Rate and uncertainty of the ^{18}Ne(α,p)^{21}Na reaction from Monte-Carlo simulations

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The reaction rate $N_A \langle \sigma v \rangle$ of the ^{18}Ne(α,p)^{21}Na reaction has been calculated from resonance energies and resonance strengths using the Monte-Carlo technique. This results in a recommended reaction rate with a well-constrained uncertainty which is below a factor of 2 in the most relevant temperature range of $0.5 \leq T_9 \leq 2$. The overall uncertainty includes contributions from the resonance energies, resonance strengths, and for the first time from uncertain J^π assignments. The new rate is based on experimental resonance energies and calculated resonance strengths which have been completely re-analyzed, leading to slightly lower strengths than in previous studies. The new recommended reaction rate is close to previous results which had to be taken as a compromise from discrepant results of different indirect approaches.

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1. Introduction

The ^{18}Ne(α,p)^{21}Na reaction is a possible route for the outbreak from hot CNO cycles to the rapid proton capture (rp) process [1, 2]. Typical temperatures are of the order of Giga-Kelvins (in usual notation: $T_9 \approx 1 - 2$). The astrophysical reaction rate $N_A\langle\sigma v\rangle$ of the ^{18}Ne(α,p)^{21}Na reaction is essentially defined by the reaction cross section at energies around $E \approx 1 - 3$ MeV which corresponds to excitation energies $E^* \approx 9 - 11$ MeV in the ^{22}Mg compound nucleus. It has been shown that the reaction rate $N_A\langle\sigma v\rangle$ is given by the sum over resonant contributions which can be calculated in good approximation using the simple narrow-resonance formalism [3].

In the recent years three approaches have been followed to determine the reaction rate of the ^{18}Ne(α,p)^{21}Na reaction. A direct measurement of resonance strengths was attempted by Groombridge *et al.* [4], and two indirect approaches used a combination of experimental data and theoretical considerations. The reverse ^{21}Na(p,α)^{18}Ne reaction was measured in [5, 6]; here the obtained result has to be corrected for excited state contributions. Alternatively, experimental resonance energies from excitation energies in the ^{22}Mg compound nucleus were combined with J^π assignments and theoretical α-particle partial widths Γ_α because for most of the levels the total width Γ is practically identical with the proton partial width Γ_p and thus the resonance strength $\omega\gamma_{\alpha p}$ can be approximated by $\omega\gamma_{\alpha p} \approx (2J+1)\Gamma_\alpha$. Unfortunately, it was found that the direct approach is in strict contradiction to the indirect approaches. The two indirect approaches have relatively large uncertainties of about a factor of two, but are discrepant by a factor of three, and the previously recommended rate had to be taken from a compromise of the indirect approaches. For a detailed discussion see [3].

In the present study we improve the indirect approach which uses experimental resonance energies and theoretical Γ_α. For this purpose we use the best available excitation energies from a high-resolution ^{24}Mg(p,t)^{22}Mg experiment [7] in combination with recently improved J^π assignments [8, 9]. All Γ_α were re-analyzed assuming the same α-particle reduced widths θ_α^2 in the ^{22}Mg and ^{22}Ne mirror nuclei when spectroscopic information in ^{22}Ne is available from the ^{18}O(^{6}Li,d)^{22}Ne α-transfer [10] or the ^{18}O(α,γ)^{22}Ne α-capture reactions [10, 11, 12, 10, 13].

2. Reaction Rates from Monte-Carlo calculations

The reaction rate $N_A\langle\sigma v\rangle$ of the ^{18}Ne(α,p)^{21}Na reaction is calculated using the Monte-Carlo technique which is presented in detail in [14, 15, 16, 17]. The Monte-Carlo approach allows to take into account the uncertainties of all parameters which in the present case are the resonance energies E and the resonance strengths $\omega\gamma_{\alpha p}$. It turns out that the resulting uncertainty of $N_A\langle\sigma v\rangle$ is dominated by the uncertainties in the resonance strengths $\omega\gamma_{\alpha p}$.

The resonance strengths $\omega\gamma_{\alpha p}$ and α-particle partial widths Γ_α were determined from reduced widths θ_α^2 in the ^{22}Ne mirror nucleus. In cases where no spectroscopic information was available, a Monte-Carlo sampling of Γ_α was made using a Porter-Thomas distribution of Γ_α and an average reduced width $\theta_\alpha^2 = 0.03 \pm 0.01$ from a recent systematic study of reduced widths [18].

