NIC-XIII school, 2014

Stellar Structure & Evolution

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Plan

- L1: Basics of stellar structure and evolution

- L2: Physical ingredients

- L3: Massive stars

- L4: Low- and intermediate-mass stars

Acknowledgements & Bibliography

- Slides in white background (with blue title) were taken from Achim Weiss' lecture slides, which you can find here: http://www.mpa-garching.mpg.de/~weiss/lectures.html

- Some graphs were taken from Onno Pols' lecture notes on stellar evolution, which you can find here:

http://www.astro.ru.nl/~onnop/education/stev_utrecht_notes/

- Some slides (colourful ones) and content was taken from George Meynet's summer school slides.

Acknowledgements & Bibliography

Recommended further reading:

- R. Kippenhahn & A. Weigert, Stellar Structure and Evolution, 1990, Springer-Verlag, ISBN 3-540-50211-4

- A. Maeder, Physics, Formation and Evolution of Rotating Stars, 2009, Springer-Verlag, ISBN 978-3-540-76948-4

- D. Prialnik, An Introduction to the Theory of Stellar Structure and Evolution, 2000, Cambridge University Press, ISBN 0-521-65937-X

- C.J. Hansen, S.D. Kawaler & V. Trimble, Stellar Interiors, 2004, Springer-Verlag, ISBN 0-387-20089-4

- M. Salaris & S. Cassisi, Evolution of Stars and Stellar Populations, 2005, John Wiley & Sons, ISBN 0-470-09220-3

L1: Basics of Stellar Structure and Evolution

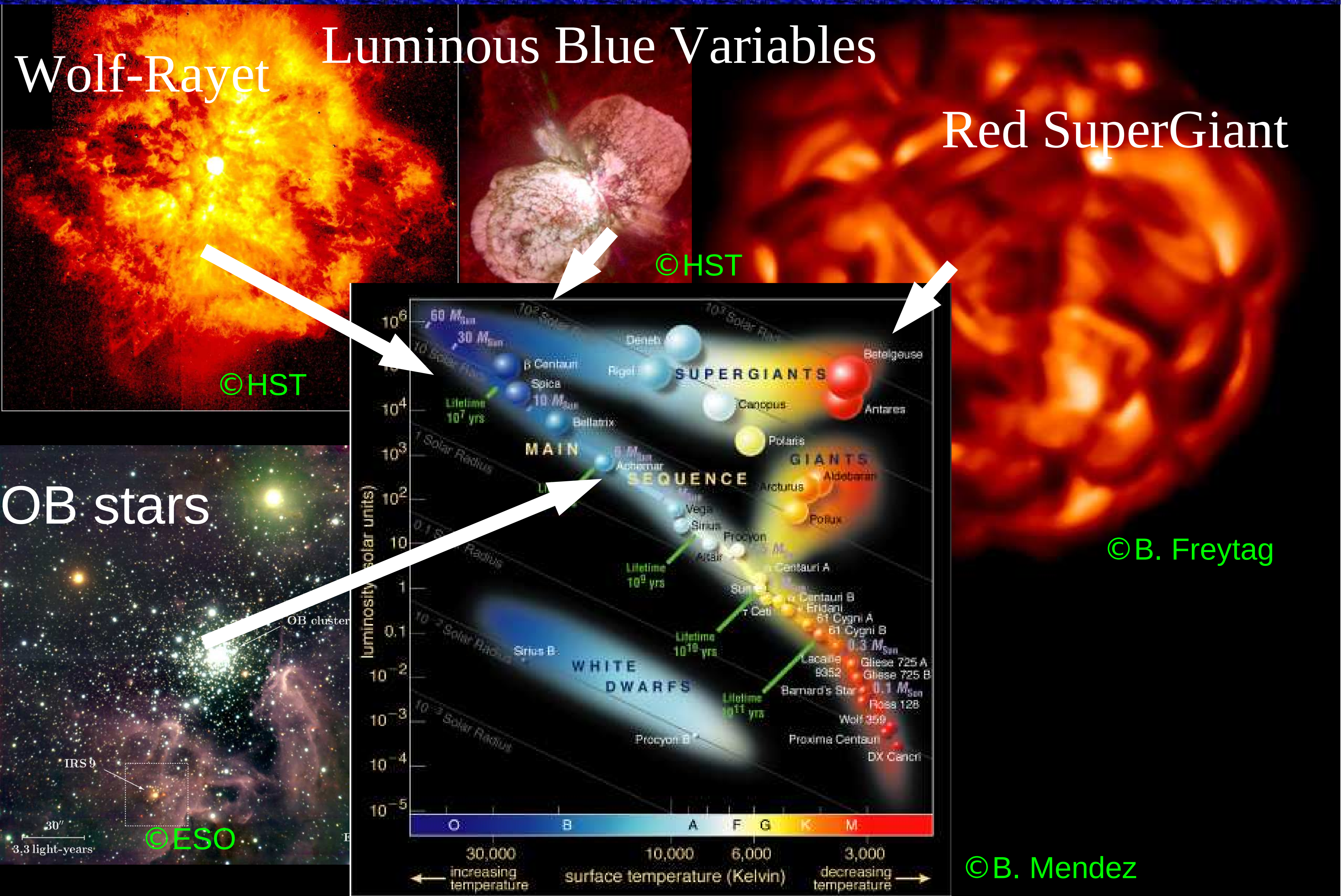
- Importance and observational constraints

