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Transverse-spin gluon distribution function

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We introduce the spin-operator representation for the gluon as well as quark distribution functions as nucleon matrix element of the gauge-invariant bilocal light-cone operators in QCD. To identify the relevant spin operators for quarks and gluons in a unified manner, we rely on the transformation properties of the quark and gluon fields in the coordinate space under the action of the generator of the Lorentz group. In particular, this approach allows us to define the transverse-spin gluon distribution function $G_T(x)$, which is the genuine counterpart of the transverse-spin quark distribution function $g_T(x)$ relevant to the transverse-spin structure function $g_2(x,Q^2)$ in the deep inelastic scattering. We show that $G_T(x)$ is given by the sum of the chromoelectric and chromomagnetic correlators associated with helicity-flip by one unit, and the treatment of the latter correlator completes the classification of the collinear parton distribution functions up to twist three. We show that $G_T(x)$ receives the three-gluon and quark-gluon correlation effects and discuss the operator product expansion for $G_T(x)$. We also discuss the relevance of the first moment of $G_T(x)$ for the partonic decomposition of the transverse nucleon spin.

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In hard processes like deep inelastic scattering (DIS), Drell-Yan process, etc., the cross sections are given by the hard partonic scattering combined with the parton distribution functions (PDFs), as the factorization formulas. The PDFs are given by the Fourier transformation of the bilocal operators instantaneous for a relevant light-cone direction, e.g., z^+. For the hard processes with one hard scale, the transverse degrees of freedom can be also integrated out in the factorization formulas, leaving the collinear PDFs as the one-dimensional light-cone Fourier transform; e.g., $\sim \int dz^- e^{i(xP^+)z^-} \langle PS|\psi^\dagger(0)\psi(z^-)|PS\rangle$ for the quark distributions in the nucleon with momentum P and spin S ($P^2 = -S^2 = M^2$); here and in the following, we omit the Wilson line operator in-between the constituent fields. Inserting the various gamma matrices in-between the quark and antiquark fields gives a complete set of the quark distribution functions [1, 2, 3] (see also [4]):

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}_\beta(0)\psi_\alpha(\lambda n)|PS\rangle = \frac{1}{2}\left[q(x)\not{P} + \Delta q(x)\,(S\cdot n)\,\gamma_5\not{P} + g_T(x)\gamma_5\not{S}_\perp\right]_{\alpha\beta} \,, \tag{1}$$

with a light-like vector $n^\mu = g_-^\mu/P^+$. Here and in the following, we treat the distribution functions up to twist three: the twist-two density and helicity quark distributions, $q(x)$ and $\Delta q(x)$, and the twist-three transverse-spin quark distribution $g_T(x)$. These chiral-even distributions are relevant in the following discussions; in (1), we omitted the terms associated with chiral-odd distributions.

The gluon distribution functions may be introduced [1, 2, 5, 7] by replacing the quark fields ψ with the gluon fields A^μ, in particular, by the formal replacement $\psi \to n^- F^{+\nu} \equiv F^{n\nu}$ with $F^{\mu\nu}$ the gluon field strength tensor, as motivated by $F^{+\nu} = \partial^+ A^\nu$ in the $A^+ = 0$ gauge:

$$\frac{-1}{x}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|F^{n\nu}(0)F^{n\sigma}(\lambda n)|PS\rangle = \frac{1}{2}\left[G(x)g_\perp^{\nu\sigma} + \Delta G(x) i\varepsilon^{\nu\sigma Pn}(S\cdot n) + 2G_{3E}(x) i\varepsilon^{\nu\sigma\alpha n} S_{\perp\alpha}\right], \tag{2}$$

where $g_\perp^{\nu\sigma} = g^{\nu\sigma} - P^\nu n^\sigma - P^\sigma n^\nu$, and $\varepsilon^{\nu\sigma Pn} \equiv \varepsilon^{\nu\sigma\alpha\beta}P_\alpha n_\beta$ with the Levi-Civita tensor of $\varepsilon_{0123} = 1$. The RHS is analogous to (1), with the twist-two density and helicity distributions, $G(x)$ and $\Delta G(x)$, and the twist-three gluon distribution $G_{3E}(x)$, which was denoted as $G_3(x)$ ($\mathscr{G}_{3T}(x)$) in [5] ([4, 7]).

