

## Microscopic Approach to Alpha-Nucleus Optical Potentials for Nucleosynthesis

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A microscopically based approach for  $\alpha$ -nucleus optical potentials at low incident energies is presented. Using a parametrised  $g$ -matrix a nuclear matter  $\alpha$ -optical potential is formulated. In addition an adapted nuclear structure component which accounts for collective states has been determined. Apart from the Skyrme-based RPA-calculations of the collective states the approach makes consistent use of the  $g$ -matrix as an effective interaction. Comparing with experimental elastic scattering data indicates the applicability of the approach for reaction calculations in nucleosynthesis.

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## 1. Introduction

The observed abundances of elements in the universe are a direct consequence of the properties of nuclei and the interplay of fundamental forces governing the stellar evolution [1]. A detailed knowledge of the relevant nuclear processes is therefore one basic prerequisite to understand the mechanisms of nucleosynthesis. Among the different processes of nucleosynthesis the generation of p-nuclei (see e.g. [2]), which takes place in explosive scenarios, is subject of intensive research in recent years. The so-called p-nuclei comprise 35 proton-rich nuclei between  $^{74}\text{Se}$  and  $^{196}\text{Hg}$  and are generated via a complex network of nuclear reactions. In this network nuclear reactions involving  $\alpha$ -particles, e.g.  $(\alpha, \gamma)$ -,  $(\alpha, n)$ - and  $(\alpha, p)$ -reactions, play an important role. The experimental determination of the corresponding reaction cross sections at the typical energies of the stellar environment is challenging.

Reaction calculations involving  $\alpha$ -particles require the knowledge of  $\alpha$ -nucleus optical potentials which are not well known for medium and heavy nuclei at low  $\alpha$ -energies [3]. The  $\alpha$ -nucleus optical potentials are usually determined via a fit of a parametrized potential to reproduce elastic differential cross sections. However, at energies below the Coulomb barrier cross sections are small and experimental elastic cross section data for medium and heavy nuclei are scarce. Therefore,  $\alpha$ -nucleus optical potentials are usually obtained via extrapolation from high to low energies. Due to the extrapolation procedure these  $\alpha$ -nucleus optical potentials have unknown uncertainties and thus an uncontrollable impact on the calculations of reaction rates occur.

In this contribution we present consistent calculations of  $\alpha$ - $^{40}\text{Ca}$  optical potential within the nuclear matter [4, 5] and the nuclear structure approach [6, 7]. We extend the nuclear matter approach to  $\alpha$ -nucleus optical potentials in section 2.1 and perform calculations for the  $\alpha$ - $^{40}\text{Ca}$  system. In section 2.2 we revive the nuclear structure approach of Osterfeld et al. [8] and perform evaluations of the imaginary part of the  $\alpha$ - $^{40}\text{Ca}$  optical potential. The capability of these almost parameter-free approaches to predict elastic  $\alpha$ -nucleus scattering cross sections is studied in sec. 3. In section 4 a summary and an outlook of future efforts is given.

## 2. Microscopic Approaches to $\alpha$ -Nucleus Optical Potentials

Optical Potentials are effective one-body interactions between collision partners and are basic ingredients of most reaction calculations. They are complex-valued and provide in their simple form an explicit description of elastic scattering while non-elastic processes are taken into account globally. Within many-body field theory the nucleon-nucleus optical potential can be identified as the mass operator of the one-particle Green function [9]. This definition includes all exchange effects and an exact calculation would require a complete solution of the many-body problem. Consequently, the mass operator can only approximately be determined. Two approximations, i.e. the *Nuclear Matter Approach* [4, 5] and the *Nuclear Structure Approach* [6, 7], are the microscopic basis of our procedure and their adaption to  $\alpha$ -nucleus scattering is discussed in the next subsections.

### 2.1 Adaption of the Nuclear Matter Approach

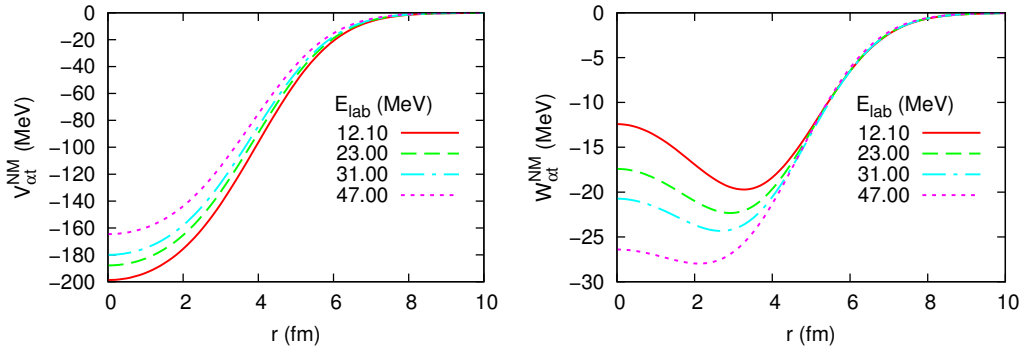
Within the nuclear matter approach one makes use of Brückner-Hartree-Fock theory [11] to