- Physics governing the structure and evolution of stars

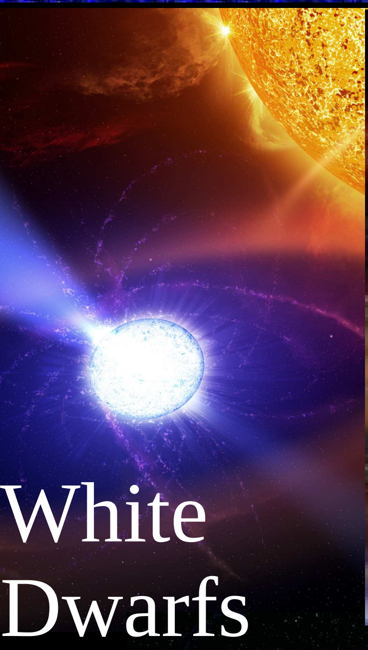
- Equations of stellar structure

- Modelling stars and their evolution

Importance as Stellar Objects



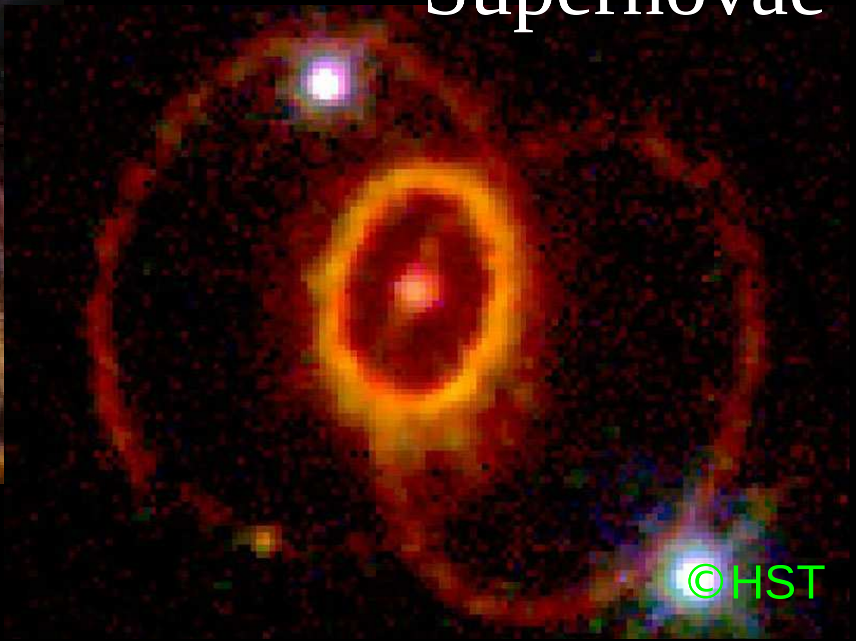
Importance as Progenitors



White Dwarfs

GRBs

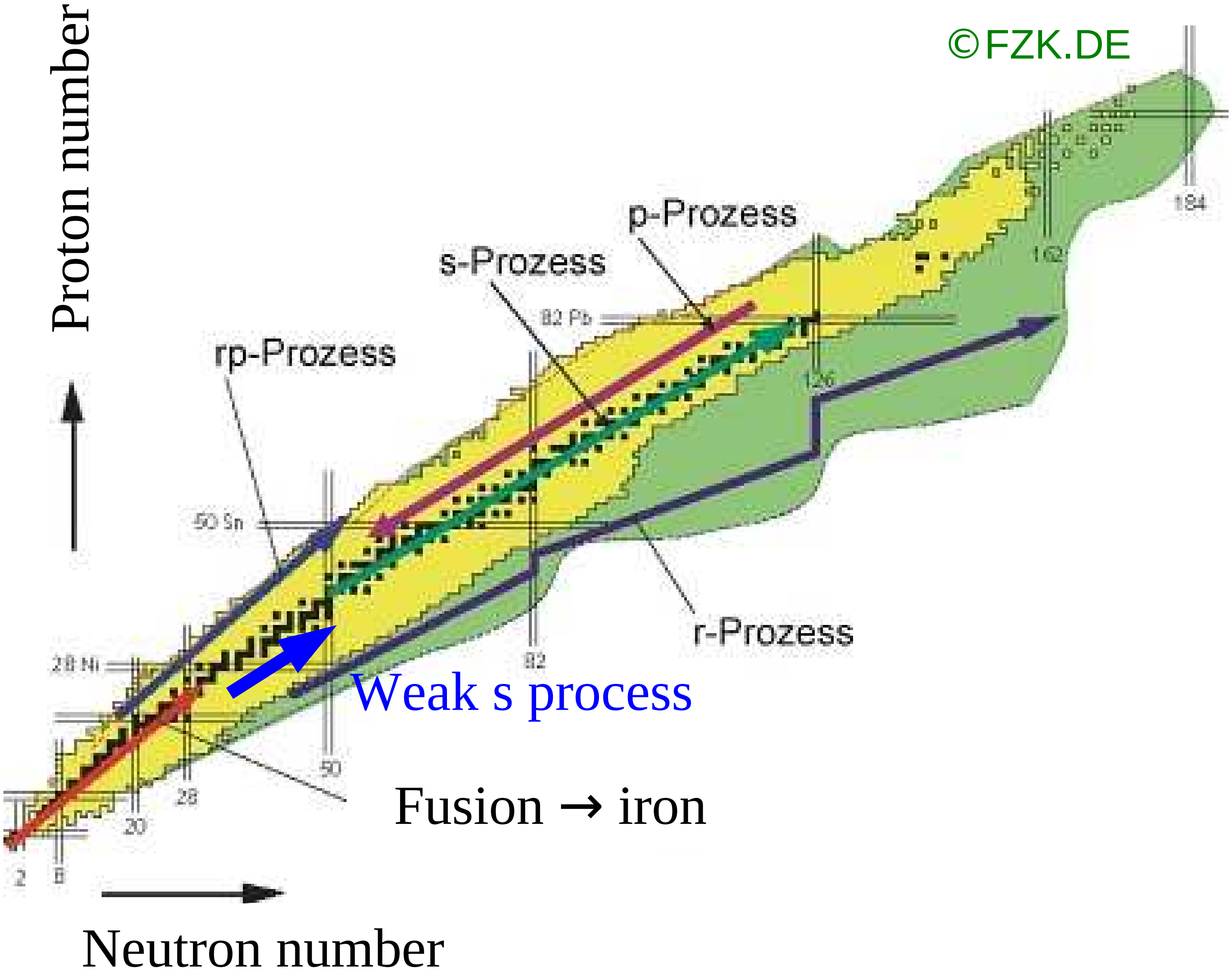
Supernovae



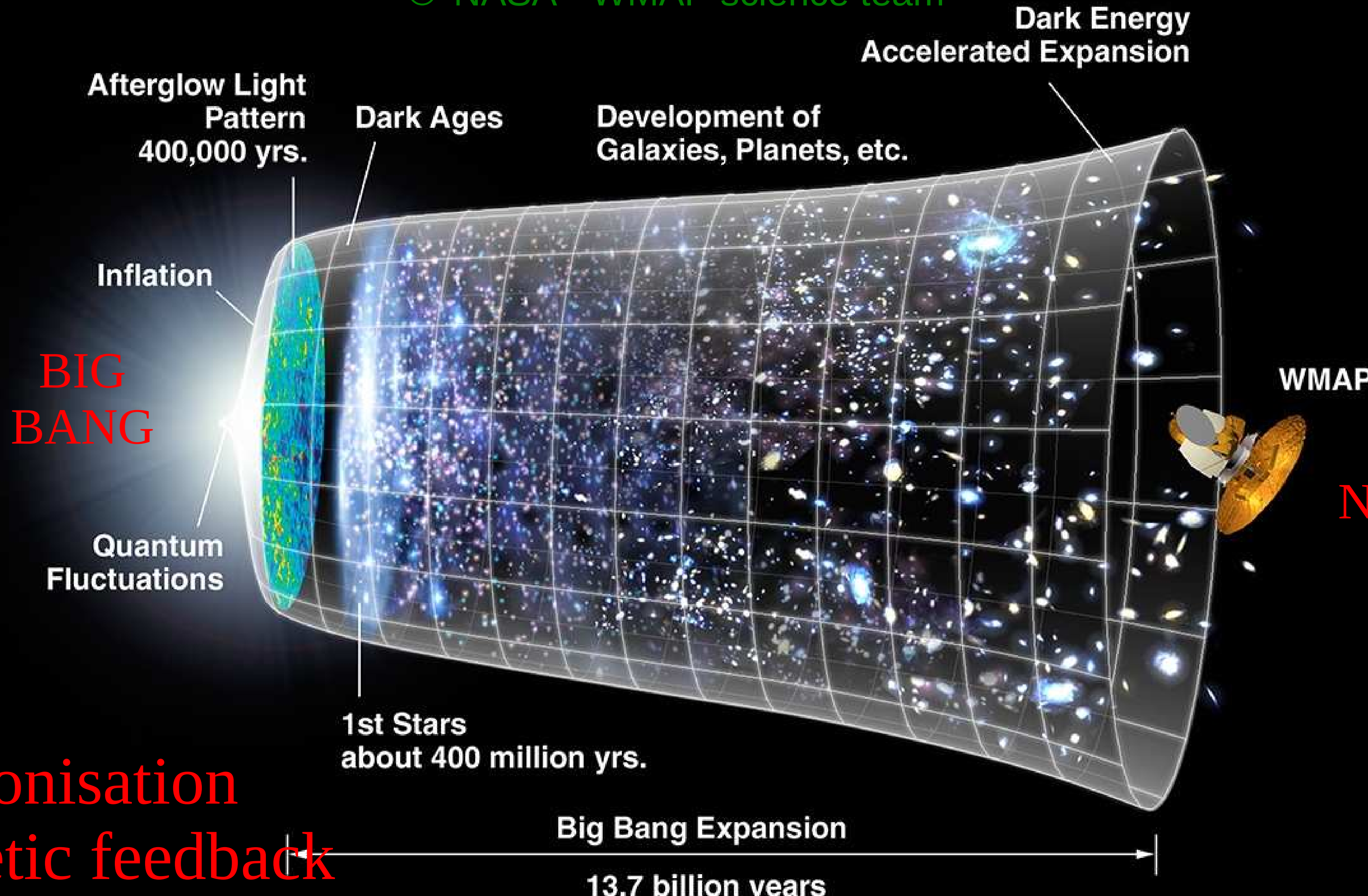
Neutron Stars

Black Holes

Importance for Nucleosynthesis

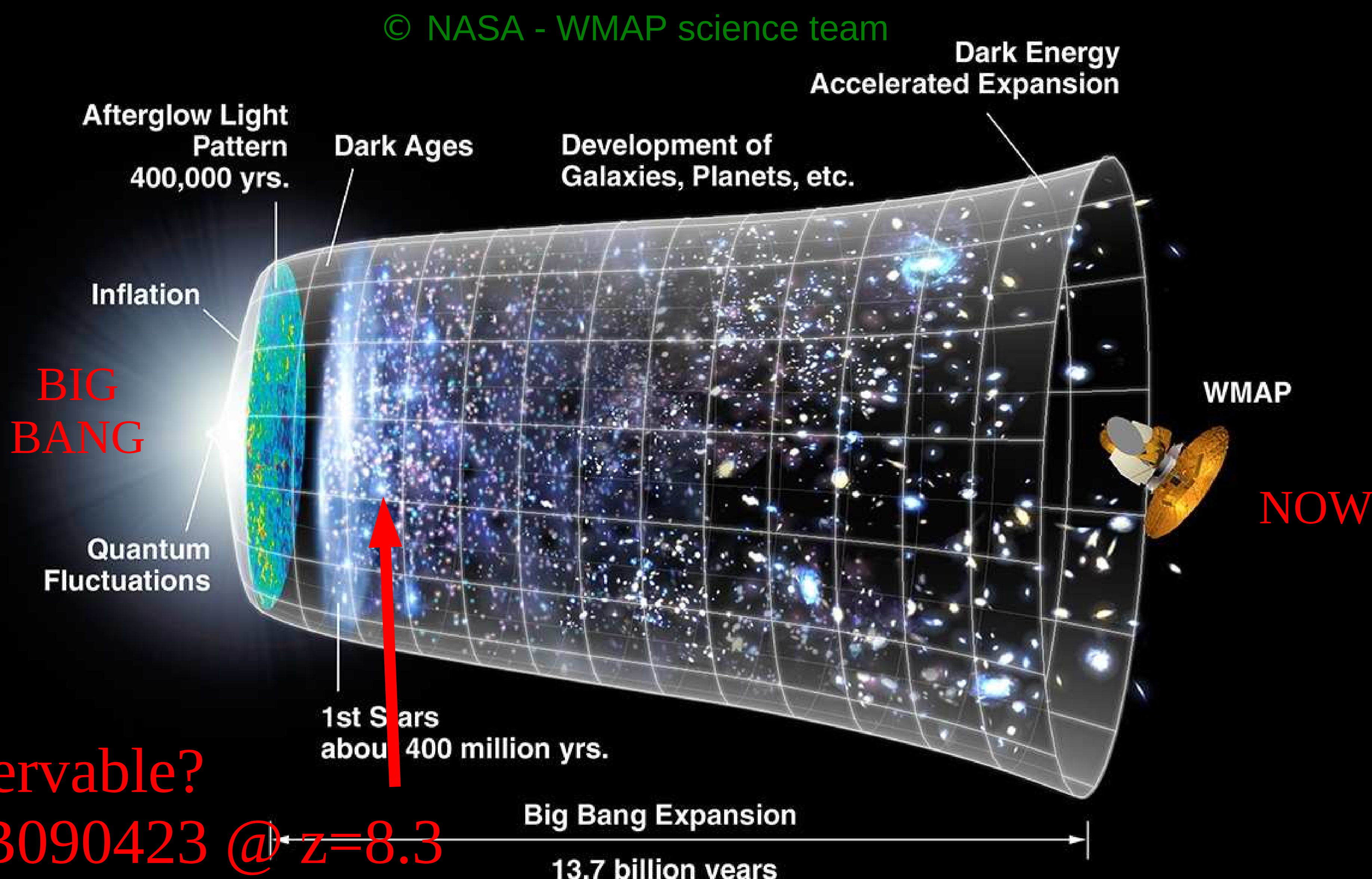


Stars Role in Universe



- Re-ionisation
- Kinetic feedback
- Chemical feedback observed in EMP stars

First Stellar Generations: Importance



- Observable?
- GRB090423 @ z=8.3
Universe age ~ 600 Myr (Tanvir et al 09: arXiv:0906.1577)

Observational Data

- Photometry → brightness and colour

- Spectroscopy → T_{eff}, g, X_i (composition), mass loss

- Astrometry → distance

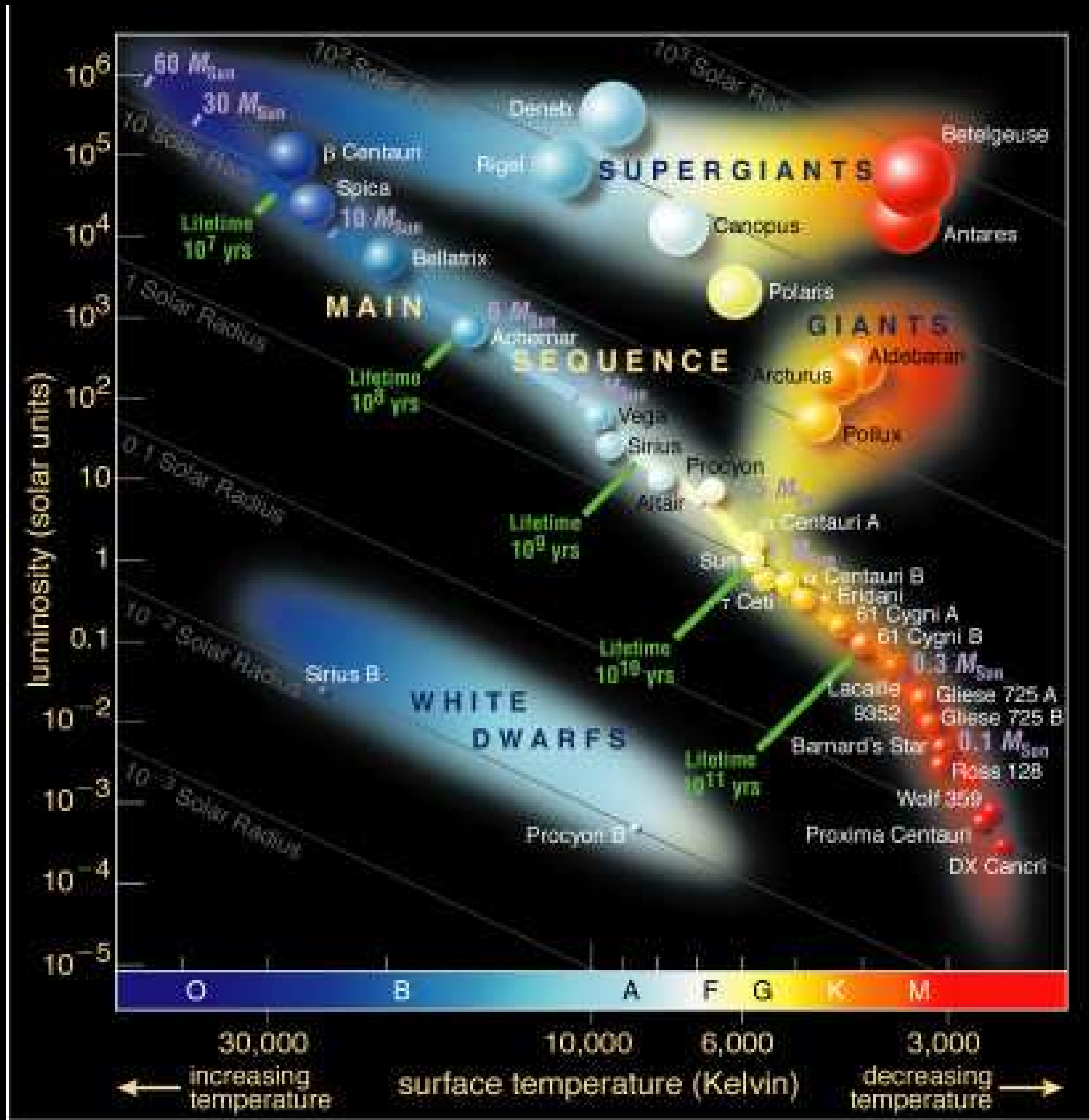
- Eclipsing binaries → mass ratios, orbital info

- Occultations & interferometry → angular diameter, R

- Seismology → interior structure: c_s, ρ, ...

Observational Data: HRD

Hertzsrpung-Russell (HR) Diagram:



Stellar Luminosity

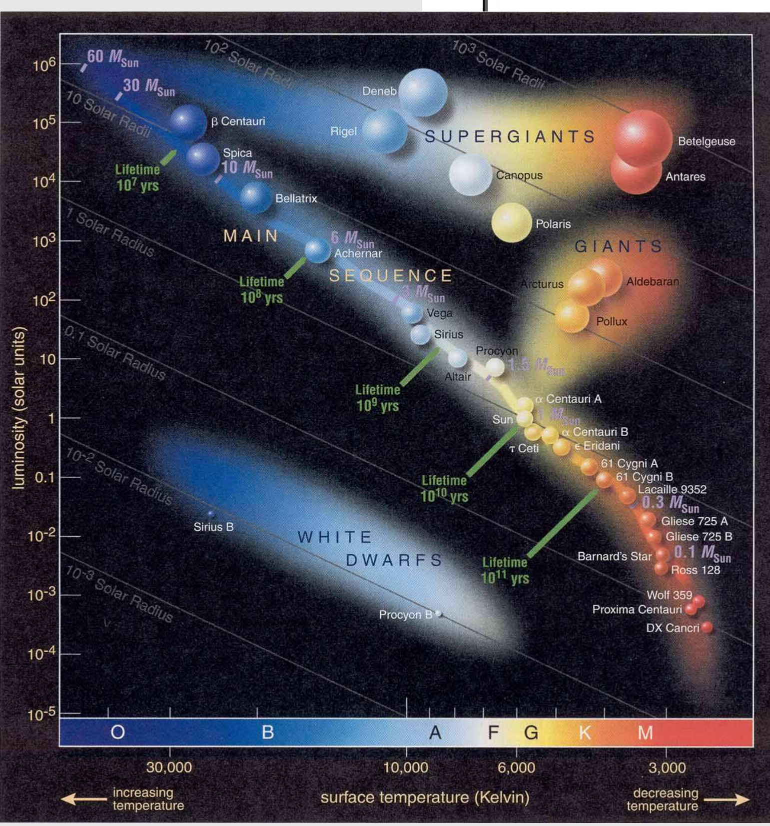
How can two stars have the same temperature, but vastly different luminosities?

- The luminosity of a star depends on 2 things:
  - surface temperature
  - surface area (radius)
  - $L = \sigma T^4 4 \pi R^2$ (σ=5.67e-8 $Wm^{-2}T^{-4}$)
  - The stars have different sizes!!
- The largest stars are in the upper right corner of the H-R Diagram.

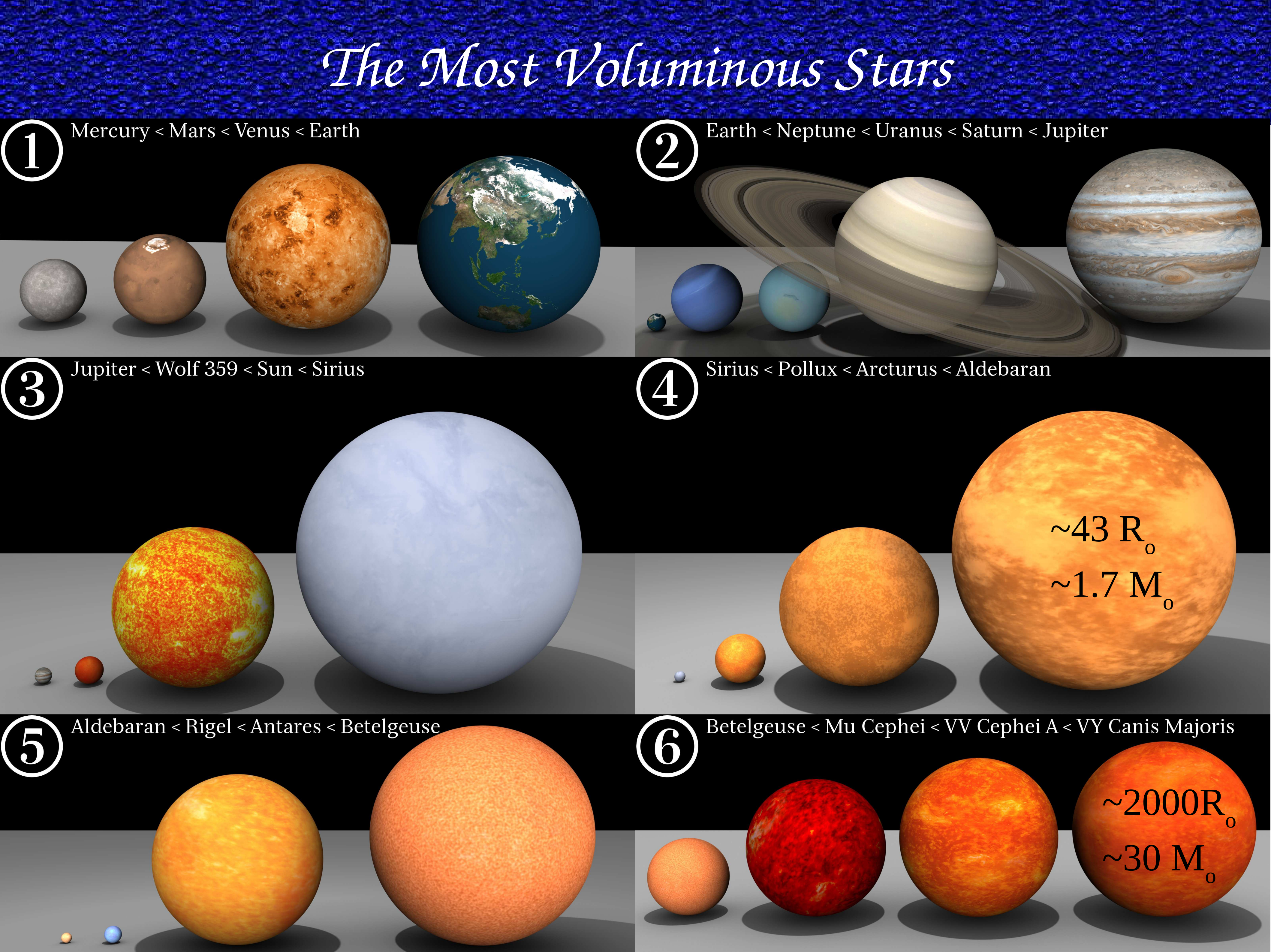
Stellar Luminosity Classes

Table 15.2 Stellar Luminosity Classes

Class	Description
I	Supergiants
II	Bright giants
III	Giants
IV	Subgiants
V	Main sequence