Now, in order to go beyond the conventional treatment (2), we consider the Lorentz tensor decomposition of the most general gluonic correlator and find, to the twist-three accuracy,

$$\begin{aligned}
&-\frac{1}{x}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|F^{\mu\nu}(0)F^{\xi\sigma}(\lambda n)|PS\rangle \\
&= \frac{1}{2}G(x)\left(P^\mu P^\xi g_\perp^{\nu\sigma} - P^\nu P^\xi g_\perp^{\mu\sigma} - P^\mu P^\sigma g_\perp^{\nu\xi} + P^\nu P^\sigma g_\perp^{\mu\xi}\right) \\
&+ \frac{1}{2}\Delta G(x) i(S\cdot n)P_\alpha\left(P^\mu \varepsilon^{\nu\xi\sigma\alpha} - P^\nu \varepsilon^{\mu\xi\sigma\alpha}\right) \\
&+ G_{3E}(x) i S_{\perp\alpha}\left(P^\mu \varepsilon^{\nu\xi\sigma\alpha} - P^\nu \varepsilon^{\mu\xi\sigma\alpha}\right) + G_{3H}(x) i P_\alpha\left(S_\perp^\mu \varepsilon^{\nu\xi\sigma\alpha} - S_\perp^\nu \varepsilon^{\mu\xi\sigma\alpha}\right) \,,
\end{aligned} \tag{3}$$

consistent with PT-invariance and hermiticity [6]. When contracted with $n_\mu n_\xi$, the tensor structure with $G_{3H}(x)$ vanishes, while the other terms reproduce the formula (2). The new distribution $G_{3H}(x)$ is of twist three and is associated with the transverse spin of the nucleon. This new contribution seems to violate the correspondence between the quark and gluon cases, as suggested in (1) and (2), and thus it is desirable to clarify the physical meaning of $G_{3H}(x)$ as well as of $G_{3E}(x)$.

For this purpose, we first note that (1) with $P = (P^0, 0, 0, P^3)$, $P^\pm = \frac{P^0 \pm P^3}{\sqrt{2}}$ can be recast as

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\psi_\beta^\dagger(0)\psi_\alpha(\lambda n)|PS\rangle = \frac{1}{\sqrt{2}}\left[q(x)P^+\mathscr{P}_{(+)} + 2\Delta q(x)\,(S\cdot n)\,P^+\mathscr{P}_{(+)}\hat{s}^3\right.$$

$$+\sqrt{2}\, g_T(x)\left(\mathscr{P}_{(-)}\boldsymbol{S}_\perp\cdot\hat{\mathbf{s}}\mathscr{P}_{(+)}+\mathscr{P}_{(+)}\boldsymbol{S}_\perp\cdot\hat{\mathbf{s}}\mathscr{P}_{(-)}\right)\Big]_{\alpha\beta}\ , \qquad (4)$$

where $\mathscr{P}_{(\pm)}=\frac{1}{2}\gamma^{\mp}\gamma^{\pm}$ are the projection operators with $\mathscr{P}_{(+)}\mathscr{P}_{(-)}=0$, $\mathscr{P}_{(+)}+\mathscr{P}_{(-)}=1$, such that $\mathscr{P}_{(+)}\psi$ and $\mathscr{P}_{(-)}\psi$ are the "good" and "bad" components of the quark fields in the light-cone quantization formalism, and $\hat{s}^i=\frac{1}{4}\eta^{ijk}\sigma^{jk}$ are the quark spin operators.[1] From (4), we find $(n^-/\sqrt{2})\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P|\psi^\dagger(0)\psi(\lambda n)|P\rangle=q(x)$, $\sqrt{2}n^-\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle PS_\parallel|\psi^\dagger(0)\hat{s}^3\psi(\lambda n)|PS_\parallel\rangle=\Delta q(x)$, and $(1/M)\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle PS_\perp|\psi^\dagger(0)\hat{s}^\perp\psi(\lambda n)|PS_\perp\rangle=g_T(x)$, up to the twist-four corrections suppressed for the fast-moving nucleon by powers of M/P^+. These representations demonstrate that the "quantum mechanical" expectation values in the spin space, $\psi^\dagger\psi$, $\psi^\dagger\hat{s}^i\psi$, are relevant for the quark distributions of low twist. We note that the twist-three distribution $g_T(x)$ is a complicated object, because $\mathscr{P}_{(-)}\psi=-(2n\cdot\partial)^{-1}\not{n}\not{D}_\perp\mathscr{P}_{(+)}\psi$, using the QCD equations of motion with $A^+=0$, and the good components $\mathscr{P}_{(+)}\psi$ are the independent degrees of freedom in the light-cone quantization: $q(x)$, $\Delta q(x)$ are literally the distributions, while $g_T(x)$ represents the quark-gluon three-body correlations.

