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Gravitational Redshift In The Post-Newtonian Potential Field: The Schwarzschild Problem

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Considering both the special and general relativistic approximations, we analyze the gravitational redshift in the Schwarzschild problem (i.e. the two body problem associated to a potential of the form $A/r+C/r^3$, where r is the distance between photon and the center of a star, and A,C well-established positive constants). We present the difference between the redshift for Newton's potential and the one for Schwarzschild's potential in the third order terms. In both cases (special and general relativistic approximations) the difference value reaches the ratio γ/R^3, where R is the geometrical radius of the star and $\gamma = C/c^2$, c being speed of the light). In the general relativistic approximation, we show that a black hole effect appears at a ratio noted by us ρ_s which is larger than R_s - Schwarzschild gravitational radius.

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1. Introduction

Taking into account a nonrelativistic law of gravitation, Mioc and Ureche have studied the topic of post-Newtonian potentials and their implications in diverse astronomic experience (see [5]). Roman and Mioc [4] developed the three-body problem study using the Schwarzschild gravitational field. Ureche [6] computed free fall collapse time and gravitational redshift (see [7]) for a post-Newtonian potential, namely Maneff's field. Recently, Lupu [2] extended the applications of the Schwarzschild potential at the study of the satellite dynamics.

In the two body Schwarzschild problem, the force function is:

$$U(r) = m(\frac{A}{r} + \frac{C}{r^3}) = GM\frac{m}{r}(1 + \frac{L^2}{c^2r^2}), \tag{1.1}$$

where $A = GM$, $C = \frac{GML^2}{c^2}$, and G is Newtonian gravitational constant; M, m are the masses of two interacting bodies in the field (e.g. a massive cosmic object and a test particle); r is the distance between M and m; c is the speed of light; L is constant angular momentum (see [3] for details).

In this paper, we extend Ureche's work [7] and analyze the gravitational redshift in the two-body problem using the Schwarzschild gravitational field. In this certain case the Φ potential which describes the effects is:

$$\Phi(r) = -\frac{GM}{r} - \frac{GML^2}{c^2r^3} = -\frac{A}{r} - \frac{C}{r^3}. \tag{1.2}$$

We calculate the gravitational redshift computed in the field described by the potential in the form (1.2) in two frameworks: (i) in a special relativistic approximation, (ii) in a general relativistic approximation. We compare our results with the ones obtained for the Newtonian gravitational field.

2. Special Relativistic Approximation (SRA)

In the framework of the two body problem using the Schwarzschild potential in the form (1.2), we consider a celestial body with M mass and R radius, and a photon at the surface of the body with m_f relativistic mass, λ wavelength (or ν frequency). We use the index 0 for a distant observer. Taking into account the conservation of energy law for the considered photon, we have:

$$m_f c^2 + m_f \Phi = m_{f_0} c^2 + m_{f_0} \Phi_0. \tag{2.1}$$

Also,

$$m_f c^2 = h\nu, \nu = \frac{c}{\lambda}, \tag{2.2}$$

where h is the Planck's constant.

We use the following notations: $\lambda_0 - \lambda = \Delta\lambda$, $z_g = \frac{\Delta\lambda}{\lambda}$ where z_g is the gravitational redshift.

Neglecting Φ_0 in the relation (2.1) and developing this equation until the third order term, we obtain the gravitational redshift of SRA, as follows: Considering the Schwarzschild gravitational field, we obtain:

$$z_{gSch} = \frac{\alpha}{R} + \frac{\alpha^2}{R^2} + \frac{(\alpha^3 + \gamma)}{R^3} \tag{2.3}$$

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(where $\alpha = \frac{A}{c^2}, \gamma = \frac{C}{c^2}$).