The Monte-Carlo technique of [14, 15, 16, 17] was extended to take into account uncertain spin and parity assignments. In these cases a discrete probability distribution was used where each

possible J^π was assigned with a probability $p(J^\pi)$ with $\sum_i p_i(J^\pi) = 1$. The resulting reaction rate $N_A \langle \sigma v \rangle$ and its uncertainties are shown in Fig. 1.
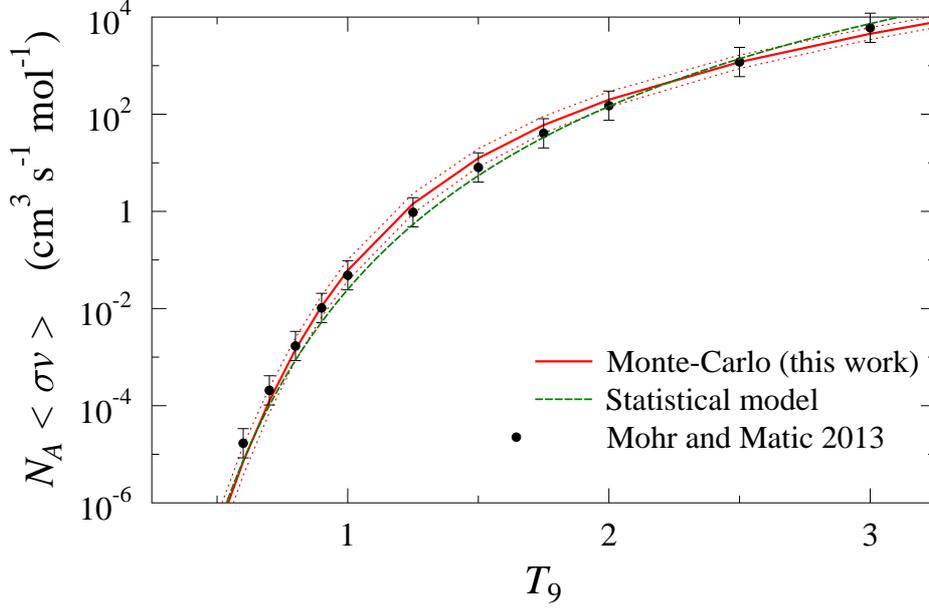

Figure 1: Reaction rate $N_A \langle \sigma v \rangle$ of the $^{18}\text{Ne}(\alpha,p)^{21}\text{Na}$ reaction from Monte-Carlo calculations (red, lower and upper rate red dotted), compared to a calculation in the statistical model (green dashed, [20]) and to the previous recommendation by Mohr and Matic [3] (shown as data points).

The recommended rate is taken from the median result of the Monte-Carlo samples, and the lower (upper) rate is taken from the 16th (84th) percentile of the Monte-Carlo samples. The new result is in good agreement with the previous recommendation [3] but provides reliable uncertainties instead of a reasonable estimate before. In addition, it should be kept in mind that the previous recommendation was a compromise between a relatively high rate calculated from experimental resonance energies E and theoretical resonance strengths $\omega\gamma_{\alpha p}$ and a relatively low rate derived from reverse reaction data [5, 6]. The new result is now based on the best available information on the resonance strengths $\omega\gamma_{\alpha p}$. The discrepancy to the reverse reaction data has been reduced significantly, thus strengthening the present approach. Further information on the new Monte-Carlo rate determination is given in [19]. Finally, it is noted that the statistical model prediction for the reaction rate $N_A \langle \sigma v \rangle$ (taken from [20]) is of the correct order of magnitude although the statistical model by definition is not able to reproduce the detailed shape of the reaction cross section wich is governed by about 30 resonances in the energy interval under study.

