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NUCLEAR EQUATION OF STATE FOR CORE-COLLAPSE SUPERNOVAE WITH REALISTIC NUCLEAR FORCES

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We propose a new nuclear equation of state (EOS) for numerical simulations of core-collapse supernovae (SNe) using the variational many-body theory with realistic nuclear forces. Starting from the nuclear Hamiltonian composed of the Argonne v18 two-body potential and Urbana IX three-body potential, we first construct the EOS of uniform nuclear matter by the cluster variational method. The obtained free energies of pure neutron matter and symmetric nuclear matter are in good agreement with those of Fermi hypernetted chain variational calculations. Moreover, the mass-radius relation of neutron stars derived from this EOS at zero temperature is consistent with recent observational data. Using the free energies of uniform nuclear matter based on the realistic nuclear Hamiltonian, we then construct the EOS for non-uniform nuclear matter by the Thomas-Fermi calculation. The obtained phase diagram of hot nuclear matter is reasonable as compared with the Shen EOS, and thermodynamic quantities in the non-uniform phase with the present EOS are similar to those with the Shen EOS. Finally, the present EOS is applied to a spherically symmetric adiabatic numerical simulation of SNe. We confirm that our EOS is successfully applied to this hydrodynamics simulation. It is also seen that the present EOS in a relatively large proton fraction is softer than the Shen EOS, which is consistent with the fact that the incompressibility of the present EOS is smaller than that of the Shen EOS.

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1. Introduction

The nuclear equation of state (EOS) plays a crucial role in the studies of high-energy astrophysical objects, such as neutron stars, core-collapse supernovae (SNe) and black hole formations. To understand these phenomena properly, a reliable nuclear EOS is desired. However, constructing a nuclear EOS suitable for SN simulations is a difficult task, because an EOS table must contain various thermodynamic quantities over a wide range of baryon number densities $n_{\rm B}$, proton fractions $Y_{\rm p}$, and temperatures T. The SN-EOS of Lattimer and Swesty [1] adopts the Skyrme-type effective interaction for uniform matter and the compressible liquid-drop model for non-uniform matter. Another EOS widely used in SN numerical simulations was developed by Shen et al. [2, 3]. This EOS is based on the relativistic mean field theory for uniform matter and on the Thomas-Fermi calculation for non-uniform matter. Several EOSs applicable to SN simulations are also developed recently. However, these SN-EOSs are based on phenomenological models for uniform matter.

Under these circumstances, we are constructing a new nuclear EOS based on the realistic nuclear forces with the variational many-body theory. For uniform matter, the EOS is calculated with the cluster variational method [4, 5, 6], while the Thomas-Fermi method is adopted for non-uniform phase. In this paper, we report some properties of our new SN-EOS and apply this EOS to a test hydrodynamics simulation of core-collapse SN.

2. Supernova equation of state with realistic nuclear forces

In this section, we briefly describe the construction of the nuclear EOS based on realistic nuclear forces. Starting from the nuclear Hamiltonian composed of the Argonne v18 (AV18) two-body potential and the Urbana IX (UIX) three-body potential, we first calculate the energy per particle E/N of uniform matter at zero temperature. The nuclear Hamiltonian is decomposed into the two-body Hamiltonian containing the AV18 potential and the three-body Hamiltonian containing the UIX potential. The two-body energy per nucleon E_2/N is expressed as the expectation value of the two-body Hamiltonian with the Jastrow wave function in the two-body cluster approximation. E_2/N is then minimized with respect to the spin-isospin-dependent central, tensor, and spin-orbit correlation functions included in the Jastrow wave function with appropriate constraints. The three-body energy per nucleon E_3/N is expressed as the modified expectation value of the three-body Hamiltonian with the Fermi-gas wave function. E_3/N contains four parameters, whose values are chosen so that the total energy per particle $E/N = E_2/N + E_3/N$ reproduces the empirical saturation properties. Then, we obtain the following values: the saturation density $n_0 = 0.16$ fm^{-3}, saturation energy $E_0/N = -16.09$ MeV, incompressibility $K = 245$ MeV, and symmetry energy $E_{\rm sym} = 30.0$ MeV, as reported in Ref. [6].