The spin operator for a particle can be identified in the $x\to 0$ limit of the transformation law of the corresponding field $\Phi(x)$ under the action of the generator $\mathscr{M}^{\mu\nu}$ of the Lorentz group:

$$[\mathscr{M}^{\mu\nu},\Phi(x)]=-\left(i\left(x^\mu\partial^\nu-x^\nu\partial^\mu\right)+\Sigma^{\mu\nu}\right)\Phi(x)\ , \qquad (5)$$

and this guarantees the usual $SU(2)$ algebra, $\left[\hat{s}^i,\hat{s}^j\right]=i\eta^{ijk}\hat{s}^k$, obeyed by $\hat{s}^i\equiv\frac{1}{2}\eta^{ijk}\Sigma^{jk}$. Indeed, we obtain $\Sigma^{\mu\nu}=\frac{1}{2}\sigma^{\mu\nu}$ for $\Phi=\psi$. Similarly, manifestly gauge-covariant form of the transformation law for the gluon is given by (5) with $\Phi(x)=F^{\alpha\beta}(x)$, $\Sigma^{\mu\nu}F^{\alpha\beta}=i(g^{\mu\alpha}F^{\nu\beta}-g^{\nu\alpha}F^{\mu\beta}+g^{\mu\beta}F^{\alpha\nu}-g^{\nu\beta}F^{\alpha\mu})$, so that $\hat{s}^i\equiv\frac{1}{2}\eta^{ijk}\Sigma^{jk}$ give the spin operators for a spin-1 particle. This fact allows us to extend the above spin-operator representation of the PDFs to the gluon case. The corresponding "quantum mechanical" expectation values in the spin space should be given by $(F^{\alpha\beta})^\dagger F^{\alpha\beta}$, $(F^{\alpha\beta})^\dagger\hat{s}^iF^{\alpha\beta}$; note, $(F^{\alpha\beta})^\dagger F^{\alpha\beta}\neq(F^{\alpha\beta})^\dagger F_{\alpha\beta}$, where $(F^{\alpha\beta})^\dagger F_{\alpha\beta}=F^{\alpha\beta}F_{\alpha\beta}$ is a Lorentz invariant contraction. We find, $(F^{\alpha\beta})^\dagger F^{\alpha\beta}=-2F^{+\beta}F^{+}{}_{\beta}+\cdots$, and $(F^{\alpha\beta})^\dagger\hat{s}^3F^{\alpha\beta}=2iF^{+\beta}\widetilde{F}^{+}{}_{\beta}+\cdots$, so that

$$\frac{(n^-)^2}{2x}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P|\left(F^{\alpha\beta}(0)\right)^\dagger F^{\alpha\beta}(\lambda n)|P\rangle\;=\;G(x)\ , \qquad (6)$$

$$\frac{(n^-)^2}{2x}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle PS_\parallel|\left(F^{\alpha\beta}(0)\right)^\dagger\hat{s}^3F^{\alpha\beta}(\lambda n)|PS_\parallel\rangle\;=\;\Delta G(x)\ , \qquad (7)$$

up to twist-four corrections, suppressed as $\sim(M/P^+)^2$ for the fast-moving nucleon, and similarly,

$$\frac{n^-}{2\sqrt{2}Mx}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle PS_\perp|\left(F^{\alpha\beta}(0)\right)^\dagger\hat{s}^\perp F^{\alpha\beta}(\lambda n)|PS_\perp\rangle=\frac{-n^-}{2Mx}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle PS_\perp|i\widetilde{F}^{+\perp}(0)F^{+-}(\lambda n)$$
$$+iF^{+\perp}(0)F^{12}(\lambda n)+\text{h.c.}|PS_\perp\rangle=G_{3E}(x)+G_{3H}(x)\equiv G_T(x)\ , \qquad (8)$$

with "h.c." denoting the hermitian conjugate of the preceding terms, where the contributions of operators involving F^{+-} can be expressed by G_{3E} using (2), while those involving F^{12} require the new distribution G_{3H} of (3). We denote the resulting sum $G_{3E}(x)+G_{3H}(x)$ as $G_T(x)$. The QCD equations of motion in the $A^+=0$ gauge give $F^{+-}=\frac{1}{n\cdot\partial}\left(D_{\perp j}F^{jn}+g\bar{\psi}t^a\not{n}\psi t^a\right)$, which shows