In the following we present some technical details of the Monte-Carlo calculations in [19]. For each temperature T, 10,000 samples were calculated resulting in a distribution of reaction rates $N_A \langle \sigma v \rangle$. Four examples are shown for the temperatures $T_9 = 0.6$, 1, 1.5, and 2 (see Fig. 2). Because the underlying resonance strengths $\omega\gamma_{\alpha p}$ follow a lognormal distribution, the resulting distributions of $N_A \langle \sigma v \rangle$ are also approximately lognormal and not Gaussian distributed. In analogy to the 1σ uncertainty of the Gaussian distribution which covers 68 %, we give lower and upper rates from the 16th and 84th percentile of the 10,000 Monte-Carlo samples.
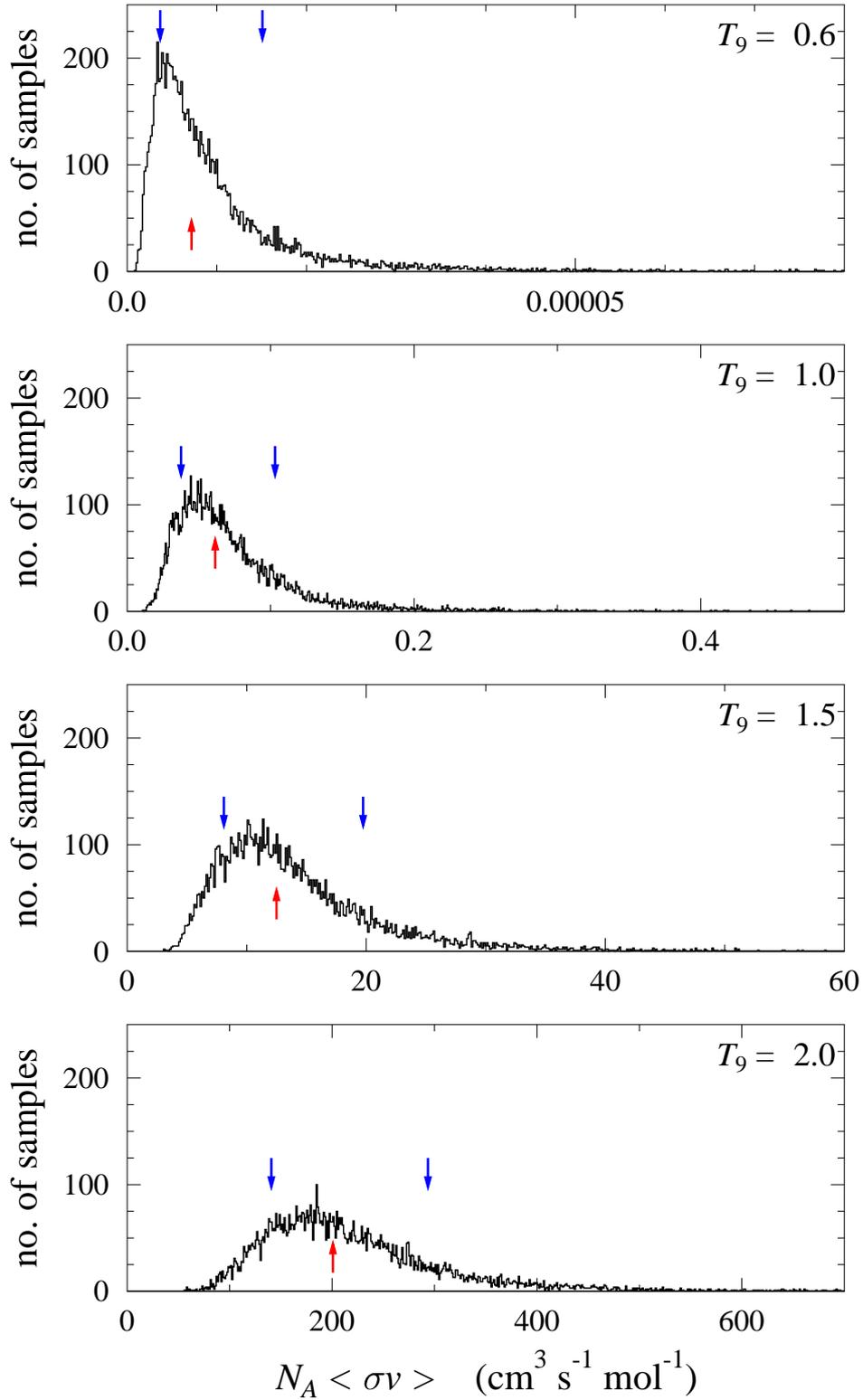
Figure 2: Distribution of reaction rates $N_A\langle\sigma v\rangle$ for the temperatures $T_9 = 0.6$, 1, 1.5, and 2 (top to bottom) from 10,000 Monte-Carlo samples. The median values and the lower and upper rates (defined as 16th and 84th percentile of the 10,000 samples) are marked by vertical red and blue arrows.

