$A_N$ in inclusive lepton-proton collisions

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Some estimates for the transverse single spin asymmetry, $A_N$, in the inclusive processes $\ell p^\uparrow \rightarrow hX$ are compared with new experimental data. The calculations are based on the Sivers and Collins functions as extracted from SIDIS azimuthal asymmetries, within a transverse momentum dependent factorization approach. The values of $A_N$ thus obtained agree in sign and shape with the data. Predictions for future experiments are also given.
1. Introduction and Formalism

We present a phenomenological analysis of recent HERMES data [1] for the single spin asymmetry (SSA) measured in the inclusive hadron production in lepton-proton collisions. This study is based on a previous paper [4], recently extended [3], where we considered the transverse SSAs for the $\ell p^+ \rightarrow hX$ process in the $\ell - p$ center of mass (c.m.) frame, with a single large $P_T$ final particle.

Such $A_N$ is the exact analogue of the SSAs observed in $pp \rightarrow hX$, the well known and large left-right asymmetries (see Ref. [3] and references therein). On the other hand, the process is essentially a semi-inclusive deep inelastic scattering (SIDIS) process, for which, at large $Q^2$ values (and small $P_T$ in the $\gamma' - p$ c.m. frame), the TMD factorization is proven to hold [5, 6]. Notice that even without the detections of the final lepton, large $P_T$ values imply large values of $Q^2$.

We computed these SSAs assuming the TMD factorization and using the relevant TMDs (Sivers and Collins functions) as extracted from SIDIS data. A first simplified study of $A_N$ in $\ell p^+ \rightarrow hX$ processes was performed in Ref. [7]. The process was also considered in Refs. [8] in the framework of collinear twist-three formalism.

In Ref. [2] (where all details can be found) we considered the process $p^+ \ell \rightarrow hX$ in the proton-lepton c.m. frame (with the polarized proton moving along the positive $Z_{cm}$ axis) with:

$$A_N = \frac{d\sigma^\uparrow(P_T) - d\sigma^\downarrow(P_T)}{d\sigma^\uparrow(P_T) + d\sigma^\downarrow(P_T)} - \frac{d\sigma^\uparrow(-P_T) - d\sigma^\downarrow(-P_T)}{2d\sigma^{unp}(P_T)},$$

(1.1)

where

$$d\sigma^\uparrow = \frac{E_h d\sigma^{p^+\ell-hX}}{d^2P_h}$$

(1.2)

is the cross section for the inclusive process $p^+\ell \rightarrow hX$ with a transversely polarized proton with spin $\uparrow$ or $\downarrow$ w.r.t. the scattering plane [2]. For a generic transverse polarization along an azimuthal direction $\phi_S$ in the chosen reference frame, in which the $\uparrow$ direction is given by $\phi_S = \pi/2$, one has:

$$A(\phi_S, S_T) = S_T \cdot (\mathbf{p} \times \mathbf{P}_T) A_N = S_T \sin \phi_S A_N,$$

(1.3)

where $\mathbf{p}$ is the proton momentum. Notice that one simply has:

$$A_{\sin\phi_S}^{\sin\phi_T} = \frac{2}{S_T} \int \frac{d\phi_S [d\sigma(\phi_S) - d\sigma(\phi_S + \pi)] \sin \phi_S}{\int d\phi_S \left[ d\sigma(\phi_S) + d\sigma(\phi_S + \pi) \right]} = A_N.$$

(1.4)

Within a TMD factorization scheme for the process $p\ell \rightarrow hX$ with a single large scale (the final hadron transverse momentum $P_T$ in the proton-lepton c.m. frame) the main contribution to $A_N$ comes from the Sivers and Collins effects [4]:

$$A_N = \sum_{q, \{\lambda\}} \int \frac{dx dz}{16 \pi^2 x z s} d^2k_\perp d^3p_\perp \delta (p_\perp \cdot \mathbf{p}^\prime_q) J(p_\perp) \delta (\bar{s} + \bar{t} + \bar{u}) [\Sigma(\uparrow) - \Sigma(\downarrow)]^{q_\ell - q_\ell}$$

$$\sum_{q, \{\lambda\}} \int \frac{dx dz}{16 \pi^2 x z s} d^2k_\perp d^3p_\perp \delta (p_\perp \cdot \mathbf{p}^\prime_q) J(p_\perp) \delta (\bar{s} + \bar{t} + \bar{u}) [\Sigma(\uparrow) + \Sigma(\downarrow)]^{q_\ell - q_\ell},$$

(1.5)

with

$$\sum_{\{\lambda\}} [\Sigma(\uparrow) - \Sigma(\downarrow)]^{q_\ell - q_\ell} = \frac{1}{2} \Delta^N f_{q/p'} \left( x, k_\perp \right) \cos \phi \left[ |\tilde{M}_1^q|^2 + |M_2^q|^2 \right] D_{h/q}(z, p_\perp)$$

$$+ h_{1q}(x, k_\perp) \tilde{M}_1^q \tilde{M}_2^q \Delta^N_{h/q}(z, p_\perp) \cos (\phi' + \phi_q')$$

(1.6)
\[
\sum_{\{\lambda\}} [\Sigma(\uparrow) + \Sigma(\downarrow)] q^{\ell-q\ell} = f_{q/p}(x,k_\perp) \left[ |M_1|^2 + |M_2|^2 \right] D_{h/q}(z,p_\perp).
\]

(1.7)

All details can be found in Refs. [4, 3]. Here we simply recall some main features.

