

## Shell-model studies of $^{22}\text{Mg}$ at excitations of interest for the thermonuclear radiative capture reactions

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A comparison of the structure properties of the  $^{22}\text{Ne}$  and  $^{22}\text{Mg}$  mirror nuclei is interesting because in nucleosynthesis  $^{22}\text{Mg}$  can be formed through the capture reaction  $^{21}\text{Na}(p,\gamma)$  for which the cross sections will depend on spin-parity assignments of the  $^{22}\text{Mg}$  states around the proton-emission threshold. We propose, based on shell model calculations using the  $(0+1)\hbar\omega$  PSDPF interaction, a one to one level correspondence between  $^{22}\text{Ne}$  and  $^{22}\text{Mg}$ . In particular what the negative parity states are concerned, three states are identified in  $^{22}\text{Ne}$ :  $2^-$  at 5.146 MeV,  $3^-$  at 5.910 MeV and  $0^-$  at  $\sim 6.234$  MeV, they correspond to the mirror states in  $^{22}\text{Mg}$ :  $2^-$  at 5.006 MeV,  $3^-$  at 5.838 MeV and  $0^-$  at 6.046 MeV.

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1

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## 1. Introduction

In the last decade, great interest is focused on the structure of the radionuclide  $^{22}\text{Mg}$  because it plays a crucial role in determining the astrophysical reaction rates of  $^{21}\text{Na}(p,\gamma)^{22}\text{Mg}$  and  $^{18}\text{Ne}(\alpha,p)^{21}\text{Na}$  reactions in explosive stellar scenarios [1]. Such processes can occur in classical novae, like in ONe ones, predominantly by nuclear activity which takes place in the NeNa cycle. In fact, nucleosynthesis in the NeNa cycle during nova outburst leads to the synthesis of the astronomically important unstable  $^{22}\text{Na}$  nucleus. This nuclide  $\beta$  decays ( $T_{1/2}=2.6$  yr) with the emission of a characteristic  $\gamma$  ray of energy 1.275 MeV.  $^{22}\text{Na}$  is thought to be produced primarily via two reaction paths [2]; the first is the so-called “cold” NeNa cycle,  $^{20}\text{Ne}(p,\gamma)^{21}\text{Na}(\beta^+, \nu_e)^{21}\text{Ne}(p,\gamma)^{22}\text{Na}$ , the second is the so-called “hot” NeNa cycle, which is associated with higher temperatures,  $^{20}\text{Ne}(p,\gamma)^{21}\text{Na}(p,\gamma)^{22}\text{Mg}(\beta^+, \nu_e)^{22}\text{Na}$ . In the  $^{21}\text{Na}+p$  compound system, three resonances at  $Ex = 5.714, 5.837, 5.962$  MeV had been thought to contribute to the reaction rate at ONe novae temperatures. Higher energy states at  $Ex = 6.046, 6.248, 6.323, 6.587$  and  $6.615$  MeV, in addition to the state at  $5.962$  MeV may contribute to the  $^{21}\text{Na}(p,\gamma)^{22}\text{Mg}$  rate for X-ray burst events [3].

In this context, properties of  $^{22}\text{Mg}$  levels in the excited energy region below and around the proton-emission threshold of 5.5 MeV have extensively been investigated by many reaction channels (see Ref.[1] and references therein). However, there is still uncertainty in the determination of their spins and parities [4]. A source for spin-parity assignments are the mirror states in  $^{22}\text{Ne}$ , which is a stable nucleus and has thus been well studied experimentally [4]. Shell model (SM) calculations are certainly of use to define and predict spin-parity values.

We present in this paper, a comparison between the mirror experimental spectra of  $^{22}\text{Mg}$  and  $^{22}\text{Ne}$  as well as the SM predictions obtained by our  $(0+1)\hbar\omega$  PSDPF interaction.

## 2. Experimental status

Several experiments using different reactions were performed in order to explore the spectroscopic properties of  $^{22}\text{Mg}$ . Nevertheless, there is still ambiguity in the determination of spin-parity assignments of several levels mainly those of astrophysical interest. The experimental spectrum of this nucleus is still not complete and a comparison with the well-studied mirror nucleus  $^{22}\text{Ne}$  as well as with SM calculations is indicated.