The numerical stability of the Monte-Carlo results was tested at $T_9 = 1.5$ by 10-fold repetition of the Monte-Carlo sampling. The result is shown in Fig. 3. It is found that the results obtained from 10,000 Monte-Carlo samples are sufficiently stable. The following numbers for $N_A \langle \sigma v \rangle$ are given in $\mathrm{cm}^3 \, \mathrm{s}^{-1} \, \mathrm{mol}^{-1}$ (without explicitly repeating the unit). The median value varies between 12.40 and 12.65 with an average of the 10 repetitions of 12.52. The lower rate has an average of 8.17; the 10 repetitions vary between 8.09 and 8.26. The corresponding numbers for the high rate are 19.76 (average) and a range of 19.56 to 19.94. The achieved statistical uncertainty is about 1 % which is practically negligible compared to the uncertainty of the reaction rate $N_A \langle \sigma v \rangle$ which is of the order of a factor 1.5 for $T_9 = 1.5$ and slightly larger at lower temperatures.
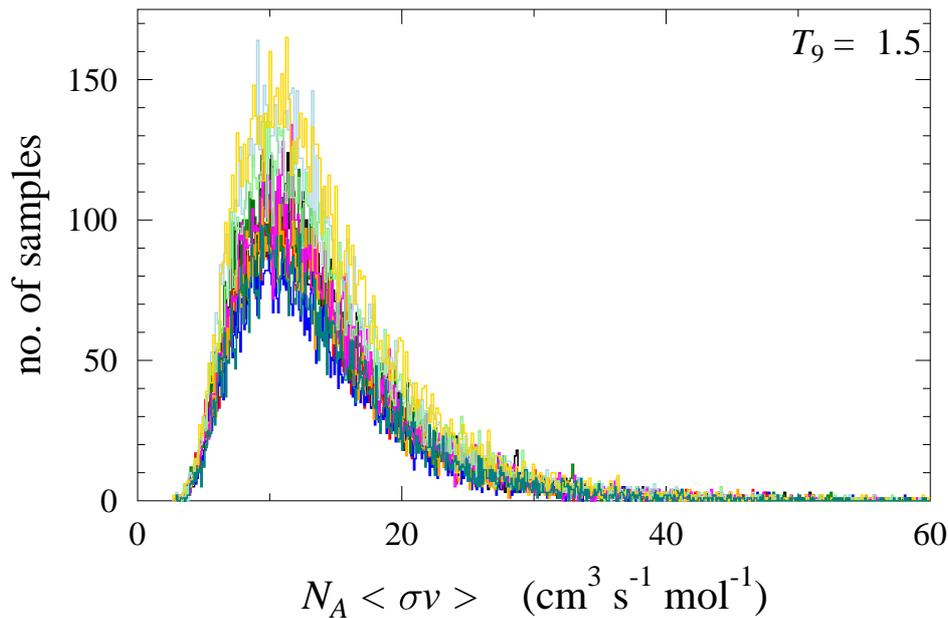

Figure 3: Distribution of reaction rates $N_A \langle \sigma v \rangle$ for the temperature $T_9 = 1.5$: the 10,000 Monte-Carlo samples were repeated 10 times. Each of the 10 calculations is shown in a different color. The resulting median values and low and high rates (16th and 84th percentile) are stable within about 1 %.

3. Summary and Outlook

The reaction rate $N_A \langle \sigma v \rangle$ of the $^{18}Ne(\alpha,p)^{21}Na$ reaction has been determined from Monte-Carlo calculations. The calculations are based on a re-determination of all resonance strengths $\omega \gamma_{\alpha p}$ and take into account uncertainties of the resonance energies E and resonance strengths $\omega \gamma_{\alpha p}$. In addition, the uncertainties from tentative J^π assignments were taken into account for the first time.

The new result is close to the previous recommendations [3, 9]. However, the previous recommendations had to be a compromise between discrepant results from different indirect approaches. The present result reduces these discrepancies and brings the indirect approaches closer together, thus strengthening the reliability of the indirectly determined reaction rate.

Finally, it is interesting to note that a theoretical prediction of the reaction rate $N_A \langle \sigma v \rangle$ [20] using the standard α-nucleus potential of McFadden and Satchler [21] provides the correct order of

magnitude for the light ^{18}Ne target although the the statistical model obviously cannot reproduce the energy dependence of the cross section $\sigma(E)$ which is governed by about 30 resonances in the astrophysically relevant energy range. Contrary to this very reasonable result for ^{18}Ne, the statistical model typically overestimates α-induced cross sections for heavy targets at low energies. This typical behavior for heavy targets is found at least down to ^{58}Ni [22] and ^{64}Zn [23]. Contrary to this typical behavior, the statistical model slightly underestimates the ^{44}Ti$(\alpha,p)^{47}$V cross section [24, 25] and dramatically underestimates the ^{33}S$(\alpha,p)^{36}$Cl [26, 27] and ^{23}Na$(\alpha,p)^{26}$Mg [28] cross sections, but works reasonable again for ^{18}Ne. This indicates that α-induced reaction cross sections for targets with $20 \leq A \leq 50$ may be more uncertain than estimated before.

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