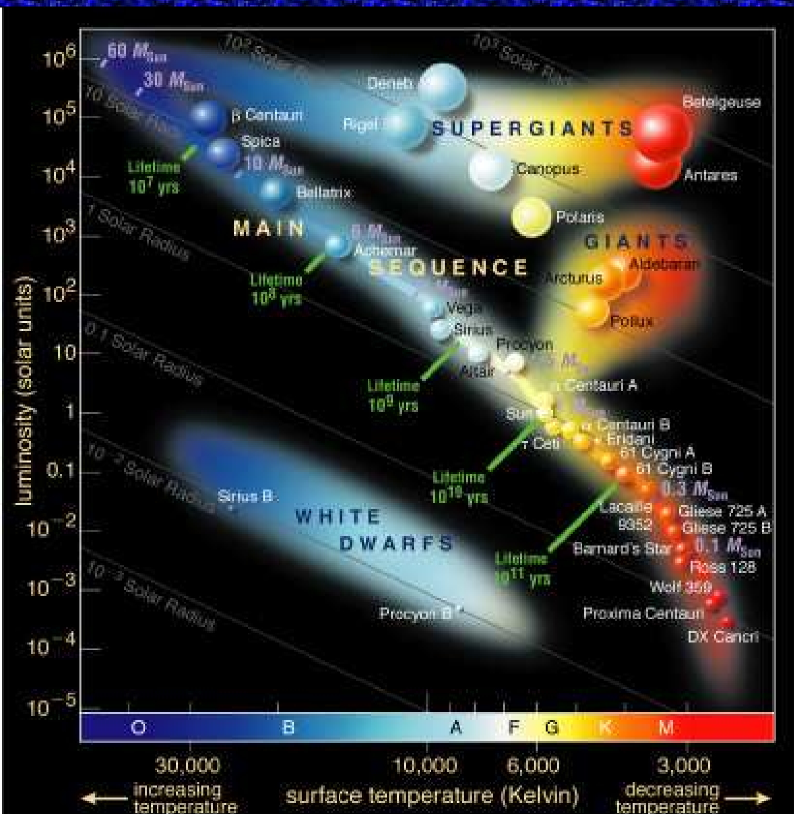


Radius of largest stars is larger than 1 AU!

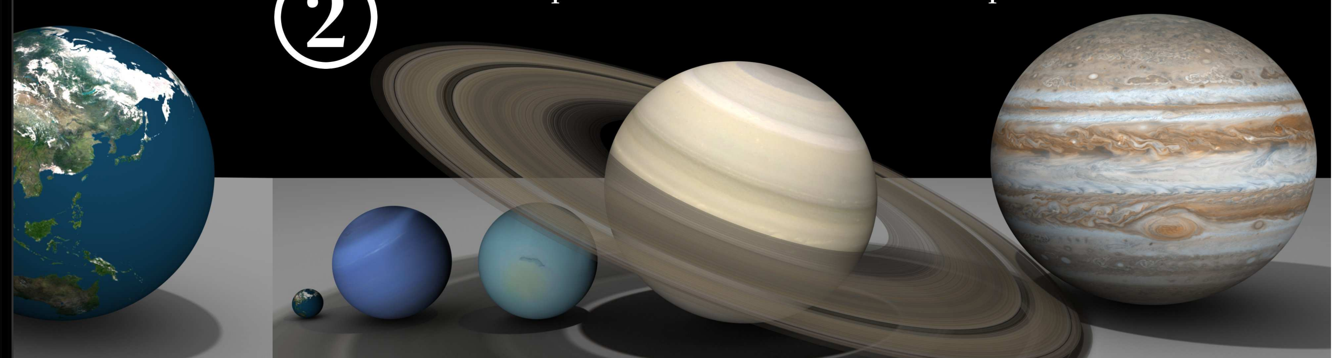


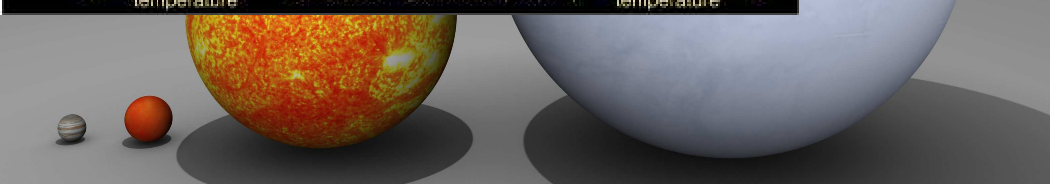
The Most Voluminous Stars
1
Mercury < Mars < Venus < Earth
2
Earth < Neptune < Uranus < Saturn < Jupiter
3
Jupiter < Wolf 359 < Sun < Sirius
4
Sirius < Pollux < Arcturus < Aldebaran
~43 R_o
~1.7 M_o
5
Aldebaran < Rigel < Antares < Betelgeuse
6
Betelgeuse < Mu Cephei < VV Cephei A < VY Canis Majoris
~2000R_o
~30 M_o

The Most Voluminous Stars



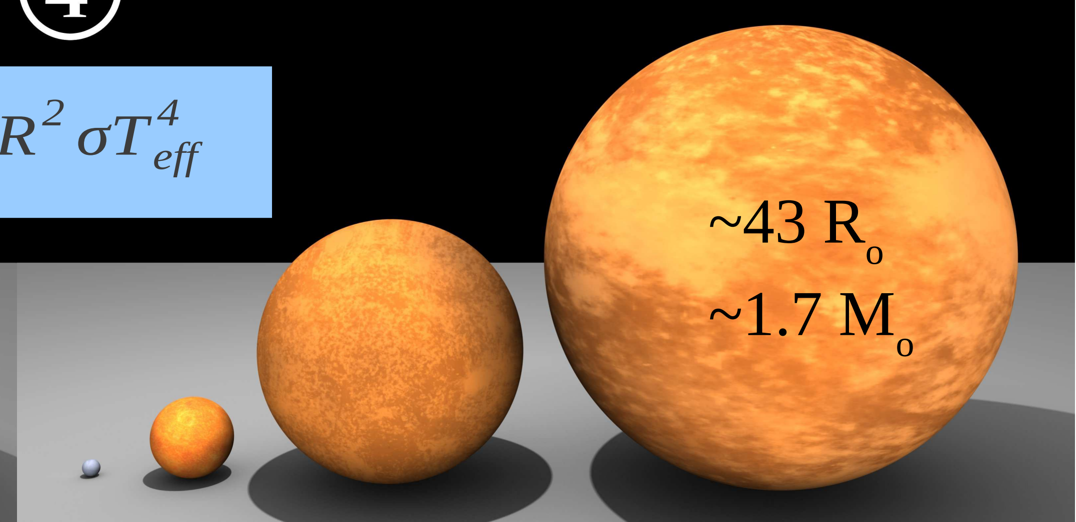
2 Earth < Neptune < Uranus < Saturn < Jupiter



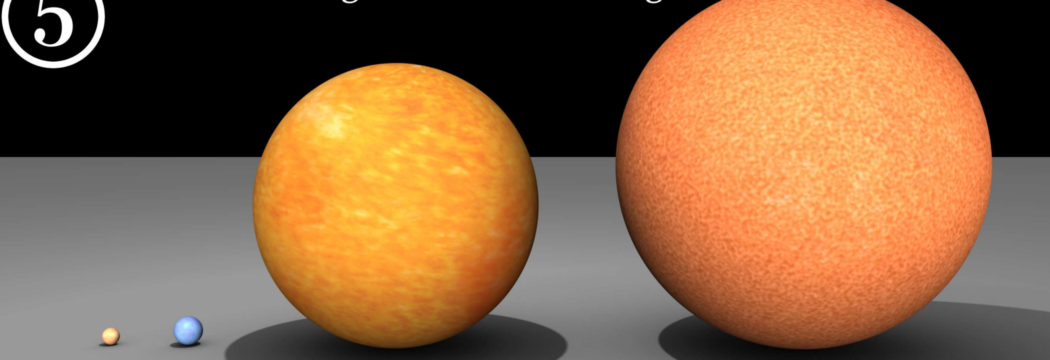


4 Sirius < Pollux < Arcturus < Aldebaran

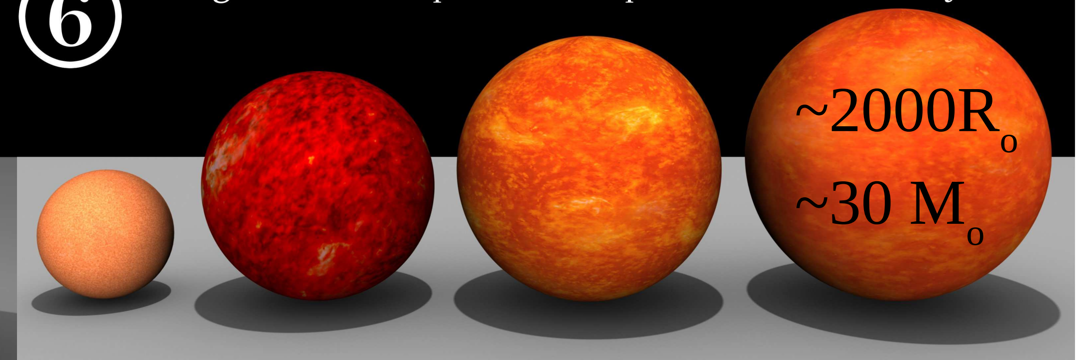
$$L = 4\pi R^2 \sigma T_{eff}^4$$



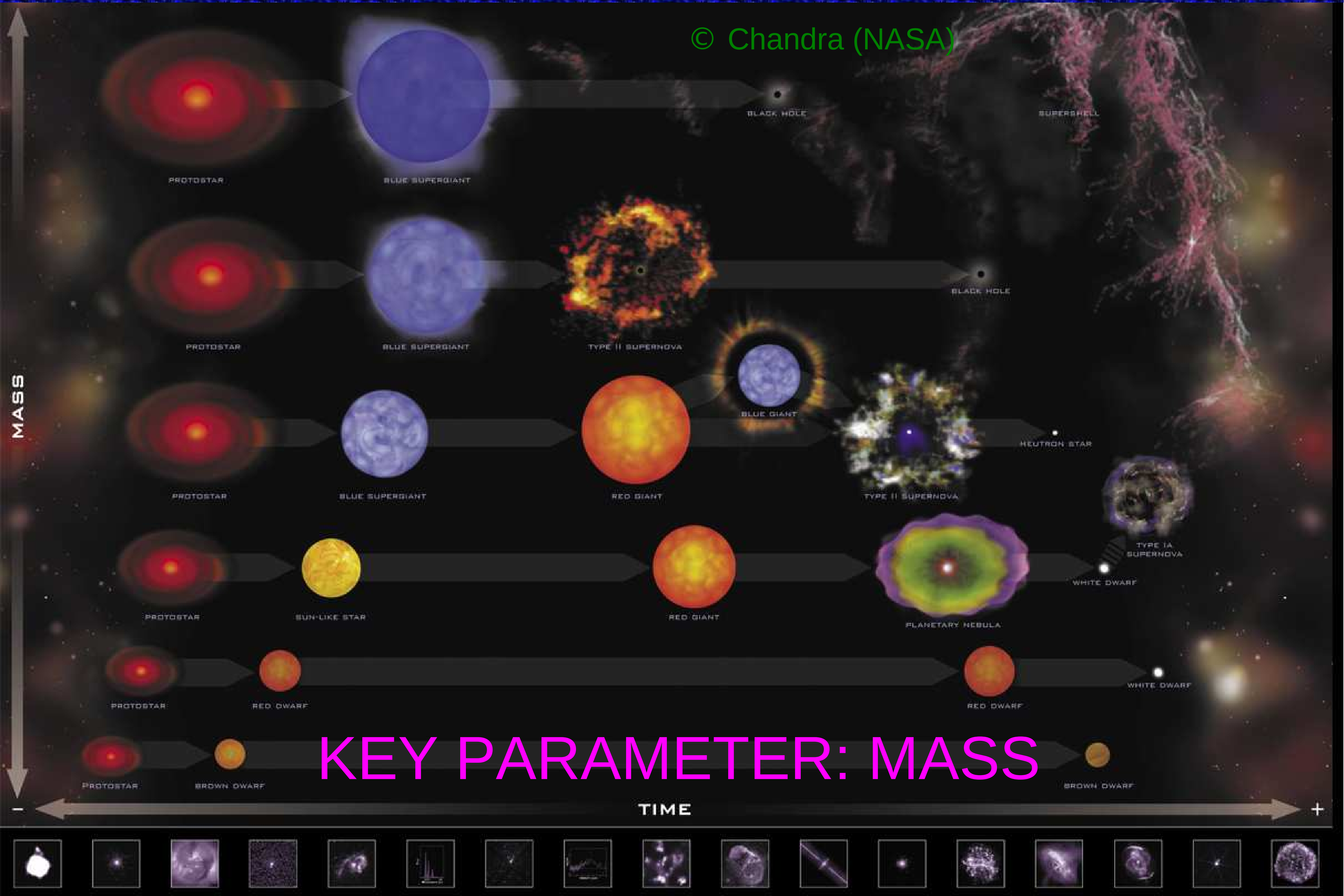
5 Aldebaran < Rigel < Antares < Betelgeuse



6 Betelgeuse < Mu Cephei < VV Cephei A < VY Canis Majoris



Stellar Evolution in 1 Slide



KEY PARAMETER: MASS

Stellar Evolution Theory

Goal of stellar evolution theory:

"To understand the structure and evolution of stars, and their observational properties, using known laws of physics"

Basic assumptions:

- Stars are self-gravitating hot plasma

- Stars emit energy in the form of photons from the surface (+ neutrinos in advanced phases)

- Stars are (approximately) spherically symmetric

→ Can be reduced to a 1D problem, with radius as the natural coordinate (Euler description)

Radius and Mass

- Euler description: radius as indep. variable

but more convenient to use mass, m_r, as

independent variable (Lagrangian descr.)