evaluate a microscopically-based effective nucleon-nucleon interaction. Especially, we refer to the work of Jeukenne, Lejeune and Mahaux [4] in which the nuclear matter optical potential was calculated and applied to finite nuclei by means of the local density approximation (LDA). In a further step a parametrisation of the nuclear matter optical potential was generated [12]. This parametrisation has simplified the generation of semi-microscopic nucleon-nucleus optical potentials based on the nuclear matter approach. Avoiding the drawbacks of this work, Bauge et al. [13] presented a parametrisation with a wide range of applicability from  $A = 30$  to 240 and implemented it into the MOM code package [14]. Using the proper target density the spherical proton- and neutron-nucleus optical potentials  $V_{pt}(r; E_N)$ ,  $V_{nt}(r; E_N)$  are calculated at a given nucleon energy  $E_N$ .

In order to obtain the  $\alpha$ -nucleus optical potential the neutron- and proton-nucleus optical potentials were folded with the  $\alpha$ -particle density  $\rho_\alpha(\mathbf{r})$ ,

$$V_{\alpha t}^{NM}(r; E_\alpha) = \int d^3 r' \rho_\alpha(\mathbf{r}') \frac{1}{2} [V_{nt}^{NM}(r'; E_{\alpha/4}) + V_{pt}^{NM}(r'; E_{\alpha/4})], \quad (2.1)$$

where  $\rho_\alpha$  was taken as a sum of Gaussian using the parameters given in [15]. The real part ( $V_{\alpha t}^{NM}$ ) and the imaginary part ( $W_{\alpha t}^{NM}$ ) of the nuclear matter  $\alpha$ - $^{40}\text{Ca}$  optical potential are displayed in Fig. 1.



**Figure 1:** The real part (left) and the imaginary part (right) of the  $\alpha$ - $^{40}\text{Ca}$  optical potential within the nuclear matter approach at different incident energies of the  $\alpha$ -particle.

## 2.2 Nuclear Structure Approach for $\alpha$ -Scattering

The nuclear matter approach for nucleon-nucleus optical potentials accounts essentially for particle-hole excitations. At low energies ( $\leq 30$  MeV) the impact of correlations becomes important. Therefore we evaluated contributions of collective states to the imaginary part of the optical potential with the nuclear structure approach formulated by Osterfeld et al. [8]. Assuming the  $\alpha$ -particle to be a fundamental particle, exchange terms vanish and the imaginary part  $W(\mathbf{r}, \mathbf{r}')$  is simply given [8, 16] by the sum over intermediate states,

$$W^{NS}(\mathbf{r}, \mathbf{r}') = \text{Im} \left[ \sum_{N, \ell, \ell_c, L} \delta_{L, J_N} \frac{\hat{\ell}^2 \hat{\ell}_c^2}{16\pi^2 \hat{L}^2} \langle \ell 0 \ell_c 0 | L 0 \rangle^2 F_{J_N, LOL}^{dir}(r) g_{\ell_c}(r, r'; E - E_x^N) F_{J_N, LOL}^{dir}(r') P_\ell(\cos\theta) \right], \quad (2.2)$$

where  $\hat{\ell} = \sqrt{2\ell + 1}$  and the Green's function  $g_{\ell_c}(r, r'; E - E_x^N)$  describes the propagation of the  $\alpha$ -particle while the target is excited to the  $N - th$  intermediate state with angular momentum  $J_N$

and excitation energy  $E_x^N$ . Describing the intermediate target states via RPA, the direct transition formfactor and the nuclear transition density are given by

$$F_{J_N,LOL}^{dir}(r) = \int dr_a r_a^2 \rho_{LOL}^{J_N}(r_a) v_L(r, r_a), \quad (2.3)$$

$$\rho_{LSJ}^{J_N}(r) = \sum_{n_1, \ell_1, j_1} \sum_{n_2, \ell_2, j_2} \left[ X_{j_1 j_2}^{J_N} + Y_{j_1 j_2}^{J_N} \right] \frac{\hat{s}_1 \hat{S}_{j_1} \hat{j}_2 \hat{L} \hat{J}}{\sqrt{4\pi}} \langle \ell_1 0 L 0 | \ell_2 0 \rangle \begin{Bmatrix} \ell_2 & s_2 & j_2 \\ \ell_1 & s_1 & j_1 \\ L & S & J \end{Bmatrix} R_{n_1, \ell_1, j_1}(r) R_{n_2, \ell_2, j_2}(r). \quad (2.4)$$

