system around the $2m_{\pi^+}$ threshold [9]. Neglecting potential (but small) common systematic contributions to the experimental errors, the results from both $K_{3\pi}$ and K_{e4}^{+-} analyses have been combined in more precise scattering lengths measurements:

$$a_0^0 = 0.2210 \pm 0.0047_{stat} \pm 0.0040_{syst}, \quad a_0^0(\text{ChPT}) = 0.220 \pm 0.005_{th},$$
$$a_0^2 = -0.0429 \pm 0.0044_{stat} \pm 0.0028_{syst}, \quad a_0^2(\text{ChPT}) = -0.0444 \pm 0.0010_{th}$$
$$a_0^0 - a_0^2 = 0.2639 \pm 0.0020_{stat} \pm 0.0015_{syst}, \quad a_0^0 - a_0^2(\text{ChPT}) = 0.264 \pm 0.004_{th},$$

where the corresponding values of scattering lengths calculated by ChPT are quoted for comparison. These results provide a stringent test of ChPT predictions.

References

[1] J. Bijnens, G. Colangelo, J. Gasser, *Nucl.Phys.***B427** (1994) 427

[2] J.Bijnens and I. Jemos, *Nucl.Phys* **B854** (2012) 631

[3] NA48: V. Fanti et al., *Nucl. Instrum. Meth.* **A574** (2007) 433

[4] PDG: J. Beringer et al., *Phys. Rev.* **D86** (2012) 010001

[5] NA48/2: Batley, J.R. et al., *Phys. Lett.* **B715** (2012) 105

[6] NA48/2:Batley, J.R. et al., CERN-PH-EP-2014-145, [arXiv:1406.4749] (2014)

[7] N. Cabibbo and A. Maksymowicz, *Phys. Rev.* **137** (1965) B438; *Phys. Rev.* **168** (1968) 1926

[8] NA48/2: Batley, J.R. et al., *Eur.Phys.J.* **C70** (2010) 635

[9] NA48/2: Batley, J.R. et al., *Eur.Phys.J.* **C64**, (2009) 589