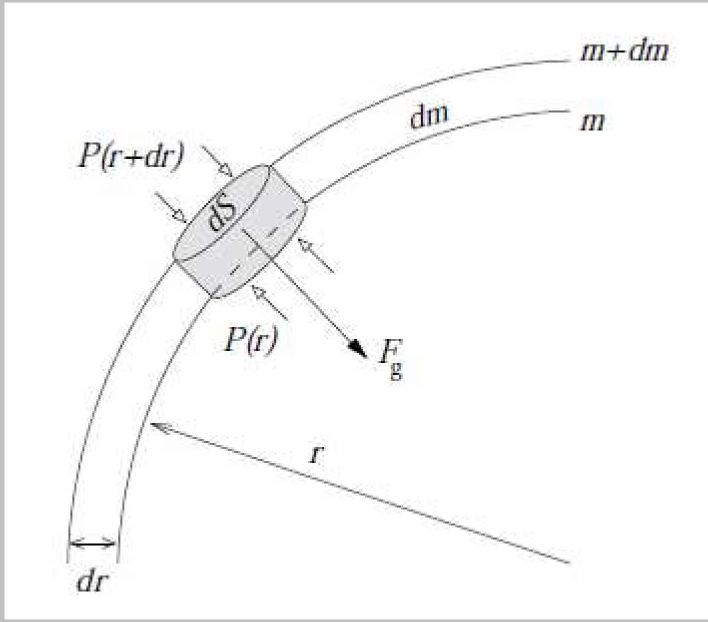


Figure 2.1. Mass shell inside a spherically symmetric star, at radius r and with thickness dr. The mass of the shell is $dm = 4\pi r^2 \rho\, dr$. The pressure and the gravitational force acting on a cylindrical mass element are also indicated.

From SE notes, O. Pols (2009)

Conservation of mass applied to a spherical

shell in a steady state ($\partial()/\partial t = 0$) gives:

$dm = \rho\, dV = 4\pi r^2 \rho\, dr$ from which we can write:

$$dr/dm = (4\pi r^2 \rho)^{-1} \quad (1)$$

Gravity and Equation of Motion

The gravitational field is given by Poisson's equation: $\nabla^2 \Phi = 4\pi G \rho$.

In spherical symmetry, we get:

$$g = d\Phi/dr = Gm_r/r^2$$

Conservation of momentum applied to a small gas element (see figure) gives:

$$d^2r/dt^2\, dm = - g\, dm + P(r)\, dS – P(r+dr)dS,$$

which we can re-write as the eqn. of motion:

$$d^2r/dt^2 = - Gm/r^2 – 1/\rho\, dP/dr$$

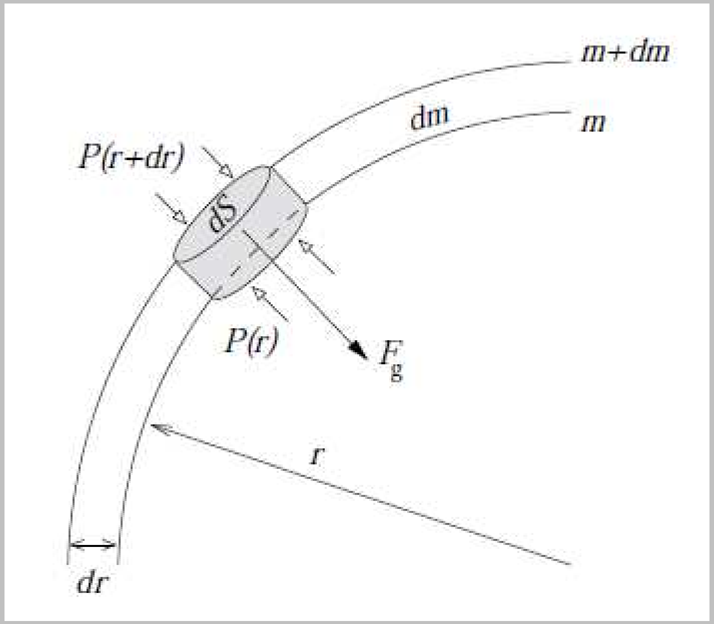


Figure 2.1. Mass shell inside a spherically symmetric star, at radius r and with thickness dr. The mass of the shell is $dm = 4\pi r^2 \rho\, dr$. The pressure and the gravitational force acting on a cylindrical mass element are also indicated.

From SE notes, O. Pols (2009)

Hydrostatic Equilibrium

Assuming hydrostatic equilibrium, we get:

$$d^2r/dt^2 = 0 = - Gm/r^2 - 1/\rho \; dP/dr$$

Combining with Eqn (1): $dr/dm = (4\pi r^2 \rho)^{-1}$, we obtain:

$$dP/dm = - Gm/(4\pi r^4) \quad (2)$$

On the other hand, we can estimate the free-fall timescale, t_{ff}, by ignoring the pressure term in the eqn. of motion. We then can write:

$$d^2r/dt^2 \approx R/t_{ff}^2 \rightarrow t_{ff} \approx \sqrt{(R/g)} \; (\approx 1/2\sqrt{(G<\rho>)} = t_{dyn})$$

Examples for t_{ff}/t_{dyn}: 30 min for the sun, 18 days for red giant ($100R_\odot$), 4.5 s for white dwarfs ($R_\odot/50$) << stars' lifetime

→ hydrostatic equil. is a good approximation

The Virial Theorem

The Virial theorem can be obtain by multiplying the hydrostatic equil. Eqn. (2) by the volume ($V=4/3\ \pi r^3$) and integrate over the total mass, M:

$$\int V\ dP/dm = -\ 1/3 \int (Gm/r)\ dm = 1/3\ E_{grav}$$

The left hand-side term is related to the internal energy, E_{int}. After some algebra, one obtains for an ideal mono-atomic gas:

$$E_{grav} = -\ 2\ E_{int} = -2\ L\ dt$$

If a star contracts, half of the energy is radiated away (L) and the other half is used to increase the internal energy (so T goes up).

Seeing it another way, the star loses energy by radiation (L) → it must contract → its internal energy/temperature goes up

negative specific heat!

Kelvin-Helmholz and Nuclear Timescales

Kelvin and Helmholz independently derived the timescale for thermal adjustments, t_{KH}. Consider a star contracting due to gravity and supported only by thermal pressure (internal energy). The timescale for contraction is given by (using the Virial theorem):

$$t_{KH} = E_{int}/L \approx -E_{grav}/(2L) \approx -GM^2/(2RL) \rightarrow$$

$$t_{KH} \approx 1.5 \times 10^7 (M/M_\odot)^2 (R_\odot/R) (L_\odot/L) \text{ yr}$$

Lifetime of Sun much longer than t_{KH}, thus something else powers stars: nuclear reactions!

Nuclear t: $t_{nuc} = E_{nuc}/L \approx 0.007_{(m\text{-->}E)} X_H f_{core} Mc^2/L \approx 10^{10} (M/M_\odot)(L_\odot/L)$ yr

$t_{nuc} >> t_{KH} >> t_{ff}$

stars are generally in hydro+thermal equilibrium

Conservation of Energy

Local energy conservation can be written as:

$$dL_r/dm = \epsilon_{nuc} + \epsilon_{grav\text{-}therm} - \epsilon_\nu, \quad (3)$$

Where ϵ are in units of erg/g/s.

ϵ_{nuc} is the nuclear energy generation rate,

$\epsilon_{gravo\text{-}therm}$ is the gravo-thermal energy generation rate (>0 for contraction)

ϵ_ν, is the neutrino energy loss rate (absolute value) emitted by the plasma (as opposed to nuclear reactions).

Energy Transport

Average temperature gradient in the Sun: $\Delta T/\Delta r$ $10^7/10^{11} = 10^{-4}$ K/cm

Energy is transported by radiation, convection and conduction.

$$dT/dm = -T/P * Gm/(4\pi r^4) * (\partial \ln T/\partial \ln P) = -T/P * Gm/(4\pi r^4)\ \nabla \quad (4)$$

- For radiation, $\nabla_{rad} = 3/(16\pi acG) * \kappa L_r P/(mT^4)$,

where κ is the (Rosseland) mean opacity (assuming radiative diffusion)

- For conduction, the opacity is modified to include heat conduction

- For convection, $\nabla_{ad} = P\delta/(T\rho c_P)$, (assuming adiabatic convection),

which is a good approximation for stellar interiors but not the convective zones in the envelope, for which thermal losses much be considered.

The mixing-length theory (Bohm-Vitense 1958) is most often used.

Convection

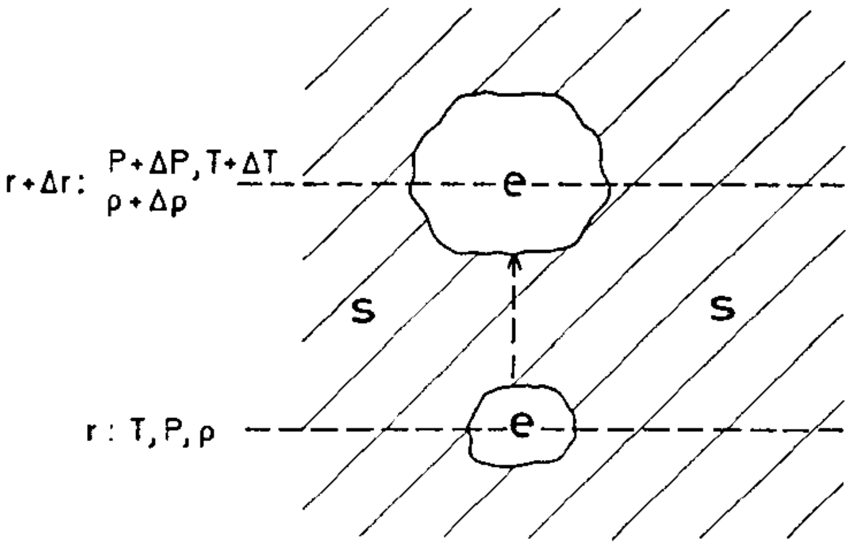
Importance:

- Transports energy and mixes composition

- Lengthens lifetime of stars (fresh fuel) if convective core is present

- Enables the enrichment of surface in giant stars with convective envelopes.

Perturbations and stability: convection



Schematic picture: the temperature excess DT is positive, if the element is hotter than its surrounding. $DP = 0$ due to hydrostatic equilibrium. If $D\rho < 0$, the element is lighter and will move upwards.

Take an element and lift it by $\triangle r$:

$$D\rho = \left[\left(\frac{\partial \rho}{\partial r} \right)_e - \left(\frac{\partial \rho}{\partial r} \right)_s \right] \triangle r$$

Stability condition

Note: The tricky bit here is to remember that these gradients are negative!

The stability condition thus is $\left(\frac{\partial\rho}{\partial r}\right)_e - \left(\frac{\partial\rho}{\partial r}\right)_s > 0.$

EOS: $\mathrm{d}\ln\rho = \alpha\mathrm{d}\ln P - \delta\mathrm{d}\ln T - \varphi\mathrm{d}\mu \rightarrow$ the stability condition changes into

$$\left(\frac{\delta}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\right)_s - \left(\frac{\delta}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\right)_e - \left(\frac{\varphi}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}r}\right)_s > 0$$

multiply the stability condition by the pressure scale height

$$H_P := -\frac{\mathrm{d}r}{\mathrm{d}\ln P} = -P\frac{\mathrm{d}r}{\mathrm{d}P} = \frac{P}{\rho g} > 0 \Rightarrow$$

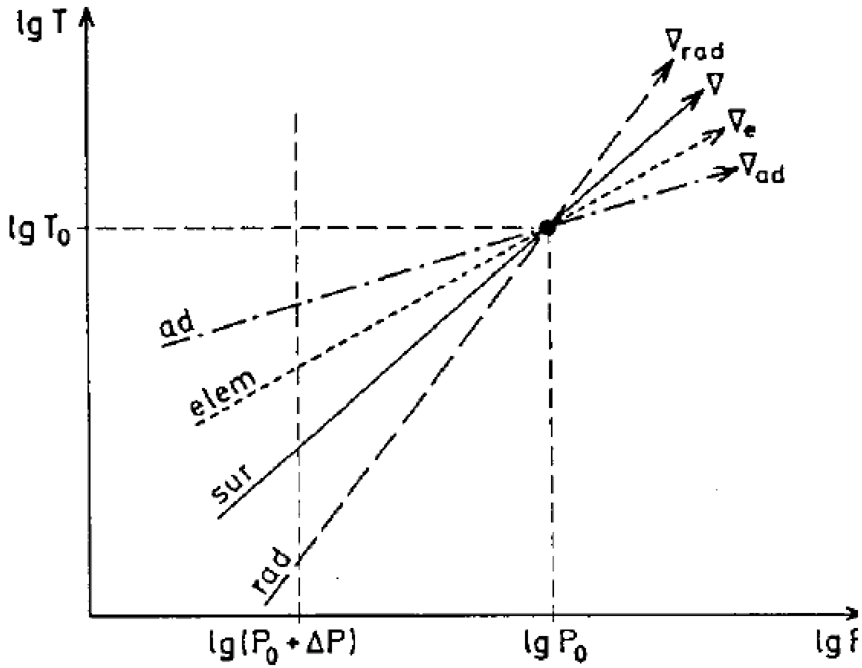
Slides taken from Achim Weiss' lectures:
http://www.mpa-garching.mpg.de/~weiss/lectures.html

Stability condition . . .

$$
\begin{aligned}
\left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_s &< \left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_e + \frac{\varphi}{\delta}\left(\frac{\mathrm{d}\ln \mu}{\mathrm{d}\ln P}\right)_s \\
\nabla_s &< \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta}\nabla_\mu \\
\nabla_{\mathrm{rad}} &< \nabla_{\mathrm{ad}} + \frac{\varphi}{\delta}\nabla_\mu
\end{aligned}
$$

The last equation holds in general cases and is called the Ledoux-criterion for dynamical stability. If $\nabla_\mu = 0$, the Schwarzschild-criterion holds. Note: $\nabla_\mu > 0$ and will stabilize.

The four ∇



In an unstable layer, the following relations hold:

$$\nabla_{\rm rad} > \nabla > \nabla_e > \nabla_{\rm ad}$$

The task of convection theory is to calculate ∇!

Convection in stars

- highly turbulent (Reynolds number $\mathrm{Re} := \frac{v \rho l_m}{\eta} \approx 10^{10}$; η viscosity; $l_m = 10^9$ cm; lab.: turbulence for $\mathrm{Re} > 100$);)
- 3-dimensional and non-local
- motion in compressible medium on dynamical timescales (v speed of blobs $\approx 10^3$ cm/s $= 10^{-5} v_{\mathrm{sound}}$)
- 3-d hydro simulations limited to illustrative cases
- 2-d hydro: larger stellar parameter range; shallow convective layers
- dynamical models: averages, simplifications, too complicated for stellar evolution

$\Rightarrow \nabla$ from simple approaches with additional extensions

The Mixing Length Theory

$$
\begin{aligned}
F &= \frac{L_r}{4\pi r^2} = F_{\rm conv} + F_{\rm rad} \\
F &=: \frac{4acG}{3}\frac{T^4 m}{\kappa P r^2}\nabla_{\rm rad} \\
F_{\rm rad} &= \frac{4acG}{3}\frac{T^4 m}{\kappa P r^2}\nabla \\
F_{\rm conv} &= \rho v c_P (DT)
\end{aligned}
$$

A blob starts somewhere with $DT > 0$ and loses identity after a typical mixing length distance l_m. On average

$$
\frac{DT}{T} = \frac{1}{T}\frac{\partial(DT)}{\partial r}\frac{l_m}{2} = (\nabla - \nabla_e)\frac{l_m}{2}\frac{1}{H_P}
$$

The mixing length parameter

- mixing length $l_m = \alpha_{\mathrm{MLT}} H_P$; α_{MLT}: mixing-length parameter
- α_{MLT} of order 1
- determined usually by solar models, $\alpha_{\mathrm{MLT}} \approx 1.6 \cdots 1.9$
- or other comparisons with observations (giant stars with CE)
- NO calibration or meaning!