[1]The repeated indices should be understood as summed over. We use Latin letters $i,j=1,2,3$ for three-dimensional space indices and also use the three-dimensional totally antisymmetric tensor $\eta^{ijk}\equiv\varepsilon^{ijk0}$, $\eta^{123}=+1$.

that $G_{3E}(x)$ is related to the three-gluon correlations and the quark-gluon correlations. Similarly, $F^{12} = \partial^1 A^2 - \partial^2 A^1 - ig[A^1, A^2]$ shows that $G_{3H}(x)$ is also related to the three-gluon correlations. Therefore, $G_T(x)$ is a complicated three-body object in contrast to $G(x)$, $\Delta G(x)$. In particular, comparing (6)-(8) with (4), $G_T(x)$ is the genuine gluonic analogue of the twist-three transverse-spin quark distribution $g_T(x)$, so that be identified as the transverse-spin gluon distribution.

As is well-known, the helicity structures in (1) can be revealed by decomposing the quark field ψ into the helicity-up and -down components, $\psi = \psi_\uparrow + \psi_\downarrow$ with $\psi_\uparrow = \frac{1}{2}(1+\sigma^{12})\psi$, $\psi_\downarrow = \frac{1}{2}(1-\sigma^{12})\psi$: $q(x)$ and $\Delta q(x)$ pick up the combinations, $\psi_\uparrow^\dagger \psi_\uparrow + \psi_\downarrow^\dagger \psi_\downarrow$ and $\psi_\uparrow^\dagger \psi_\uparrow - \psi_\downarrow^\dagger \psi_\downarrow$, respectively, in the bilinear operator in the LHS of (4), corresponding to the density and helicity distributions; on the other hand, the operator $\boldsymbol{S}_\perp \cdot \hat{\boldsymbol{s}}$ for $g_T(x)$ picks up the combinations, $\psi_\uparrow^\dagger \psi_\downarrow$, $\psi_\downarrow^\dagger \psi_\uparrow$, demonstrating that the transverse-spin distribution corresponds to helicity-flip by one unit.

To show the helicity structures of the gluon distributions, we introduce the right- and left-handed circular polarization vectors, $\varepsilon_R^\mu = (0,-1,-i,0)/\sqrt{2}$, $\varepsilon_L^\mu = (0,+1,-i,0)/\sqrt{2}$, so that the contraction with these vectors represents the helicity ± 1 states. The unit matrix in-between the gluon field strength tensors in the LHS of (6) picks up the combination, $(F^{+R})^\dagger F^{+R} + (F^{+L})^\dagger F^{+L}$, and the operator $\hat{s}^3$ of (7) picks up, $(F^{+R})^\dagger F^{+R} - (F^{+L})^\dagger F^{+L}$, clarifying that $G(x)$ and $\Delta G(x)$ are indeed the density and helicity distributions [2]. For (8), the bilocal operator $\widetilde{F}^{+\perp} F^{+-}$ relevant to $G_{3E}(x)$ is expressed by $(F^{+R})^\dagger E^3$, $(F^{+L})^\dagger E^3$ with $E^3 = F^{30} = F^{+-}$ being the third component of the chromoelectric field; because E^3 has the helicity zero, $(F^{+R})^\dagger E^3$ and $(F^{+L})^\dagger E^3$ represent the helicity-flip by ± 1. Similarly, the bilocal operator $F^{+\perp} F^{12}$ relevant to $G_{3H}(x)$ is expressed by $(F^{+R})^\dagger H^3$, $(F^{+L})^\dagger H^3$ with $H^3 = -F^{12}$ being the third component of the chromomagnetic field; because H^3 has the helicity zero, $(F^{+R})^\dagger H^3$ and $(F^{+L})^\dagger H^3$ also represent the helicity-flip by one unit. With manifest gauge invariance, we have the chromoelectric and chromomagnetic helicity-zero contributions, E^3 and H^3, for the gluon, so that we have the two types of contributions, G_{3E} and G_{3H}, for the helicity-flip by one unit relevant to the transverse-spin distribution. This explains why the transverse-spin gluon distribution G_T is given as the sum of the two distributions as in (8).