The obtained E/N for pure neutron matter and symmetric nuclear matter are in good agreement with the results of the more sophisticated Fermi hypernetted chain (FHNC) variational calculations performed by Akmal, Pandharipande and Ravenhall (APR) [7]. In addition, the mass-radius relation of neutron stars obtained with the new EOS are consistent with recent observational data as reported in Refs. [6, 8].

At finite temperature, the free energy per nucleon F/N is calculated by an extension of the variational method proposed by Schmidt and Pandharipande [9]. Following this method, F/N is expressed with the internal energy per nucleon E_{T}/N and the entropy per nucleon S/N. As in the zero-temperature case, E_{T}/N is expressed as the sum of the two-body energy E_{T2}/N and the three-body energy E_{T3}/N. E_{T2}/N is explicitly expressed by replacing the occupation probabilities of single-particle states for protons $n_{0\mathrm{p}}(k)$ and for neutrons $n_{0\mathrm{n}}(k)$ in E_2/N at zero temperature by the averaged occupation probabilities at finite temperatures for protons $n_{\mathrm{p}}(k)$ and for neutrons $n_{\mathrm{n}}(k)$, respectively. The three-body energy E_{T3}/N is assumed to be the same as E_3/N at zero temperature. The entropy S/N is also expressed in terms of $n_i(k)$ $(i = \mathrm{p,n})$, where each $n_i(k)$ is parameterized by an effective nucleon mass m_i^* $(i = \mathrm{p,n})$. F/N is then minimized with respect to the effective masses m_{p}^* and m_{n}^*.

The obtained F/N for pure neutron matter and symmetric nuclear matter are in good agreement with the results of the FHNC calculation by Mukherjee [10]. Other thermodynamic quantities derived from F/N are also reasonable.

Using this nuclear EOS for uniform matter, we then performed general relativistic, spherically symmetric numerical simulations of core-collapse SNe with and without neutrino transfer [8]. In these simulations, we adopted the Shen EOS for low-density nuclear matter, because our EOS for non-uniform matter was not yet constructed. We confirmed that the present EOS of uniform matter by the cluster variational method is successfully applied to SN simulations. Furthermore, in these simulations, it was seen that the present EOS is softer than the Shen EOS.

In order to complete an SN-EOS table, we next construct a nuclear EOS for non-uniform matter that is consistent with the EOS for uniform matter. For this purpose, we perform the Thomas-Fermi (TF) calculation following the method by Shen et al. [2, 3]. In this method, the free energy of a Wigner-Seitz (WS) cell is expressed as

$$F = \int d\mathbf{r} f(n_{\mathrm{p}}(r), n_{\mathrm{n}}(r), n_\alpha(r)) + F_0 \int d\mathbf{r} |\nabla(n_{\mathrm{p}}(r) + n_{\mathrm{n}}(r))|^2 + \frac{e^2}{2} \int d\mathbf{r} \int d\mathbf{r}' \frac{[n_{\mathrm{p}}(r) + 2n_\alpha(r) - n_{\mathrm{e}}][n_{\mathrm{p}}(r') + 2n_\alpha(r') - n_{\mathrm{e}}]}{|\mathbf{r} - \mathbf{r}'|} + \Delta E_{\mathrm{c}}. \tag{2.1}$$

Here, the local free energy density $f(n_{\mathrm{p}}(r), n_{\mathrm{n}}(r), n_\alpha(r))$ embodies the free energies of the above-mentioned uniform nuclear matter and the free energies of alpha particles. The alpha particle is treated as a non-interacting classical particle with a fixed volume. The parameter $F_0 = 68.00$ MeV fm^5 is determined so that the corresponding TF calculation for isolated atomic nuclei reproduces the gross feature of their masses and radii [5]. The last term ΔE_{c} in Eq. (2.1) is the correction for the body-centered cubic lattice. The density distributions of protons $n_{\mathrm{p}}(r)$, neutrons $n_{\mathrm{n}}(r)$, and alpha-particles $n_\alpha(r)$ are functions of r, where r is the distance from the center of the WS cell. The density distribution of electrons, $n_{\mathrm{e}} = n_{\mathrm{B}} Y_{\mathrm{p}}$, is assumed to be constant throughout the cell. Here, $n_{\mathrm{p}}(r)$, $n_{\mathrm{n}}(r)$, and $n_\alpha(r)$ are parameterized, and then the averaged free energy density of the WS cell is minimized with respect to those parameters.