Examples for ∇: Sun: $r = R_\odot/2$, $m = M_\odot/2$, $T = 10^7$, $\rho = 1$, $\delta = \mu = 1$

$$\rightarrow U = 10^{-8} \rightarrow \nabla = \nabla_{\mathrm{ad}} + 10^{-5} = 0.4$$

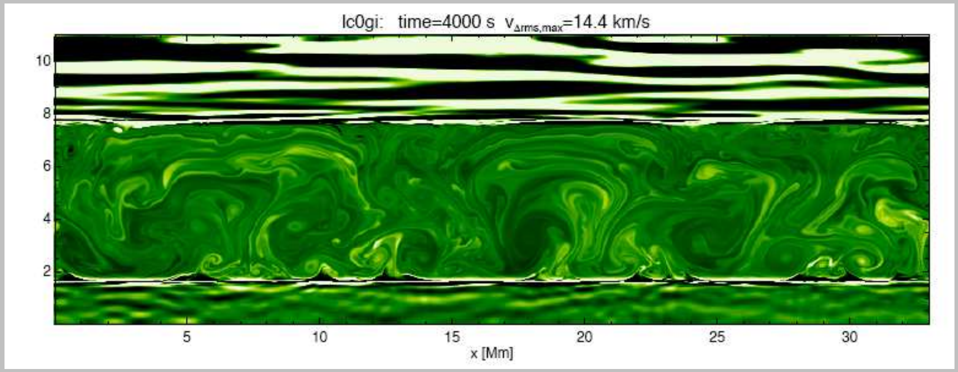
(as long as $\nabla_{\mathrm{rad}} < 100 \cdot \nabla_{\mathrm{ad}}$); at center, $\nabla = \nabla_{\mathrm{ad}} + 10^{-7}$.

Deficits of MLT

- local theory $\longrightarrow$ no overshooting from convective boundaries due to inertia of convective elements
- time-independent $\longrightarrow$ instantaneous adjustment; critical if other short timescales (pulsations, nuclear burning) present
- only one length scale, but spectrum of turbulent eddies $\longrightarrow$ improvements by Canuto & Mazzitelli
- presence of chemical gradients ignored (semiconvection) $\longrightarrow$ treatment of such layers unclear; probably slow mixing on diffusive timescale due to secular g-modes; T-gradient radiative?

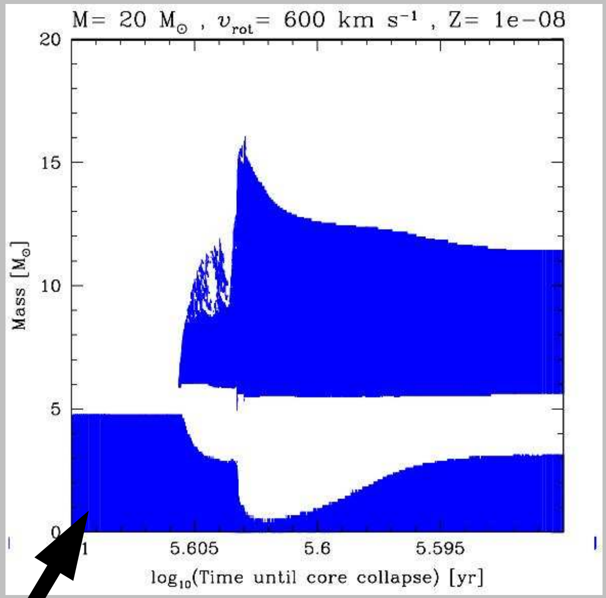
3to1D link for convection

3D simulations

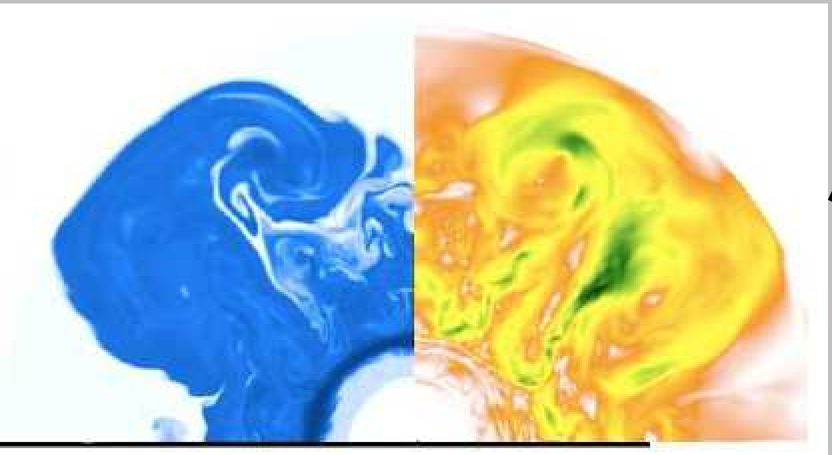


Herwig et al 06

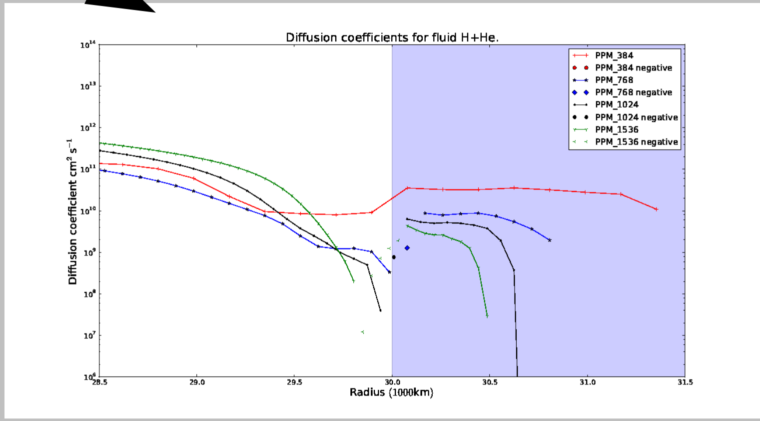
Uncertainties in 1D



e.g. Hirschi 07



e.g. Arnett & Meakin 2011
Mocak et al 2011, ...



Meakin et al 2009 ; Bennett et al in prep

Determine effective diffusion (advection?) coefficient

The overall problem

- $m = M_r$: Lagrangian coordinate
- r, P, T, L_r: independent variables
- X_i: composition variables
- ρ, κ, ϵ, ...: dependent variables
- initial value problem in time: $r(m, t = 0)$, $P(m, t = 0)$, $T(m, t = 0)$, $L_r(m, t = 0)$, $\vec{X}(m, t = 0) = \vec{X}(t = 0) \to$ integration with time
- boundary value problem in space: $r(m = 0, t) = 0$, $L_r(m = 0, t) = 0$ and $L = 4\pi\sigma R^2 T_{\mathrm{eff}}^4$, $P(m = M) = P(\tau = 2/3)$

Slides taken from Achim Weiss' lectures:
http://www.mpa-garching.mpg.de/~weiss/lectures.html

The equations

The four structure equations to be solved are:

$$\begin{aligned}
\frac{\partial r}{\partial m} &= \frac{1}{4\pi r^2 \rho} \\
\frac{\partial P}{\partial m} &= -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \\
\frac{\partial L_r}{\partial m} &= \epsilon_n - \epsilon_\nu - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \\
\frac{\partial T}{\partial m} &= -\frac{GmT}{4\pi r^4 P}\nabla
\end{aligned}$$

The equations

For energy transport, we have to find the appropriate ∇.
In case of radiative transport, this is:

$$\nabla_{\rm rad} = \frac{3}{16\pi acG}\frac{\kappa L_r P}{mT^4}$$

Finally, for the composition, we have

$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho}\left(\sum_j r_{ji} - \sum_k r_{ik}\right) + \text{diff. or similar terms}$$

Numerical Procedure

In addition: $\rho(P, T, X_i)$, $\kappa(P, T, X_i)$, $r_{jk}(P, T, X_i)$, $\epsilon_n(P, T, X_i)$, $\epsilon_\nu(P, T, X_i)$, ...

In space (mass $0 \leq m \leq M$): boundary value problem with boundary conditions:

- at center: $r(0) = 0$, $L_r(0) = 0$
- at $r = R$: P and T either from $P = 0$, $T = 0$ ("zero b.c.") or from atmospheric lower boundary and:
- Stefan-Boltzmann-law $L = 4\pi\sigma R^2 T_{\text{eff}}^4$

Central conditions

Series expansion in m around center:

$$
\begin{aligned}
r &= \left(\frac{3}{4\pi\rho_c}\right)^{1/3} m^{1/3} \\
P &= P_c - \frac{3G}{8\pi}\left(\frac{4\pi}{3}\rho_c\right)^{4/3} m^{2/3} \\
L_r &= (\epsilon_g + \epsilon_n - \epsilon_\nu)_c m
\end{aligned}
$$

Stellar atmospheres (hydrostatic, grey):

- optical depth: $\tau := \int_R^\infty \kappa\rho \mathrm{d}r = \bar{\kappa}\int_R^\infty \rho \mathrm{d}r$;
- pressure $P(R) := \int_R^\infty g\rho \mathrm{d}r \approx g_R \int_R^\infty \rho \mathrm{d}r \Rightarrow P = \frac{GM}{R^2}\frac{2}{3}\frac{1}{\bar{\kappa}}$

In time, we have an initial value problem (zero-age model).

Numerical methods

Spatial problem

1. **Direct integration** : e.g. Runge-Kutta
2. **Difference method** : difference equations replace differential equations
3. **Hybrid methods** : direct integration between fixed mesh-points; multiple-fitting method, but the variation of the guesses at the fixed points is done via a Newton-method

The Henyey-method

Write the equations in a general form:

$$A_i^j := \frac{y_i^{j+1} - y_i^j}{m_i^{j+1} - m_i^j} - f_i(y_1^{j+1/2}, \dots, y_4^{j+1/2})$$

upper index: grid-point ($j + 1/2$ mean value); lower index: i-th variable
Outer and inner boundary conditions:

$$B_i = 0 \quad i = 1, 2 \qquad C_i = 0 \quad i = 1, \dots, 4$$

where the inner ones are to be taken at grid-point $N - 1$ and the expansions around $m = 0$ have been used already.

Henyey-method (contd.)

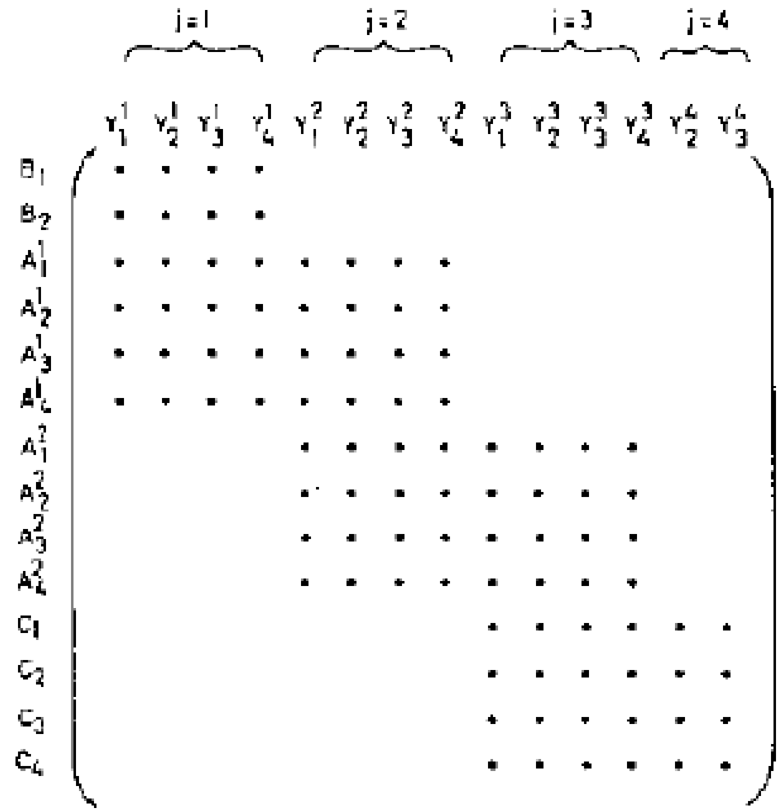
$\rightarrow 2 + 4 + (N-2) \cdot 4 = 4N - 2$ equs.
for $4 \times N$ unknowns -2 b.c.
Newton-approach for corrections δy_i

$$A_i^j + \sum_i \frac{\partial A_i^j}{\partial y_i} \delta y_i = 0$$

$$H \begin{pmatrix} \delta y_1^1 \\ \delta y_2^1 \\ \vdots \\ \delta y_3^N \\ \delta y_4^N \end{pmatrix} = \begin{pmatrix} B_1 \\ \vdots \\ A_i^j \\ \vdots \\ C_4 \end{pmatrix}$$

Henyey-scheme

Matrix H contains all derivatives and is called Henyey-matrix. It contains non-vanishing elements only in blocks. This leads to a particular method for solving it (Henyey-method).



Expresse some of corrections in terms of others, e.g.:

$\delta y_1^1 = U_1 \delta y_3^2 + V_1 \delta y_4^2 + W_1 \Rightarrow$ matrix equations for U_i, V_i, W_i.

Henyey-scheme

First block-matrix $j = 1, 2$:

$$
\begin{bmatrix}
\frac{\partial B_1}{\partial y_1^1} & \frac{\partial B_1}{\partial y_2^1} & \cdots & 0 \\
\frac{\partial B_2}{\partial y_1^1} & \frac{\partial B_2}{\partial y_2^1} & \cdots & 0 \\
\frac{\partial A_1^1}{\partial y_1^1} & \frac{\partial A_1^1}{\partial y_2^1} & \cdots & \frac{\partial A_1^1}{\partial y_2^2} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{\partial A_1^4}{\partial y_1^1} & \frac{\partial A_1^4}{\partial y_2^1} & \cdots & \frac{\partial A_1^4}{\partial y_2^2}
\end{bmatrix}
\begin{bmatrix}
U_1 & V_1 & W_1 \\
U_2 & V_2 & W_2 \\
U_3 & V_3 & W_3 \\
\vdots & \vdots & \vdots \\
U_6 & V_6 & W_6
\end{bmatrix}
=
\begin{bmatrix}
0 & 0 & -B_1 \\
0 & 0 & -B_2 \\
-\frac{\partial A_1^1}{\partial y_3^2} & -\frac{\partial A_1^1}{\partial y_4^2} & -A_1^1 \\
\vdots & \vdots & \vdots \\
-\frac{\partial A_4^1}{\partial y_3^2} & -\frac{\partial A_4^1}{\partial y_4^2} & -A_4^1
\end{bmatrix}
$$

Integration in time

$$X_i(t + \triangle t) = X_i(t) + \frac{\partial X_i}{\partial t}(T(t), P(t), \ldots)\triangle t$$

Improvement:
backward differencing (e.g. nuclear network)

$$X_i(t + \triangle t) = X_i(t) + \triangle t \sum_i r_{ij}(t) X_i(t + \triangle t) X_j(t + \triangle t)$$

done for chemical evolution, mixing, diffusion, etc.