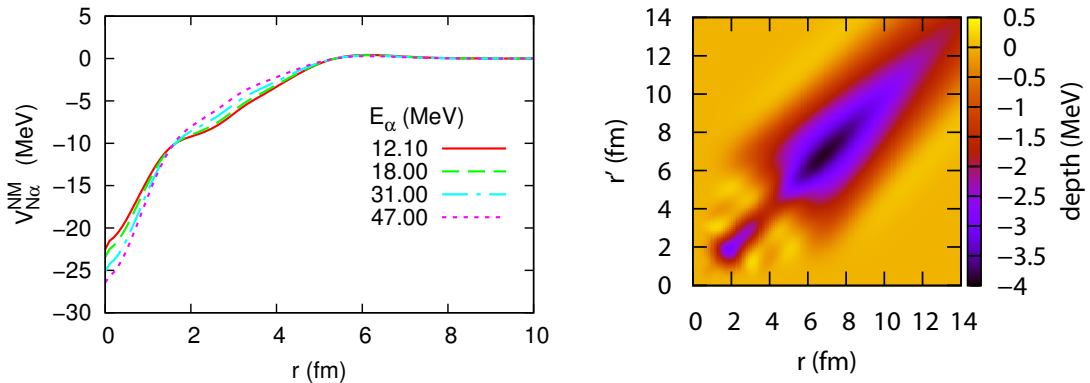
The transition density depends on the RPA particle-hole amplitudes,  $X_{j_1 j_2}^{J_N}$ ,  $Y_{j_1 j_2}^{J_N}$ , of the N-th excitation and the corresponding radial single particle wave functions,  $R_{n_1, \ell_1, j_1}$ ,  $R_{n_2, \ell_2, j_2}$ . The coupling between the ground and the intermediate states is mediated by the  $\alpha$ -nucleon potential,  $V_{\alpha N}$ , which enters into  $F^{dir}$  via its multipole expansion

$$v_L(r_1, r_2) = (4\pi)^2 \int_0^\infty dq q^2 \tilde{V}_{\alpha N}(q) j_{\ell_1}(qr_1) j_{\ell_2}(qr_2). \quad (2.5)$$

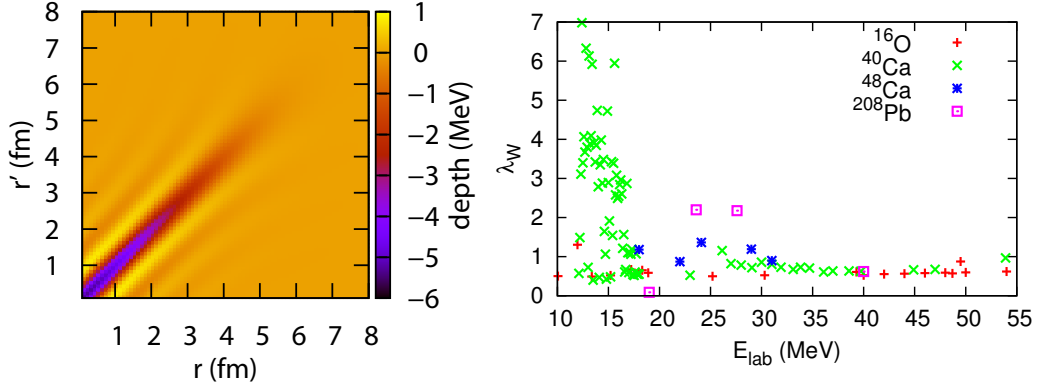
Here,  $\tilde{V}_{\alpha N}$  denotes the Fourier transformation of  $V_{\alpha N}(r)$ .

With regard to a consistent use of the NN interaction, we determine  $V_{\alpha N}$  from the real part of  $V_{\alpha t}^{NM}$  via defolding with the target density and simultaneously smoothing the long range behaviour with Gaussian terms. Typical results for  $V_{\alpha N}$  and its multipole expansion are displayed in Fig. 2.

The target structure is described by RPA-states evaluated with the program HOSPHE [17] which is based on Skyrme interactions. We include 25 oscillator shells and assume the single-particle states  $R_{n\ell j}(r)$  as oscillator wave functions with a width parameter  $b = 45A^{-1/3} - 25A^{-2/3}$ . Thus we can evaluate the transition density  $\rho_{LOL}^{J_N}(r)$  via Eq. (2.4) and subsequently the direct transition formfactor  $F_{J_N,LOL}^{dir}$  using the evaluated  $V_{\alpha N}$ . Finally, the imaginary part of the optical potential at a given  $E_\alpha$  is determined by summing over all energetically open intermediate states. Results for  $^{40}\text{Ca}$  are shown in Fig. 3. For all multipoles the potential is peaked at  $\mathbf{r}' \approx \mathbf{r}$ . Hence an equivalent local description as outlined in [8] should provide a fair description.