- \(k_\perp, p_\perp\) are respectively the transverse momenta of the parton in the proton and of the final hadron w.r.t. the direction of the fragmenting parton, with momentum \(p_{T}\). \(\phi\) is the azimuthal angle of \(k_\perp\).

- The first term on the r.h.s. of Eq. (1.7) shows the contribution of the Sivers effect [4, 11],

\[
\Delta f_{q/p,S}(x,k_\perp) \equiv \Delta N f_{q/p}(x,k_\perp) S_T \cdot (\hat{p} \times \hat{k}_\perp) = -2 \frac{k_\perp}{M} \Delta f_{q/p}(x,k_\perp) S_T \cdot (\hat{p} \times \hat{k}_\perp).
\]

(1.8)

It couples to the unpolarized elementary interaction (\(\propto (|M_1|^2 + |M_2|^2)\)) and the unpolarized fragmentation function \(D_{h/q}(z,p_\perp)\); the \(\cos \phi\) factor arises from the \(S_T \cdot (\hat{p} \times \hat{k}_\perp)\) factor.

- The second term on the r.h.s. of Eq. (1.7) represents the contribution to \(A_N\) of the unintegrated transversity distribution \(h_{1q}(x,k_\perp)\) coupled to the Collins function \(\Delta N D_{h/q}(z,p_\perp)\) [12, 13],

\[
\Delta \hat{H}_{h/q}(z,p_\perp) \equiv \Delta N D_{h/q}(z,p_\perp) s_q \cdot (\hat{p}_q \times \hat{p}_\perp) = \frac{2 p_\perp}{z m_h} H_{1}(z,p_\perp) s_q \cdot (\hat{p}_q \times \hat{p}_\perp).
\]

(1.9)

This effect couples to the spin transfer elementary interaction (\(d\hat{\sigma} q^{\ell-q\ell} - d\hat{\sigma} q^{\ell-q\ell} \propto \hat{M}_1^0 \hat{M}_2^0\)). The factor \(\cos(\phi' + \phi_0')\) arises from phases in the \(k_\perp\)-dependent transversity distribution, the Collins function and the elementary polarized interaction.

In HERMES paper [11] the lepton moves along the positive \(Z_{cm}\) axis. In this reference frame the \(\uparrow (\downarrow)\) direction is still along the \(+Y_{cm} (-Y_{cm})\) axis as in Ref. [12] and only the sign of \(x_F = 2p_L/\sqrt{s}\) is reversed. More precisely the HERMES azimuthal dependent cross section is defined as [11]:

\[
d\sigma = d\sigma_{UU} \left[ 1 + ST A_{UU}^{\sin \psi} \sin \psi \right], \text{ where } \sin \psi = S_T \cdot (\hat{P}_T \times \hat{k})
\]

(1.10)

coincides with our \(\sin \phi_3\) of Eq. (1.8), as \(p\) and \(k\) (the lepton momentum) are opposite vectors in the lepton-proton c.m. frame. Taking into account that “left” and “right” are interchanged in Refs. [12] and [11] (being defined looking downstream along opposite directions, \(p\) and \(k\)) and the definition of \(x_F\), one has:

\[
A_{UU}^{\sin \psi}(x_F, P_T) = A_N^{\ell-hX}(-x_F, P_T),
\]

(1.11)

where \(A_N^{\ell-hX}\) is the SSA in Eq. (1.3) [2], and \(A_{UU}^{\sin \psi}\) is the quantity measured by HERMES [11].

2. Results

In the following, adopting the HERMES notation, we show our estimates based on two representative extractions of the Sivers and Collins functions: \(i\) the Sivers functions from Ref. [12] (only up and down quarks), together with the first extraction of the transversity and Collins functions of Ref. [13] (SIDIS 1 in the following). In such studies the Kretzer set for the collinear fragmentation functions (FFs) [13] was adopted; \(ii\) the Sivers functions from Ref. [15], where also the sea quark contributions were included, together with an updated extraction of the transversity
and Collins functions \cite{10} (SIDIS 2 in the following); in these cases we adopted another set for the FFs, namely that one by de Florian, Sassot and Stratmann (DSS) \cite{11}.

We consider both the fully inclusive measurements \(\ell p \rightarrow \pi X\) at large \(P_T\), as well as the sub-sample of data in which also the final lepton is tagged (SIDIS category). In the first case the only large scale is the \(P_T\) of the final pion, and for \(P_T \simeq 1\) GeV, to avoid the low \(Q^2\) region, one has to look at pion production in the backward proton hemisphere, \((x_F > 0)\) in the HERMES conventions. For the tagged-lepton sub-sample data \(Q^2\) is always bigger than 1 GeV\(^2\) and \(P_T\) is still defined w.r.t. the lepton-proton direction.