Nowadays matrices can be inverted without splitting them into small sections and without decoupling of space and time, see e.g. MESA code: Paxton et al 2011

Massive Stars: Evolution of the chemical composition

Burning stages (lifetime [yr]):

Hydrogen ($10^{6\text{-}7}$): $^{1}H \rightarrow {}^{4}He$

& ^{12}C, $^{16}O \rightarrow {}^{14}N$

Helium ($10^{5\text{-}6}$): $^{4}He \rightarrow {}^{12}C$, ^{16}O

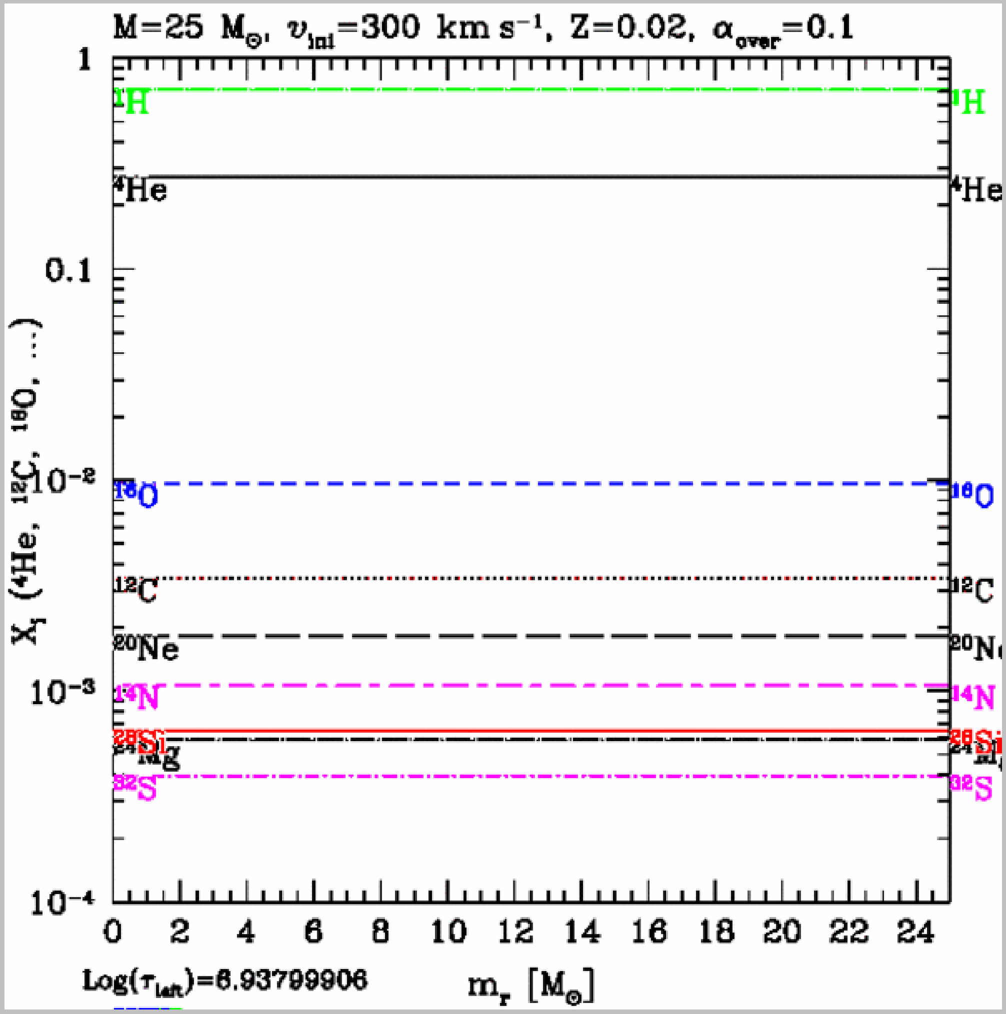
& $^{14}N \rightarrow {}^{18}O \rightarrow {}^{22}Ne$

Carbon ($10^{2\text{-}3}$): $^{12}C \rightarrow {}^{20}Ne$, ^{24}Mg

Neon (0.1-1): $^{20}Ne \rightarrow {}^{16}O$, ^{24}Mg

Oxygen (0.1-1): $^{16}O \rightarrow {}^{28}Si$, ^{32}S

Silicon (10^{-3}): ^{28}Si, $^{32}S \rightarrow {}^{56}Ni$



http://www.astro.keele.ac.uk/~hirschi/animation/anim.html

L1: Basics of Stellar Structure and Evolution

- Importance and observational constraints

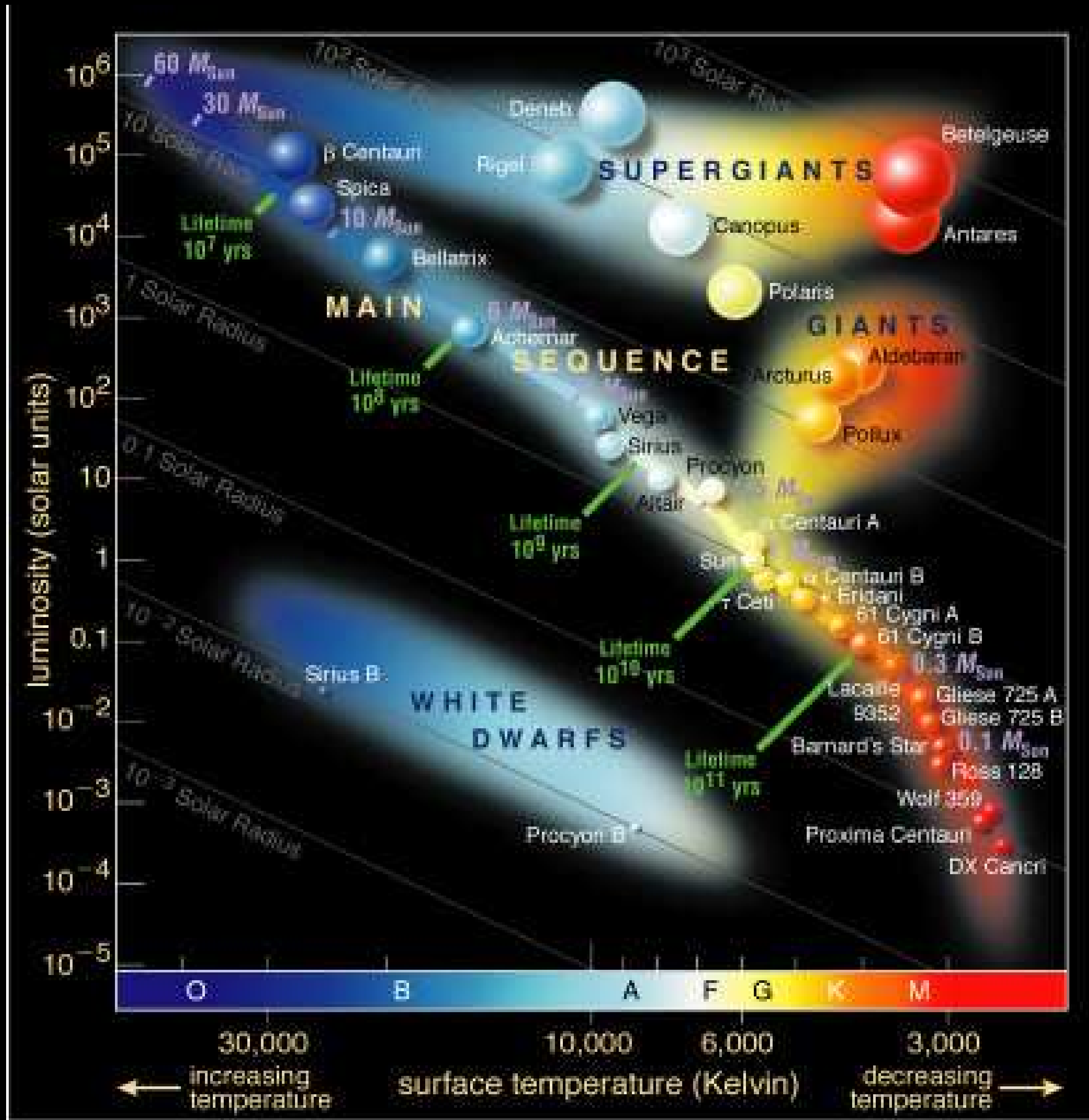
- Physics governing the structure and evolution of stars

- Equations of stellar structure

- Modelling stars and their evolution

Importance as Stellar Objects

Hertzsrpung-Russell (HR) Diagram:



Physical Ingredients

Acknowledgements & Bibliography

- Slides in white background (with blue title) were taken from Achim Weiss' lecture slides, which you can find here: http://www.mpa-garching.mpg.de/~weiss/lectures.html

- Some graphs were taken from Onno Pols' lecture notes on stellar evolution, which you can find here:

http://www.astro.ru.nl/~onnop/education/stev_utrecht_notes/

- Some slides (colourful ones) and content was taken from George Meynet's summer school slides.

Acknowledgements & Bibliography

Recommended further reading:

- R. Kippenhahn & A. Weigert, Stellar Structure and Evolution, 1990, Springer-Verlag, ISBN 3-540-50211-4

- A. Maeder, Physics, Formation and Evolution of Rotating Stars, 2009, Springer-Verlag, ISBN 978-3-540-76948-4

- D. Prialnik, An Introduction to the Theory of Stellar Structure and Evolution, 2000, Cambridge University Press, ISBN 0-521-65937-X

- C.J. Hansen, S.D. Kawaler & V. Trimble, Stellar Interiors, 2004, Springer-Verlag, ISBN 0-387-20089-4

- M. Salaris & S. Cassisi, Evolution of Stars and Stellar Populations, 2005, John Wiley & Sons, ISBN 0-470-09220-3

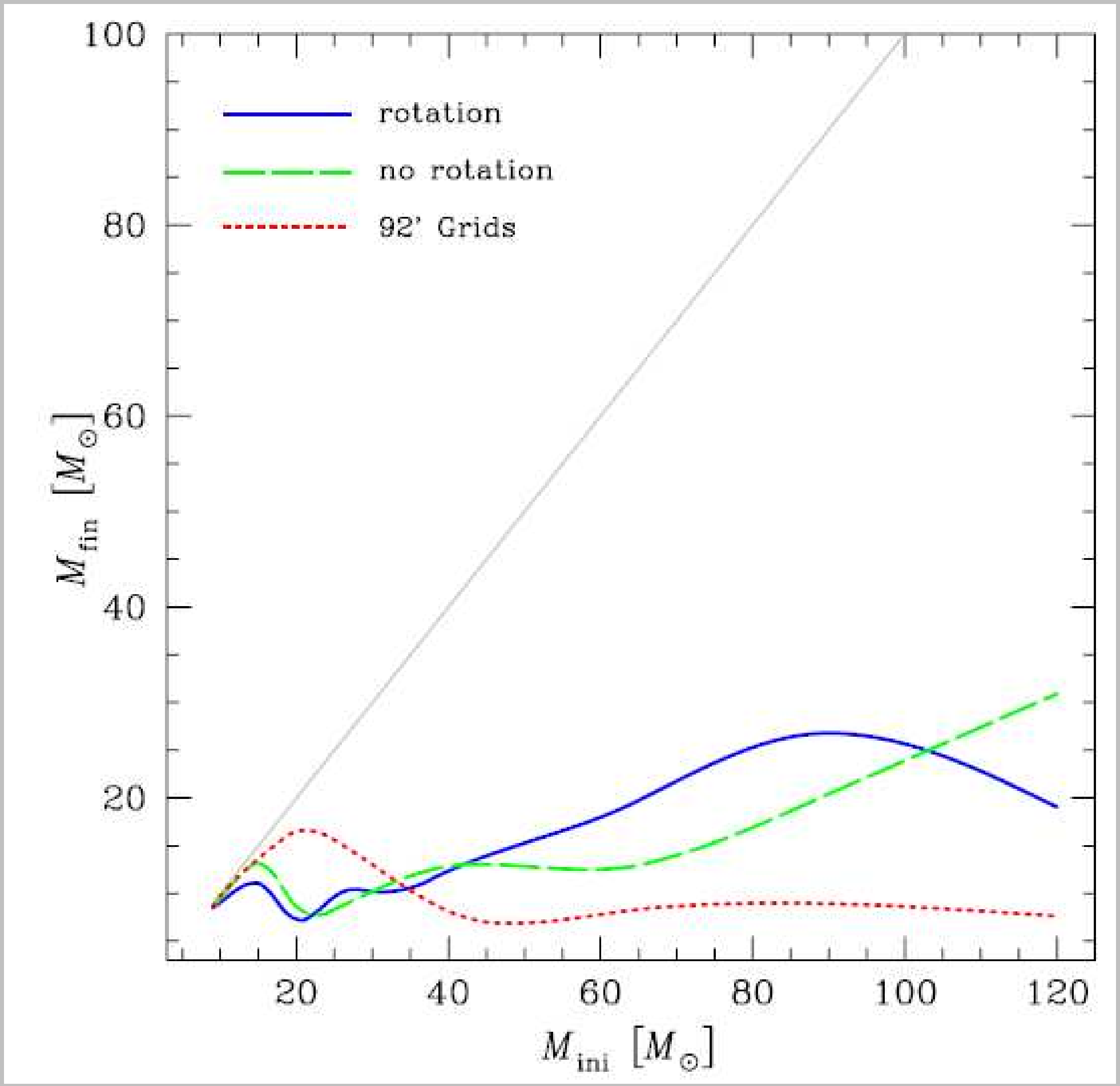
L2: Physical Ingredients

Importance, basics, effects, uncertainties of:

- Nuclear reactions → B. Meyer
- Convection in L1
- Mass loss
- Rotation
- Magnetic fields
- Binarity
- Equation of state, opacities & neutrino losses