In the same time, considering the Newtonian gravitational field we obtain:

$$z_{gN} = \frac{\alpha}{R} + \frac{\alpha^2}{R^2} + \frac{\alpha^3}{R^3}. \tag{2.4}$$

Therefore, computing the difference between z_{gSch} and z_{gN} in SRA, we obtain:

$$\Delta S = z_{gSch} - z_{gN} = \frac{\gamma}{R^3}. \tag{2.5}$$

3. General Relativistic Approximation (GRA)

We continue the previous analysis with the general relativistic approximation, where the metric associated to the potential Φ is:[1]

$$ds^2 = (1+\frac{2\Phi}{c^2})c^2dt^2 - \frac{dr^2}{(1+\frac{2\Phi}{c^2})} - r^2(d\theta^2 + sin^2\theta d\phi^2). \tag{3.1}$$

The relation between the proper time τ and the time t of distant observer is:

$$\tau = \sqrt{1+\frac{2\Phi}{c^2}}t \tag{3.2}$$

and the relation between λ, λ_0 corresponding wavelengths is:

$$\lambda = \sqrt{1+\frac{2\Phi}{c^2}}\lambda_0 \tag{3.3}$$

(λ_0 is the wavelength measured by distant observer).

We have from the formulas (3.1), (3.2) the following existential condition:

$$1+\frac{2\Phi}{c^2} > 0. \tag{3.4}$$

Also, from the form (1.2), we have this form:

$$\frac{2\Phi(r)}{c^2} = -\frac{\alpha}{r} - \frac{\gamma}{r^3}. \tag{3.5}$$

Hence, we must solve the inequality:

$$r^3 - 2\alpha r^2 - 2\gamma > 0. \tag{3.6}$$

According to Viete's formulas for the solutions of this inequalities: ρ_s (the one which is real), x and $\bar{x}$, we obtain the relations:

$$2\gamma + {\rho_s}^2(2\alpha - \rho_s) = 0. \tag{3.7}$$

Because of $\gamma, {\rho_s}^2 > 0$, we obtain the condition for the real solution:

$$2\alpha - \rho_s < 0. \tag{3.8}$$

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Computing z_g based on the relation (3.3) and developing until the third order term, we obtain the gravitational redshift of GRA, as follows: In the frame of Schwarzschild gravitational field, we obtain the Schwarzschild gravitational redshift:

$$z_g{}^{Sch} = \frac{\alpha}{R} + \frac{3}{2}\frac{\alpha^2}{R^2} + (\frac{5}{2}\alpha^3 + \gamma)\frac{1}{R^3}. \quad (3.9)$$

Also, in the Newton gravitational field framework, we compute the Newton gravitational redshift :

$$z_g{}^N = \frac{\alpha}{R} + \frac{3}{2}\frac{\alpha^2}{R^2} + \frac{5}{2}\frac{\alpha^3}{R^3}. \quad (3.10)$$

Then, if we compute the difference between the $z_g{}^{Sch}$ and $z_g{}^N$ in GRA, we obtain:

$$\Delta G = z_g{}^{Sch} - z_g{}^N = \frac{\gamma}{R^3}. \quad (3.11)$$

4. Results

After developing the gravitational redshift until the third order term, we obtain the following computational results:

The differences are the same in both the SRA and GRA, namely:

$$\Delta G = \Delta S = \frac{\gamma}{R^3}. \quad (4.1)$$

Anew, if we take $z_g{}^N \sim \frac{\alpha}{R}$, we obtain the relative difference:

$$\frac{\Delta G}{(z_g{}^N)^3} = \frac{\gamma}{\alpha^3} \quad (4.2)$$

In the general relativistic approximation case, we also obtain a qualitative result: Considering the existence condition (3.4),we obtain from the inequality (3.6) a Black hole effect in the Schwarzschild gravitational field, namely:$r > \rho_s$. From the qualitative condition (3.8), this entity ρ_s is the associated radius of Schwarzschild problem for which

$$\rho_s > 2\alpha = R_s, \quad (4.3)$$

R_s being the Schwarzschild gravitational radius in the Newton gravitational field.

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