The obtained phase diagram of hot nuclear matter at $Y_{\mathrm{p}} = 0.3$ is shown in Figure 1. The shaded region corresponds to the non-uniform phase and the dashed line shows the boundary where the mass fraction of alpha-particles $X_\alpha = 10^{-4}$. It is seen that, at $T \lesssim 0.5$MeV, the non-uniform phase appears at the densities $n_{\mathrm{B}} \lesssim 10^{-1}$ fm^{-3}. As T increases, the density region of the non-uniform phase narrows, and heavy nuclei disappear at $T \gtrsim 12$ MeV. The obtained phase diagram is close to that obtained by the Shen EOS [3]. In the non-uniform phase, the obtained thermodynamic

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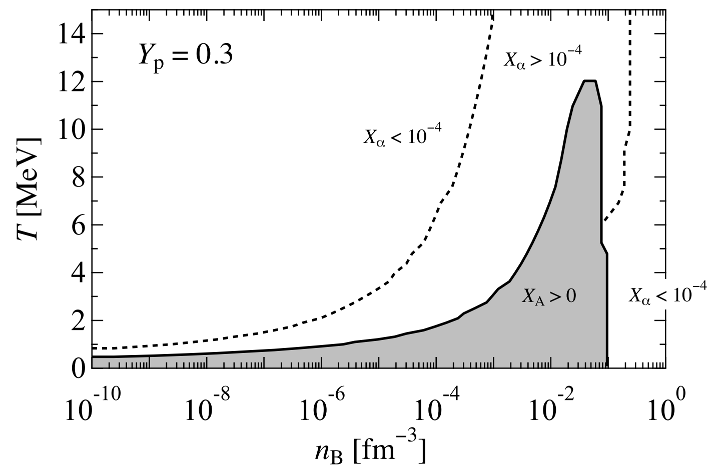

Figure 1: Phase diagram of hot nuclear matter at $Y_p = 0.3$. The shaded region represents the non-uniform phase with mass fraction of heavy nuclei X_A. The dashed line is the boundary at which the mass fraction of alpha particles becomes $X_\alpha = 10^{-4}$.

quantities, such as the free energy, internal energy, entropy and pressure are also close to those of the Shen EOS, but the two EOSs provide different mass numbers of nuclei appearing in the non-uniform phase and particle fractions [11].

3. Application to adiabatic core-collapse supernovae

Previously, we applied the uniform EOS that we constructed by the cluster variational method to SN simulations, by supplementing it with the Shen EOS for non-uniform low-density matter [8]. In this section, we apply the present EOS, which includes the non-uniform EOS being consistent with the uniform EOS by the variational method, to a hydrodynamics simulation of core-collapse SNe. As a test, we perform general relativistic, spherically symmetric adiabatic core-collapse SN simulations without neutrino transfer. In this calculation, Y_p of each fluid element does not vary during the simulation, fixed at the initial value, about 0.4-0.5, given in the pre-supernova model. Starting from the 15 $M_\odot$ pre-supernova model calculated by Woosley and Weaver [12], the iron core collapses adiabatically because the simulation does not account for the energy exchange by neutrino transfer. The dynamics of the collapsed core obtained in this simulation are then compared with those of the Shen EOS.

Figure 2 shows the radial trajectories of the mass elements in the core obtained with the present EOS and the Shen EOS. The time at the bounce is chosen to be zero. Before the bounce, the trajectory of each fluid element calculated with the present EOS and the Shen EOS are highly similar, because the relevant matter is non-uniform at this stage (As mentioned above, both EOSs give similar thermodynamic quantities in the non-uniform phase). After the bounce, the trajectories calculated with the present EOS deviate from those with the Shen EOS: specifically, the outward movements of the fluid elements are faster in the present EOS, indicating that the present EOS with relatively large values of Y_p is softer than the Shen EOS as discussed below. We note that, though it is not shown in this paper, the red curves shown in Figure 2 slightly differ from those