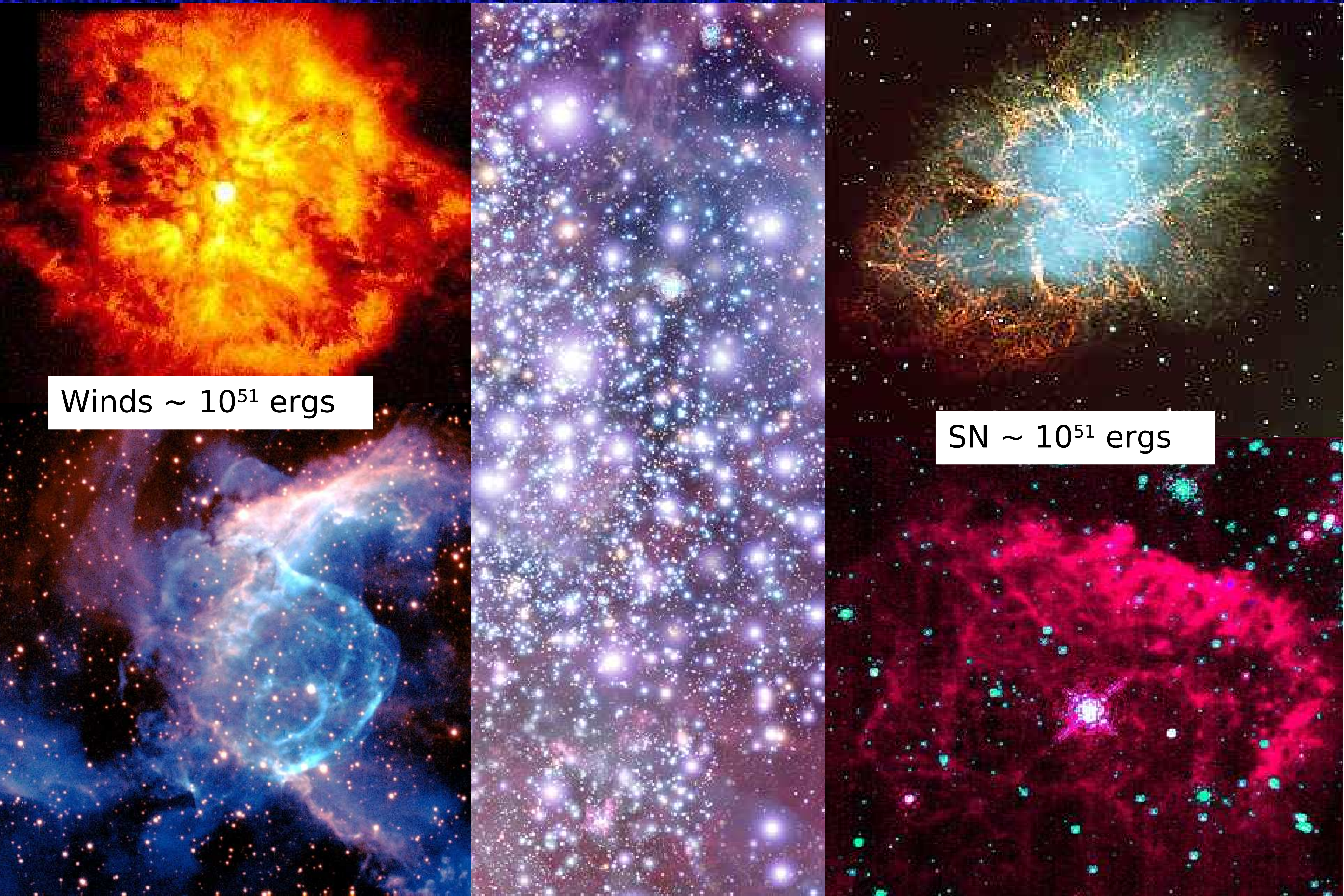
including metallicity dependence

Importance of Mass Loss



Ekström et al 12, see also Chieffi & Limongi 13

Injection of Mechanical Energy



Mass Loss: General Dependence on Stellar Mass

More massive stars have stronger winds because they are much more luminous:

For low-mass stars:

$$L \propto \frac{\beta^4 \mu^4 M^3}{\kappa}$$

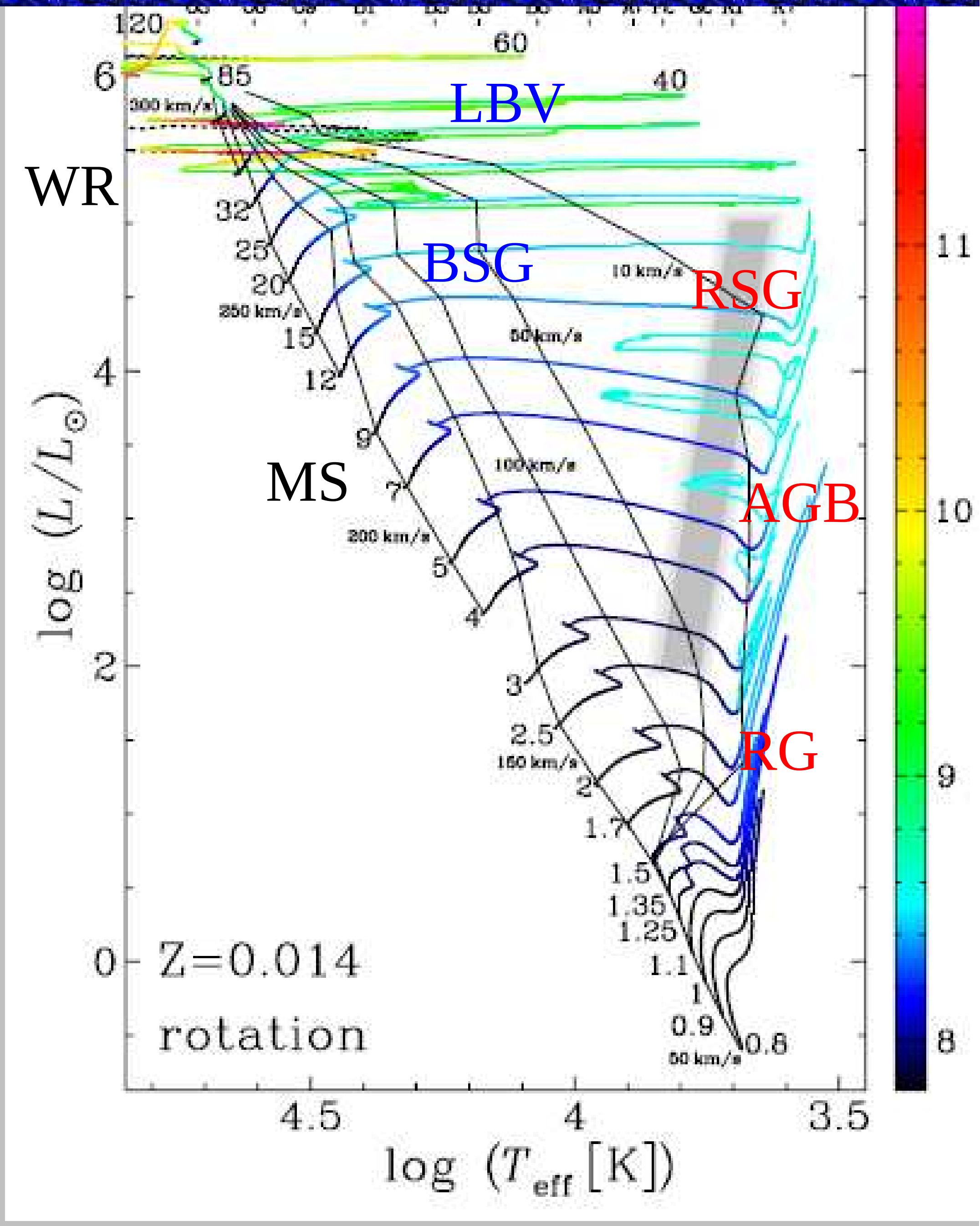
For high-mass stars:

$$L \propto \frac{\mu^4 M}{\kappa}$$

Where β is the ratio of gas to total pressure: 1 → 0 from low to high-mass stars

Main Phases of Stellar Evolution

Mass loss driving mechanism and prescriptions are very different for different evolutionary stages

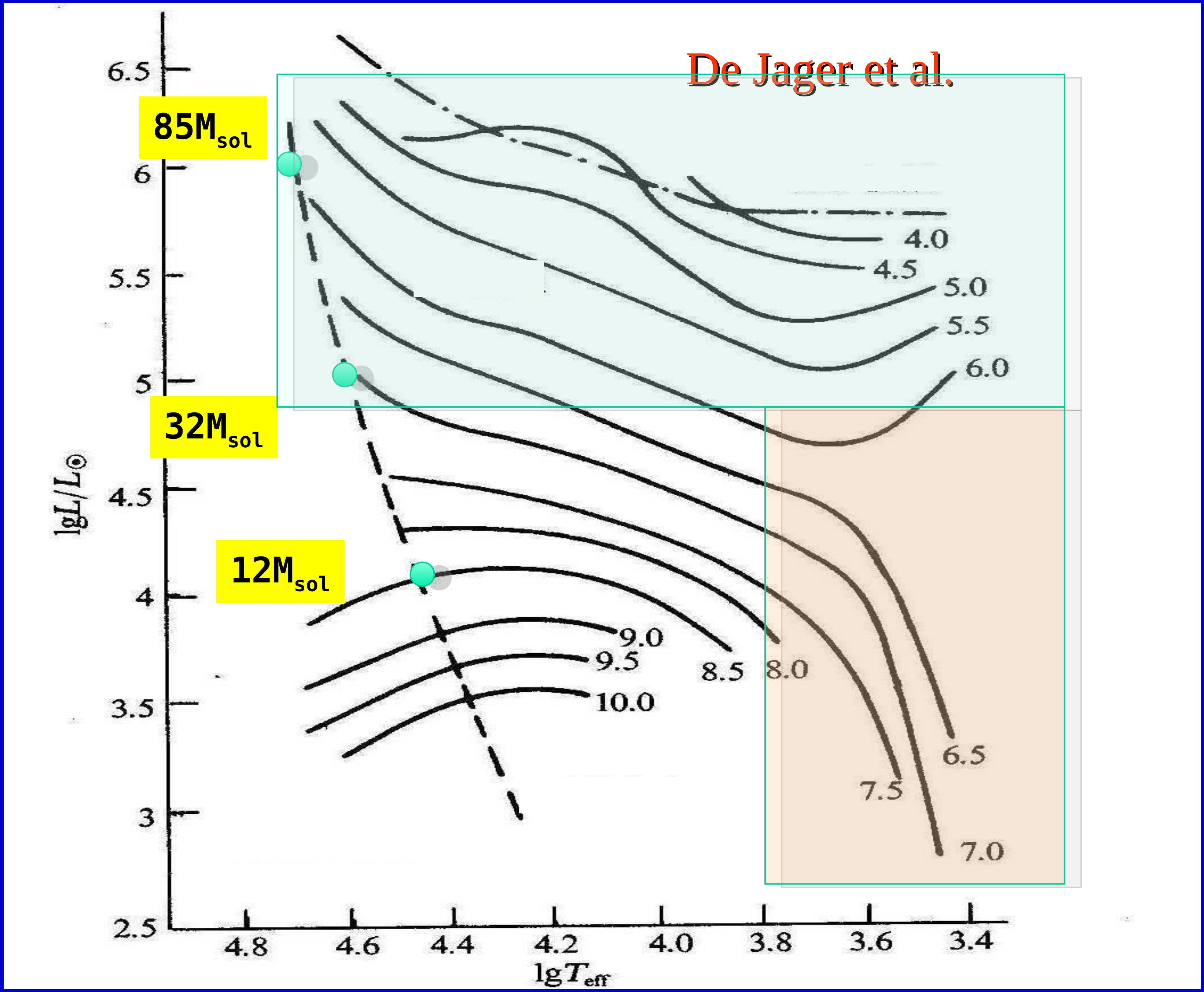


Ekstroem et al 12

Mass Loss: Types, Driving & Recipes

Mass loss driving mechanism and prescriptions at different stages:

- O-type & “LBV” stars (bi-stab.): line-driven Vink et al 2000, 2001
- WR stars (clumping effect): line-driven Nugis & Lamers 2000, Gräfener & Hamann (2008)
- RSG: Pulsation/dust? de Jager et al 1988
- RG: Pulsation/dust? Reimers 1975,78, with η=~0.5
- AGB: Super winds? Dust Bloecker et al 1995, with η=~0.05
- LBV eruptions: continuous driven winds? Owocki et al
- ...



De Jager et al.
85M$_{sol}$
32M$_{sol}$
12M$_{sol}$
$\lg L/L_{\odot}$
$\lg T_{eff}$
6.5
6
5.5
5
4.5
4
3.5
3
2.5
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0
9.5
10.0

What changes at low Z?

- Stars are more compact: $R \sim R(Z_o)/4$ (lower opacities) at $Z=10^{-8}$
- Mass loss weaker at low Z: → faster rotation

$$\dot{M}(Z)=\dot{M}(Z_o)(Z/Z_o)^{\alpha}$$

- α = 0.5-0.6 (Kudritzki & Puls 00, Ku02) (Nugis & Lamers, Evans et al 05)

- α = 0.7-0.86 (Vink et al 00,01,05)

Z(LMC)~Z_o/2.3 => Mdot/1.5 – Mdot/2

Z(SMC)~Z_o/7 => Mdot/2.6 - Mdot/5

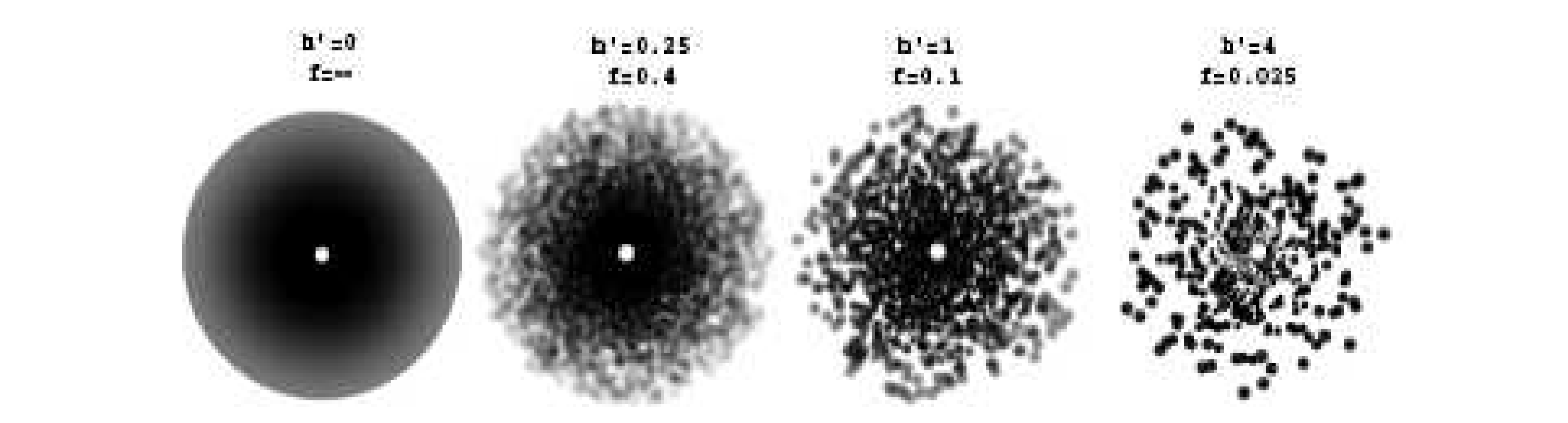
Mass loss at low Z still possible?