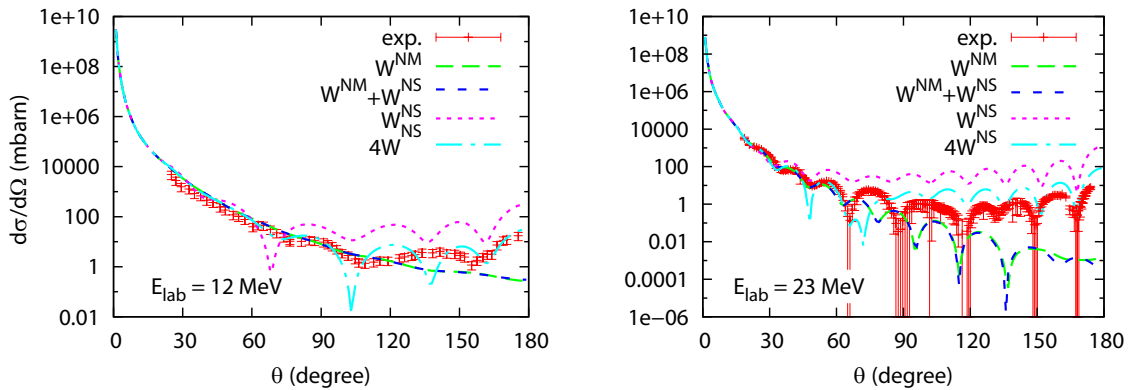
**Figure 2:** The effective  $\alpha$ -nucleon potential (left) and its  $L = 4$  multipole term at  $E_{\text{lab}} = 23$  MeV (right) derived from the real part of the  $\alpha$ - $^{40}\text{Ca}$  optical potential within the nuclear matter approach.



**Figure 3:** Left: The total imaginary part  $W^{NS}$  of the  $\alpha$ - $^{40}\text{Ca}$  optical potential at  $E_{\text{lab}} = 23$  MeV and  $\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}' = 0$ . Right: Required scaling factor  $\lambda_W$  for the imaginary part of the nuclear matter optical potential to achieve best agreement with experiment.

### 3. Implementation and Results

In a first step we studied the applicability of  $V^{NM}(r; E_\alpha)$  for the description of elastic scattering cross sections at different energies for  $^{16}\text{O}$ ,  $^{40}\text{Ca}$ ,  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$ . Similarly to nucleon-nucleus scattering the nuclear matter approach yields a fair description of differential elastic cross sections at energies above  $E_\alpha = 40$  MeV (Fig. 4). Fits for several nuclei show that the agreement can be improved, if one scales the imaginary part  $W_{\text{oi}}^{NM}$  with a factor  $\lambda_W$  about 0.65 (Fig. 3, right part). At energies below 20 MeV the agreement is still fairly good, but  $\chi^2$ -fits to experimental data do not show a systematic behaviour of  $\lambda_W^{NM}$ . The preference of  $\lambda_W^{NM} > 1$  might be a hint for missing collective contributions, which are explicitly included in  $W^{NS}$  outlined in Sec. 2.2. In order to account for these contributions we used  $W^{NM}(r) + W^{NS}(r, r)$  to evaluate in a first approach the differential elastic cross section. However, the impact of the inclusion of  $W^{NS}$  on the cross sections is almost negligible because its effective range is too small.



**Figure 4:** Comparison of calculated differential elastic  $\alpha$ - $^{40}\text{Ca}$  cross sections using different microscopic approaches with experimental data.

In principle the nuclear structure approach should provide the complete imaginary part. Therefore we used an optical model consisting of the real part of the nuclear matter approach  $V^{NM}$  and the imaginary part of the nuclear structure approach  $W^{NS}$  to evaluate again the differential elastic cross sections. In this case we can only account for about 50% of the absorption which is in agreement with previous findings [8]. Scaling of  $W^{NS}$  by a factor about 4 is required to gain reasonable absorption.

#### 4. Summary and Outlook

In this contribution we have constructed a nuclear matter alpha-nucleus optical potential based on the parameterized g-matrix of [13]. Despite its simplicity the approach provides a surprisingly fair description of  $\alpha$  elastic scattering data even at low energies. In principle collective contributions at low energies are missing and therefore we have also evaluated the imaginary part within the nuclear structure approach. It should be remarked that apart from the RPA calculation which was based on a Skyrme force, we made consistent use of the g-matrix as an effective interaction. Although the results are reasonable the obtained contributions are of too small range and cannot account for the full absorption. First applications of the potential to  $\alpha$ -involving reactions for nucleosynthesis are in progress.

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