We discuss the nucleon spin sum rules using our results. The usual spin sum rule expresses the total angular momentum for the longitudinally-polarized nucleon, $J_\parallel = \frac{1}{2}$, as the sum of the orbital angular momentum contribution L, the quark spin contribution $\Delta\Sigma$, and the gluon spin contribution ΔG, and reads, using the above-mentioned helicity PDFs,

$$\frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G\,, \qquad \Delta\Sigma \equiv \int dx \Delta q(x)\,, \qquad \Delta G \equiv \int dx \Delta G(x)\,. \tag{9}$$

Here and below, the summation over all quark and antiquark flavors for the quark spin contribution is implicit. Using (4), $\Delta\Sigma$ is given as matrix element of local operator: $\Delta\Sigma = (\sqrt{2}/P^+)\langle PS_\parallel|\psi^\dagger(0)\hat{s}^3 \psi(0)|PS_\parallel\rangle$. Substitution of (7) into (9) gives,

$$\Delta G = \frac{(n^-)^2}{2}\int dx \frac{1}{x}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle PS_\parallel|\left(F^{\alpha\beta}(0)\right)^\dagger \hat{s}^3 F^{\alpha\beta}(\lambda n)|PS_\parallel\rangle\,, \tag{10}$$

where the factor $1/x$ in the integrand prevents from obtaining the local operator. The similar sum rule for the total angular momentum of the transversely-polarized nucleon, $J_T = \frac{1}{2}$, should read,

$$\frac{1}{2} = L_T + \frac{1}{2}\Delta_T\Sigma + \Delta_T G\,, \qquad \Delta_T\Sigma \equiv \int dx g_T(x)\,, \qquad \Delta_T G \equiv \int dx G_T(x)\,. \tag{11}$$

and, substituting (4) and (7), we find, $\Delta_T\Sigma = (1/M)\langle PS_\perp|\psi^\dagger(0)\hat{s}^\perp\psi(0)|PS_\perp\rangle$, and

$$\Delta_T G = \frac{n^-}{2\sqrt{2}M}\int dx \frac{1}{x}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle PS_\perp|\left(F^{\alpha\beta}(0)\right)^\dagger \hat{s}^\perp F^{\alpha\beta}(\lambda n)|PS_\perp\rangle\ , \tag{12}$$

i.e., the result for the gluon spin contribution is again given by the integral of the bilocal operator. Apparently, the two matrix elements of the local operators for $\Delta\Sigma$ and $\Delta_T\Sigma$ are related by the space rotation in the nucleon rest frame. We note that the above formulas are obtained for the fast-moving nucleon. For the quark spin contributions, we can immediately derive the similar formulas in the rest frame with $\boldsymbol{P}=0$: $\Delta_T\Sigma$ is given by the same formula as above, while $\Delta\Sigma = (1/M)\langle PS_\parallel|\psi^\dagger(0)\hat{s}^3\psi(0)|PS_\parallel\rangle$. Thus, rotation symmetry allows us to conclude $\Delta\Sigma = \Delta_T\Sigma$. Now, the remaining question is whether ΔG and $\Delta_T G$ are equal or not. This is a nontrivial question, because both ΔG and $\Delta_T G$ are given as matrix elements of the nonlocal operators depending explicitly on a fixed light-like vector n^μ, as in (10), (12).