The  $^{22}\text{Ne}$  spectroscopy has been extensively studied experimentally. Below an excitation energy of  $\sim 6.35$  MeV, fourteen states are reported in the “Adopted Levels” [4] of NNDC with well-defined spin and parity. Twelve of them have positive parity and two of them have negative parity. On the other hand, the mirror nucleus  $^{22}\text{Mg}$  has, in the same excitation energy range, sixteen states reported in Refs. [4,5], seven of them have no fixed spin and parity. Many attempts have been made to determine the  $J^\pi$  values of the reported states at 5.006, 5.293, 5.296, 5.318, 5.452, 5.838 and 6.045 MeV. The level at 5.006 MeV is suggested to have positive parity [6], other experiments disregarded its existence [7,8]. The different proposed  $J^\pi$  values of the state at 5.293 MeV, are  $3^+$  [6,9],  $3^-$  [7],  $(2^+, 3^-)$  [3,10] and  $4^+_2$  [8,11], this state is considered to be degenerated with that at 5.296 MeV with  $J^\pi = 2^-$  [8,11]. The  $1^+_1$  state in  $^{22}\text{Mg}$  is assumed to be at

5.318 MeV [6,7,9]. For the state at 5.452 MeV, observed in other experiments at 5.454, 5.455 and 5.464 MeV, the following suggestions have been made  $J^\pi = 4^+_{2}$  [6,7,9],  $(2^+, 3^-, 4^+)$  [3,10] and  $3^+$  [8,11]. While a positive parity is assumed for the level at 5.838 MeV with  $J = 2-4$  [3,10], or 3 [7], there exists also a  $J^\pi = 3^-$  proposal [6,9]. There is a long debate about the  $J^\pi$  attribution for the degenerated states at 5.962 and 6.045 MeV, some propose  $0^+$  for one and  $1^-$  for the other [2,9,12,13] and vice versa, others suggest  $(0^+, 1^-)$  for the state at 5.962 MeV and  $0^+$  for that at 6.045 MeV [3,10].  $J^\pi = 0^+$  and  $3^-$  values are also attributed to the levels at 5.962 MeV and also 6.045 MeV, respectively [8,11], but others attribute a  $0^+$  to the 5.962 MeV [6] and to the 6.045 MeV states [7,14]. The  $J^\pi$  suggested from the isobaric triplet component ( $T=1$ ) in  $A=22$  nuclei [5] for the excitation energies at 5.006, 5.318 and 5.962 MeV are  $2^-$ ,  $1^+$  and  $0^+$ , respectively.

In this paper, we will propose a complete spectrum of  $^{22}\text{Mg}$  up to excitation energy of  $\sim 6.35$  MeV based on a comparison with its mirror  $^{22}\text{Ne}$  and our SM calculations, taking into account the previous studies.

### 3.Theoretical status

To make a 0 and 1  $\hbar\omega$  SM description of sd shell nuclei, we have recently developed the PSDPF interaction [15]. With this new interaction, the spectroscopic properties of positive and negative parity states can be calculated. What the positive parity states are concerned, the main building block of our interaction is USDB [16]. We used the PSDPF interaction to describe energy spectra of both  $^{22}\text{Mg}$  and  $^{22}\text{Ne}$  mirror nuclei. Our main aim is to make predictions for the ambiguous states in  $^{22}\text{Mg}$ .

We performed SM calculations through PSDPF without Coulomb interaction using the SM code Nathan [17,18]. For the pair  $^{22}\text{Ne}$ - $^{22}\text{Mg}$ , PSDPF predicts fifteen states of excitation energies between 0 and 6.35 MeV, three of them have negative parity. We will present a one to one level correspondence between our calculations and the  $^{22}\text{Ne}$ - $^{22}\text{Mg}$  spectra based on the comparison of their excitation energies and electromagnetic transition (EMT) properties. This will allows us to predict the spin-parity assignments of the  $^{22}\text{Mg}$  ambiguous states important in nucleosynthesis.

### 4.Results and discussion

A comparison of the two experimental  $^{22}\text{Ne}$ - $^{22}\text{Mg}$  spectra as well as the PSDPF calculations is shown in Table 1. There is good agreement between our results and the  $^{22}\text{Ne}$  experimental spectrum with an RMS deviation of 125 keV. We confirm here the  $J^\pi$  assignments of the levels at 5.524 MeV and 6.310 MeV, which have  $J^\pi = 4^+_{2}$  and  $6^+_{1}$ , respectively. The well-known states in  $^{22}\text{Mg}$  are also adequately described with PSDPF and the  $J^\pi$  assignments of levels at 6. 254 MeV and 6.247 MeV can be confirmed too as  $6^+_{1}$  and  $4^+_{3}$ , respectively. This allows us to make predictions for the not so well known states in this nucleus with respect to the mirror  $^{22}\text{Ne}$  and calculated spectrum. We start with the  $2^-$  state which is the fifth level in  $^{22}\text{Ne}$ . Previous works, as mentioned before, attributed this  $J^\pi$  to the  $^{22}\text{Mg}$  state at 5.296 MeV, however, Firestone [5] proposed it to the state at 5.006 MeV by comparison within the  $A = 22$  nuclei isobaric triplet component ( $T=1$ ). It is difficult to choose between the two suggestions since the