In both cases (inclusive or SIDIS events) the Sivers and Collins effects are not separable.

- Fully inclusive case

Only one HERMES data bin covers moderately large \(P_T\) values, with \(1 < P_T < 2.2\) GeV, and \((P_T) \simeq 1-1.1\) GeV. In Fig. 1 we show the results for \(\pi^+\) (first and second panel) and \(\pi^-\) (third and fourth panel) production coming from SIDIS 1 and SIDIS 2 sets, for the Sivers (dotted blue lines) and Collins (dashed green lines) effects separately, together with their sum (solid red lines) and the envelope of the statistical error bands (shaded area): i) here the Collins effect is almost zero, as the partonic spin transfer in the backward proton hemisphere is dynamically suppressed \cite{12}, and the azimuthal phase (in the second term on the r.h.s. of Eq. (12)) oscillates strongly; ii) the Sivers effect does not suffer from any dynamical or azimuthal phase suppression. Indeed, in contrast to \(pp \rightarrow \pi X\) processes in \(\ell p \rightarrow \pi X\) only one partonic channel is at work and, for such moderate \(Q^2\) values, the Sivers phase (\(\phi\)) in the first term on the r.h.s. of Eq. (12) is still effective in the elementary interaction; iii) at this moderate c.m. energy, even in the backward hemisphere of the polarized proton, one probes its valence region, where the extracted Sivers functions are sizeable and well constrained; iv) in the backward proton hemisphere at large \(P_T\), \(Q^2\) is predominantly larger than 1 GeV\(^2\) and we can neglect any contribution from quasi-real photo-production.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Theoretical estimates for \(A_{\text{UT}}^{\sin \psi}\) vs. \(x_F\) at \(P_T = 1\) GeV for inclusive \(\pi^+\) (first and second panel) and \(\pi^-\) (third and fourth panel) production in \(\ell p \rightarrow \pi X\) processes, computed according to Eqs. (11,12) and (13) of the text and compared with the HERMES data \cite{13}. See the legend and text for details.}
\end{figure}

- Tagged or semi-inclusive category

We consider also the HERMES sub-sample data where the final lepton is tagged \cite{13} with \(Q^2 > 1\) GeV\(^2\), \(W^2 > 10\) GeV\(^2\), \(0.023 < x_F < 0.4\), \(0.1 < y < 0.95\) and \(0.2 < z_b < 0.7\) (standard SIDIS variables). We keep focusing only on the large \(P_T\) region, namely \(P_T > 1\) GeV.

We show our estimates compared with HERMES data in Fig. 4, for positive and negative pion production as a function of \(P_T\) at fixed \(x_F = 0.2\). Again, we show the contributions from the Sivers (dotted blue line) and Collins (dashed green line) effects separately and added together (solid red lines).
approach [ii] phase integration. In the forward region both sets give tiny values; the backward region is totally negligible due to a strong suppression coming from the azimuthal
serve the following:
set (able to reproduces the behaviour of $A$)
show some estimates of $p$
the forward hemisphere of $Z$
to the configuration where the polarized
SSAs in
cess manifests some of the features of the
larger energies in a TMD scheme this pro-
x set (4th panel), the same large
coupled to the non-leading FF, with the more suppressed down quark distribution. For the SIDIS 2
almost zero due to the strong cancellation between the unsuppressed Sivers up quark distribution
Another interesting aspect is that at $x_F$
Even if there is only one partonic channel, the weak dependence on the azimuthal phase of the ele-
able and increasing with $x_F$; while negligible in the negative $x_F$ region.
Even if there is only one partonic channel, the weak dependence on the azimuthal phase of the elementary interaction at the large $Q^2$ values reached at these energies implies a strong suppression at $x_F < 0$. Notice that this behaviour is similar to that observed at various energies in $A_N$ in $p^1 p \rightarrow hX$ processes, being negligible at negative $x_F$ and increasing with positive values of $x_F$; iii) when one exploits the relation between the Qiu-Sterman function and the Sivers function the twist-3 approach for $A_N$ in $\ell p^1 \rightarrow jet + X$ [20] gives results similar, in sign and size, to those obtained in a TMD approach [2]. However, the twist-3 collinear scheme, using the SIDIS Sivers functions, leads to
values of $A_N$ in $pp \rightarrow \pi X$ collisions opposite to those measured \cite{21}. A recent analysis of $A_N$ in $pp$ scattering in the twist-3 formalism \cite{22} aiming at solving this problem introduces new large effects in the fragmentation. It is not clear how much these same effects would change the value of $A_N$ in $\ell p$ processes when going from jet to $\pi^0$ production; iv) the measurements of SSAs at such large energies, possible at a future Electron-Ion-Collider (EIC) \cite{23} would be an invaluable tool to test the TMD factorization and discriminate among different approaches.

References

\[23\] A. Accardi et al., arXiv:1212.1701 [nucl-ex].