To analyze this problem, we treat the bilocal operators in (8), corresponding to $G_{3E}(x)$ and $G_{3H}(x)$, based on the operator product expansion, which manifestly satisfies Lorentz as well as rotation symmetry. The operator product expansion for $G_{3E}(x)$ is obtained in [7] recently, and allows us to obtain the formula, $G_{3E}(x) = \int_x^1 dy \frac{\Delta G(y)}{y} +$ [genuine twist-three]; here, the first term corresponds to the Wandzura-Wilczek contribution as an integral of the gluon helicity distribution $\Delta G(x)$, and the second term denotes the genuine twist-three contributions which are expressed by certain integrals of the three-gluon correlations $\sim \langle PS_\perp|F^{+\perp}F^{+\perp}F^{+\perp}|PS_\perp\rangle$ and the quark-gluon three-body correlations $\sim \langle PS_\perp|\bar{\psi}F^{+\perp}\psi|PS_\perp\rangle$. On the other hand, the bilocal operators for $G_{3H}(x)$ in (8) prove to have appeared in the intermediate stage of the operator product expansion (of the flavor-singlet part) of the structure function $g_2(x,Q^2)$, which is the twist-three structure function in the DIS of the transversely-polarized nucleon off the longitudinally-polarized lepton. Using the results in [8, 4, 9] for the operator product expansion of the corresponding flavor-singlet part, we find that $G_{3H}(x)$ is given solely by the genuine twist-three contributions in terms of $\langle PS_\perp|F^{+\perp}F^{+\perp}F^{+\perp}|PS_\perp\rangle$, $\langle PS_\perp|\bar{\psi}F^{+\perp}\psi|PS_\perp\rangle$. It is straightforward to see that the first moment of those genuine twist-three contributions vanishes, so that we obtain, $\int dx G_{3E}(x) = \Delta G$ [7], and $\int dx G_{3H}(x) = 0$ [6]. As a result, we find, $\Delta_T G = \Delta G$, similarly as the quark spin contributions. This result coincides with the gluon spin contributions calculated in [7] using gauge-invariant decomposition of the QCD angular momentum tensor into the quark/gluon contributions (see [10]).

We finally mention about the orbital angular momentum contribution to the spin sum rules. For the longitudinally-polarized case (9), the orbital angular momentum contribution can be further decomposed into the well-defined quark and gluon contributions in a frame- and model-independent way as $L = L_q + L_g$, but in many ways as discussed by many authors [10, 11]. On the other hand, for the transversely-polarized case (11), the corresponding quark/gluon contributions $(L_T)_{q,g}$, such that $L_T = (L_T)_q + (L_T)_g$, receive the terms $\bar{C}_{q,g}P^3/[2(P^0+M)]$, respectively, which are frame dependent [7]; here $\bar{C}_{q,g}$ arise in matrix element of the QCD angular momentum tensor $\mathscr{M}^{\lambda\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}$, using the well-known parameterization of the off-forward matrix element of the (Belinfante-improved) energy-momentum tensor of quarks/gluons,

$$\langle P'S'|T^{\mu\nu}_{q,g}|PS\rangle = \bar{u}(P',S')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \bar{C}_{q,g}Mg^{\mu\nu}\right]u(P,S)\ , \tag{13}$$

with $\bar{P}^\mu = \frac{1}{2}(P^\mu + P'^\mu)$, $\Delta^\mu = P'^\mu - P^\mu$. The frame-dependent terms disappear in L_T because $\bar{C}_q + \bar{C}_g = 0$. Thus, we do not have a well-defined decomposition of the orbital angular momentum contribution L_T into the quark and gluon contributions for the transversely-polarized case (11).

To summarize, we have given a QCD definition of the transverse-spin gluon distribution function based on the spin-operator representation for bilocal operator definitions of the PDFs. This definition has the structure of the quantum mechanical expectation value in the spin space, $\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\Phi^\dagger(0)\hat{O}\Phi(\lambda n)|PS\rangle$ with $\hat{O} = 1, \hat{\mathbf{s}}$, in a unified form for the quark ($\Phi = \psi$) and gluon ($\Phi = F^{\mu\nu}$) distribution functions. For both quark and gluon cases, $\hat{O} = 1$ leads to the density PDFs, $\hat{O} = \hat{s}^3$ leads to the helicity PDFs, and $\hat{O} = \hat{s}^\perp$ leads to the transverse-spin PDFs associated with helicity-flip by one unit. We have shown that the new transverse-spin distribution function, $G_T(x)$, is given as the sum of the two twist-three gluon distribution functions, $G_{3E}(x)$ and $G_{3H}(x)$, which arise in the most general decomposition of the gluonic correlator, (3). The operator product expansions relevant to these new gluon distributions are available, and allow us to show that the first moments of $\Delta G(x)$ and $G_T(x)$, which respectively represent the gluon spin contributions to the nucleon spin for the longitudinally- and transversely-polarized cases, are equal. The operator product expansion can be exploited also to analyze the higher moments, $\int dx x^{n-1} G_T(x)$ [6]. Finally, to seek hard processes which allow direct access to $G_T(x)$ is an interesting future problem.

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