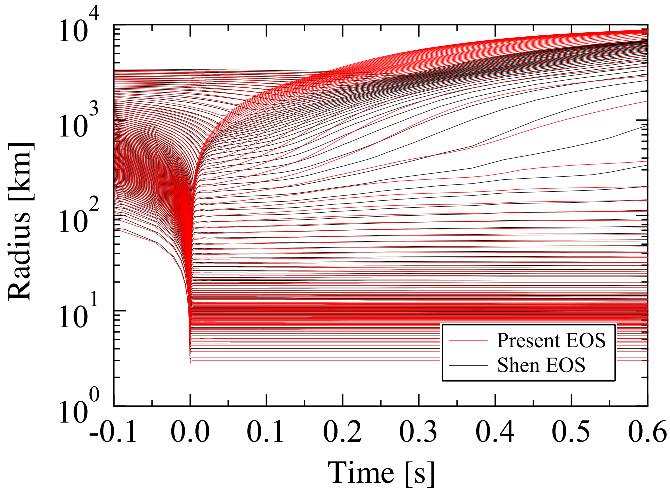

Figure 2: Radial trajectories of mass elements in the core of a 15 $M_{\odot}$ star as functions of time calculated with the present EOS (red) and the Shen EOS (black).

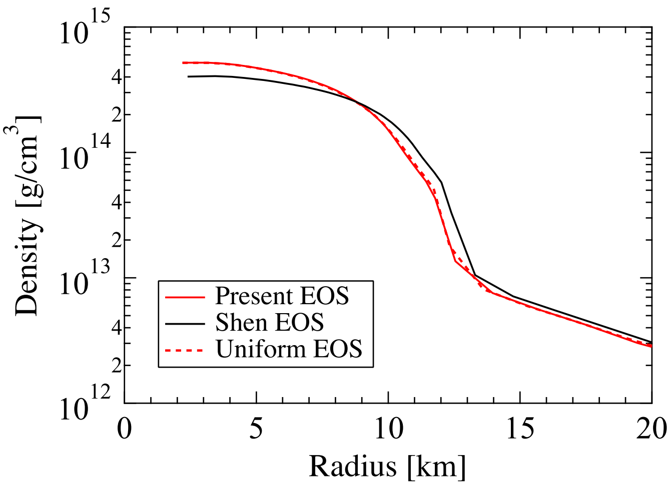

Figure 3: Density profiles at the bounce as functions of radius computed with the present EOS (red solid line) and the Shen EOS (black solid line). Also shown is the result computed with the present EOS for uniform matter supplemented by the Shen EOS for non-uniform matter (red dotted line) [8].

in our previous study given in Ref. [8], because, here we employ the SN-EOS constituted by our non-uniform EOS as well as our uniform EOS.

Figure 3 shows the density profiles in the central region of the core at the bounce, computed with the present and Shen EOSs. The present EOS predicts a larger central density, and correspondingly, a smaller high-density core radius, than the Shen EOS. Namely, the central high-density core is more compact and releases more gravitational energy in the present EOS, because the present EOS is softer than the Shen EOS in this density region at relatively large values of Y_p. Since the simulation is adiabatic, the released gravitational energy is converted into kinetic energy of the outgoing fluid elements in the outer region, as seen in Figure 2. This implies that SN explosions are more favored in the present EOS than in the Shen EOS for the case of the adiabatic collapse, where the nuclear EOS with Y_p being 0.4 - 0.5 governs the dynamics of SN. Here, we note that this relative softness of the present EOS is consistent with the result that the incompressibility of the present EOS is smaller than that of the Shen EOS.

Figure 3 also shows the density profile obtained in the hydrodynamics simulation with the uniform EOS supplemented by the Shen EOS for non-uniform matter reported in Ref. [8]. This density profile is quite similar to that obtained by the present SN-EOS, implying that the dynamics of the central core are mainly governed by the EOS of high-density nuclear matter as discussed in Ref. [8].

While the application of the present EOS to a hydrodynamics simulation of core-collapse SN is successful, calculation of several quantities related to the chemical composition of non-uniform phase in a new SN-EOS table has not been completed yet. A complete SN-EOS table and its application to more realistic neutrino-radiation hydrodynamics simulations of core-collapse SNe will be reported in a separate paper [11].

Acknowledgments

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