two states have the same  $\gamma$ -decay [4], which is somewhat different from that of the mirror state in  $^{22}\text{Ne}$ . We propose that the Firestone suggestion is correct, because all the well known levels in  $^{22}\text{Mg}$  are lower in energy, due to Coulomb effects, than those in  $^{22}\text{Ne}$  and the 5.006 MeV is the fifth level as in  $^{22}\text{Ne}$ . We thus think that the existence of the 5.296 MeV is doubtful. The second uncertain state is at 5.293 MeV which is proposed in the literature to have  $(2-4)^+$  and  $3^-$  assignments. This state decays mainly to the  $4^+_1$  level like the  $4^+_2$  in  $^{22}\text{Ne}$ , while the  $2^+$ ,  $3^+$  and  $3^-$  states decay mainly to the  $2^+_1$ . We thus suggest that the 5.293 MeV corresponds to the  $4^+_2$ . The same argument is used to attribute  $J^\pi = 3^+$  to the state at 5.452 MeV. The level at 5.318 MeV is assumed to be the first  $1^+$  state as it has the same  $\gamma$ -decay as the  $1^+_1$  in  $^{22}\text{Ne}$ . We now treat the case of states of astrophysical interest. The first one is at 5.838 MeV for which we suggest a  $3^-$  assignment, this  $J^\pi$  corresponds to the level at 5.910 MeV in  $^{22}\text{Ne}$  and their EMT are very similar. The other astrophysical interesting levels are at 5.962 and 6.045 MeV, for these states no known EMT have been reported, they are degenerated and close in energy with the  $0^+_2$  state in  $^{22}\text{Ne}$ . On the other hand, PSDPF predicts the existence of two degenerated states at 6.273 and 6.288 MeV with  $J^\pi = 0^+$  and  $0^-$ , respectively. We attribute these  $J^\pi$  assignments to the states in question at 5.962 and 6.045 MeV, respectively. Concerning the  $^{22}\text{Ne}$  case, we propose that there is a doublet of  $0^+$  and  $0^-$  at  $\sim 6.234$  MeV.

According to these assumptions, we now propose a complete spectrum for  $^{22}\text{Mg}$ ; the RMS deviation between experimental and calculated spectra is 198 keV.

$^{22}\text{Ne}$		$^{22}\text{Mg}$		PSDPF	
Ex	$J^\pi$	Ex	$J^\pi$	Ex	$J^\pi_n$
0	$0^+$	0	$0^+$	0	$0^+_1$
1.275	$2^+$	1.247	$2^+$	1.363	$2^+_1$
3.358	$4^+$	3.308	$4^+$	3.418	$4^+_1$
4.456	$2^+$	4.402	$2^+$	4.295	$2^+_2$
5.146	$2^-$	5.006	$0^+, 1, 2, 3, 4^+$	4.987	$2^-_1$
5.332	$1^+$	5.318	$2^+, 3, 4^+$	5.408	$1^+_1$
5.363	$2^+$	5.035	$2^+$	5.187	$2^+_3$
5.524	$(4)^+$	5.293	$(3^+, 4^+, 5^+)$	5.473	$4^+_2$
5.641	$3^+$	5.452	$(2^+, 3^+)$	5.468	$3^+_1$
5.910	$3^-$	5.838	$2^+, 3, 4^+$	5.632	$3^-_1$
6.120	$2^+$	5.711	$2^+$	6.243	$2^+_4$
6.234	$0^+$	5.962	$0^+$	6.273	$0^+_2$
		6.046		6.288	$0^-_1$
6.310	$(6)^+$	6.254	$(6^+)$	6.326	$6^+_1$
6.347	$4^+$	6.247	$(4^+)$	6.287	$4^+_3$

**Table 1:** Experimental [4] versus calculated with the PSDPF interaction  $^{22}\text{Ne}$  and  $^{22}\text{Mg}$  energy spectra. The energies are given in MeV.

## 5. Conclusion

The  $^{22}\text{Mg}$  structure is of nuclear astrophysical interest, especially the  $J^\pi$  assignments of levels in the vicinity of the proton threshold. Experimentally, the  $^{22}\text{Mg}$  spectrum is by far not so well established as the one of  $^{22}\text{Ne}$ . A comparison with the well-studied mirror nucleus  $^{22}\text{Ne}$  is thus important as well as with SM calculations using our 0 and 1  $\hbar\omega$  PSDPF interaction. This comparison leads to the proposed existence of a  $0^-$  state in the two mirror nuclei and to a one to one level correspondence of  $^{22}\text{Ne}$  and  $^{22}\text{Mg}$  between 0 and  $\sim 6.35$  MeV of excitation. Finally, we propose a complete spectrum of  $^{22}\text{Mg}$  up to an excitation energy of  $\sim 6.35$  MeV.

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