RSG (and LBV?): no Z-dep.; CNO? (Van Loon 05, Owocky et al)

Mechanical mass loss ← critical rotation

(e.g. Hirschi 2007, Ekstroem et al 2008, Yoon et al 2012)

CLUMPING



ρ^2

diagnostics

If wind clumped in reality but supposed to be homogeneous

Excess emission from inhomogeneities →
incorrectly interpreted as
arising from a smooth but denser medium

MASS LOSS OVERESTIMATED

Fullerton et al 05: Mdot/10

Bouret et al 05: Mdot/3 or smaller

Surlan et al 13: problem resolved?

RSG/YSG/WR – SN II, IIb, Ibc

Observational constraints:

- RSG Upper Luminosity: Log (L/L_{SUN}) ~ 5.2-5.3 *(median value of the most 5 L_{SUN} stars)* (Levesque et al 05)

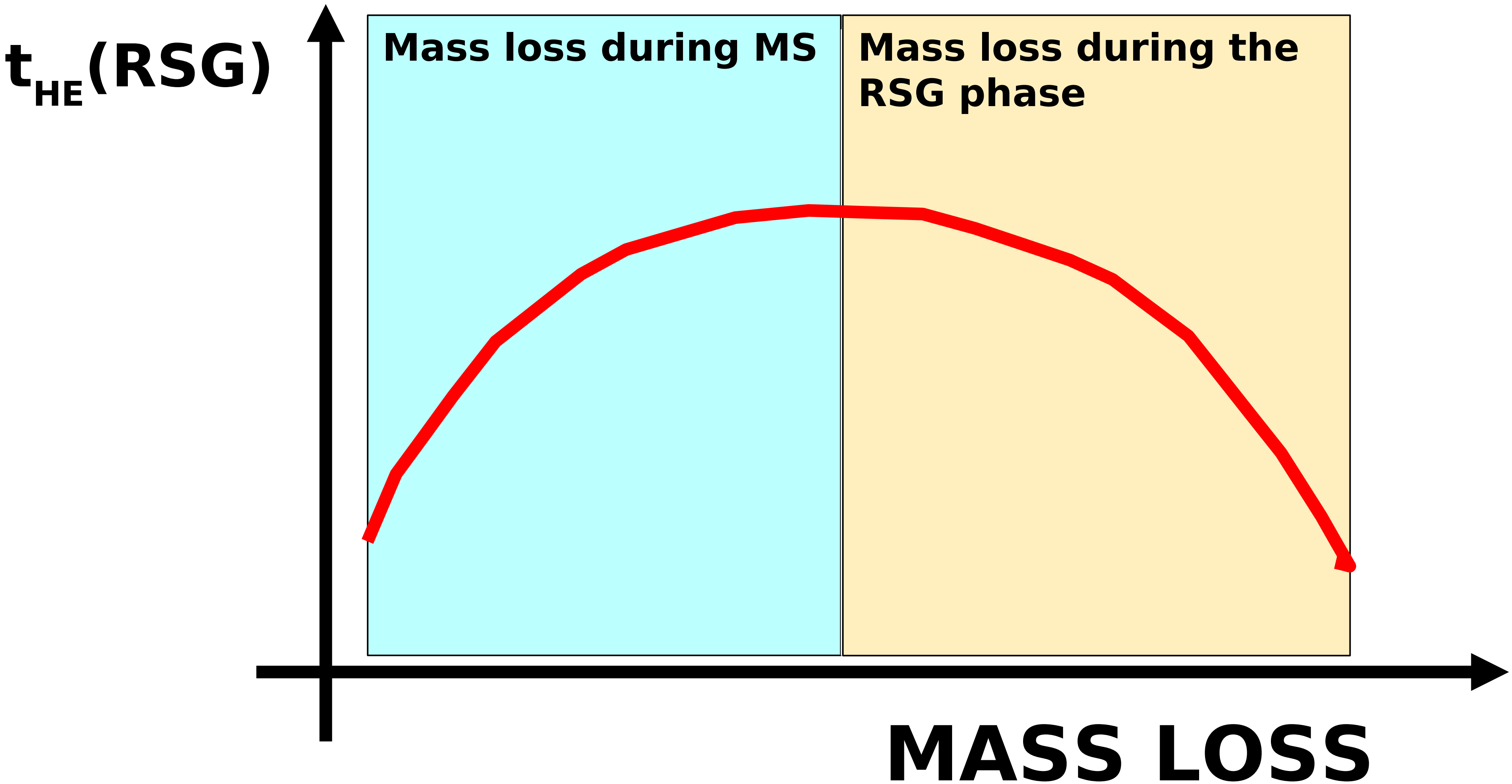
- SNII-P Log (L/L_{SUN}) <~5.1 (Smartt et al. 2009)

- No clear dependence on Z for these upper limit

- WR/O, RSG/BSG ratios vary with Z

CHANGE OF MASS LOSS

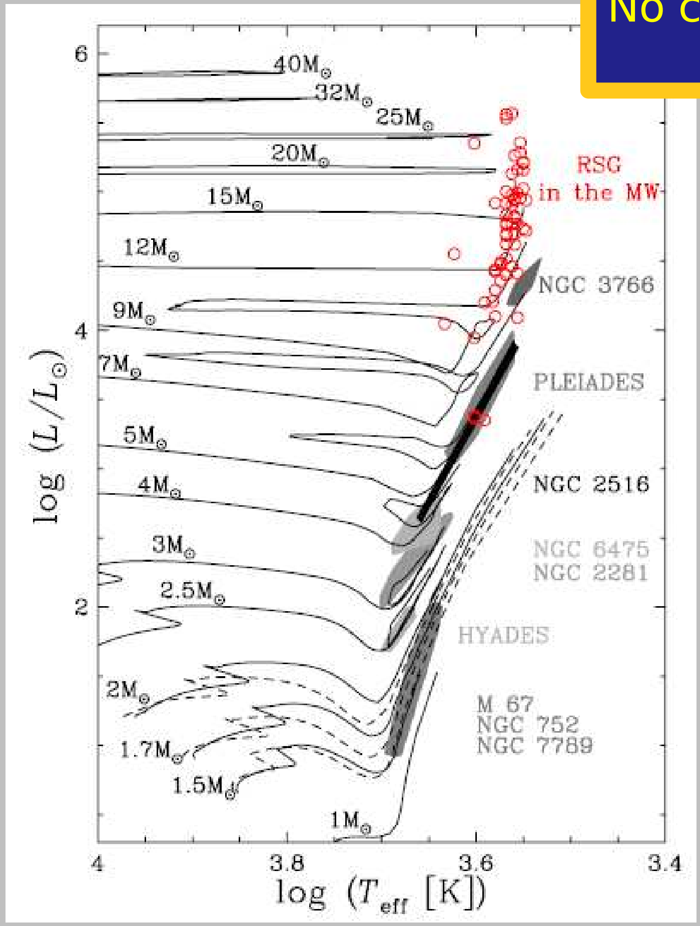
For a given initial mass

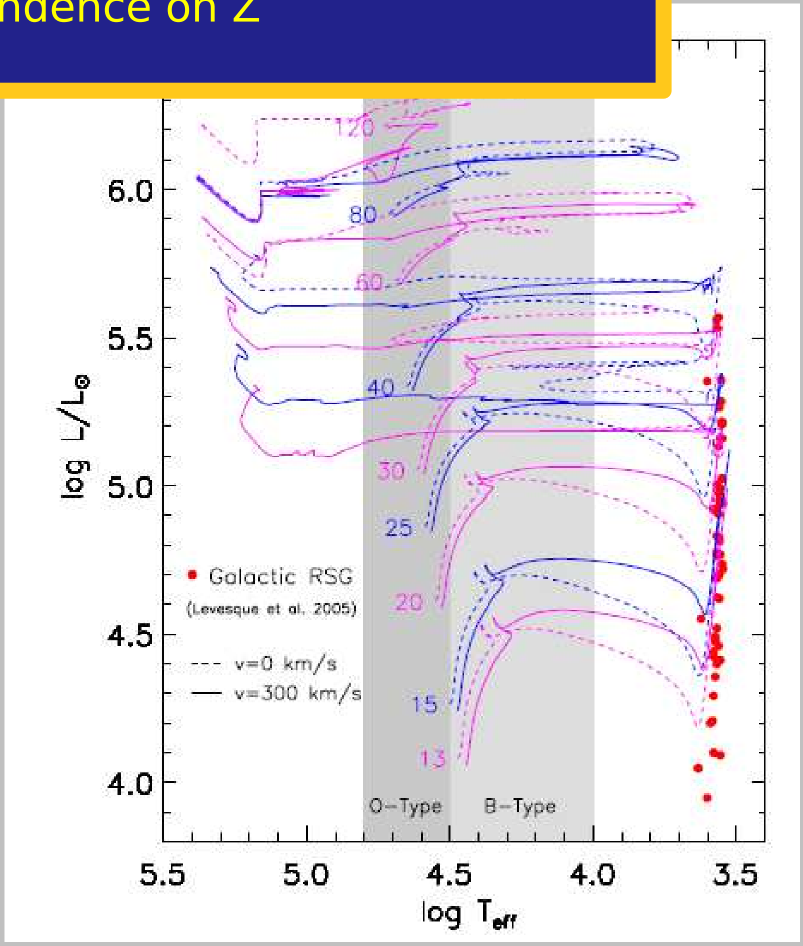


RSG/YSG/WR – SN II, IIb, Ib, Ic

RSG Upper Luminosity ~ 5.2-5.3
(median value of the most 5 L_{SUN} stars)
SNII-P ~ 5.1 (Smartt et al. 2009)

No clear dependence on Z





- Tracks: Ekstroem et al 12
- grey areas: obs. See MM89
- red circles: Levesque et al 05

- Tracks: Chieffi & Limongi 13 (CL13)

Rotation

Importance:

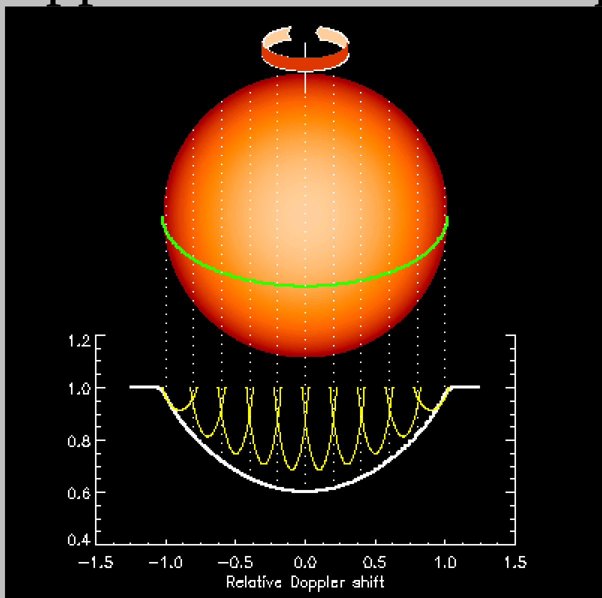
- Induces instabilities in radiative zones (none otherwise) → additional mixing of composition

- Changes properties of stars: shape, L, T_{eff}

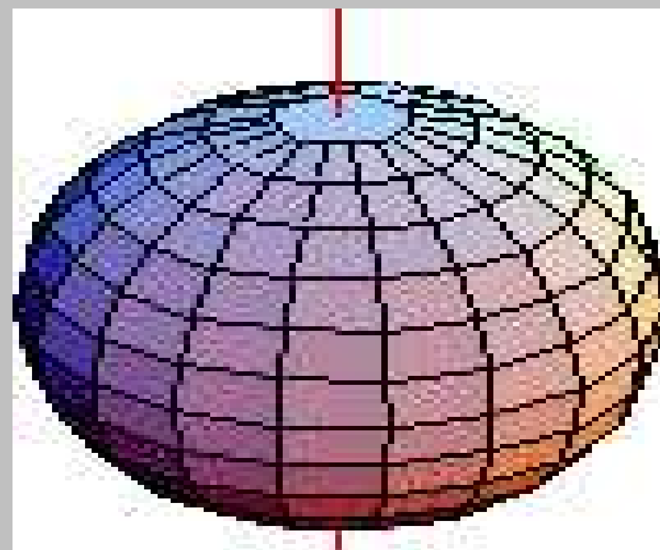
- Powers some stellar explosions (e.g. GRB, magnetars).

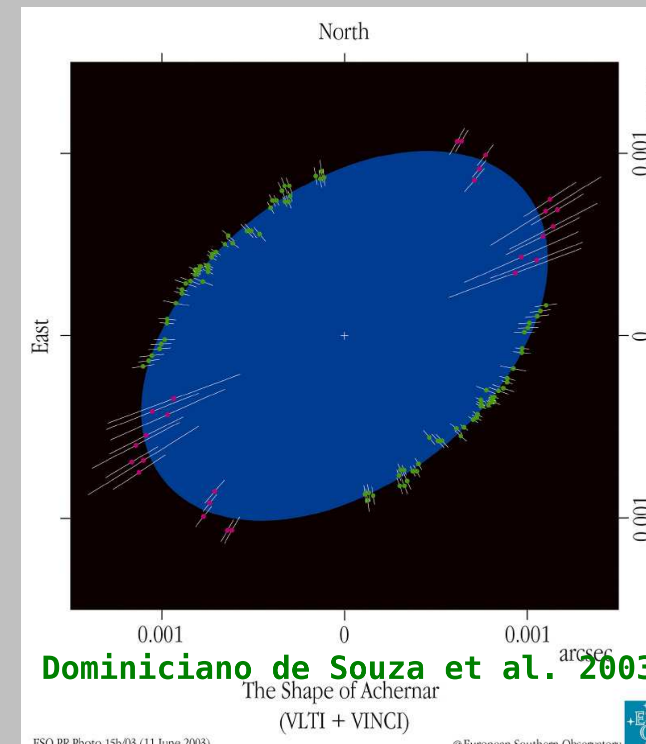
Rotational Effects on Surface

Doppler-broadened line profile

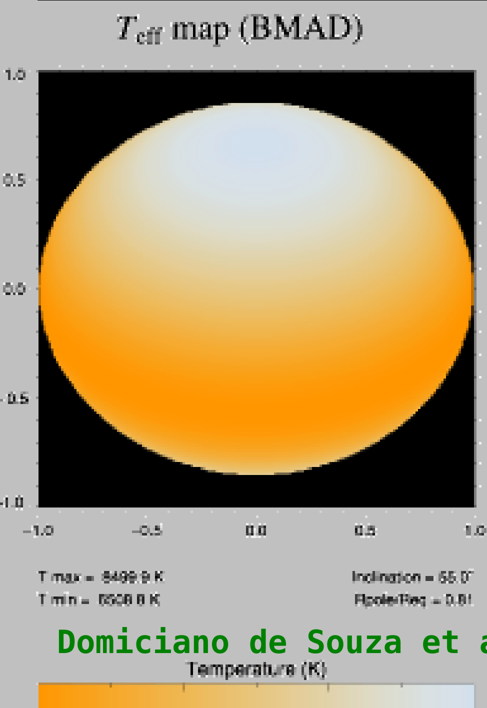


Fast rotators –> oblate shape:





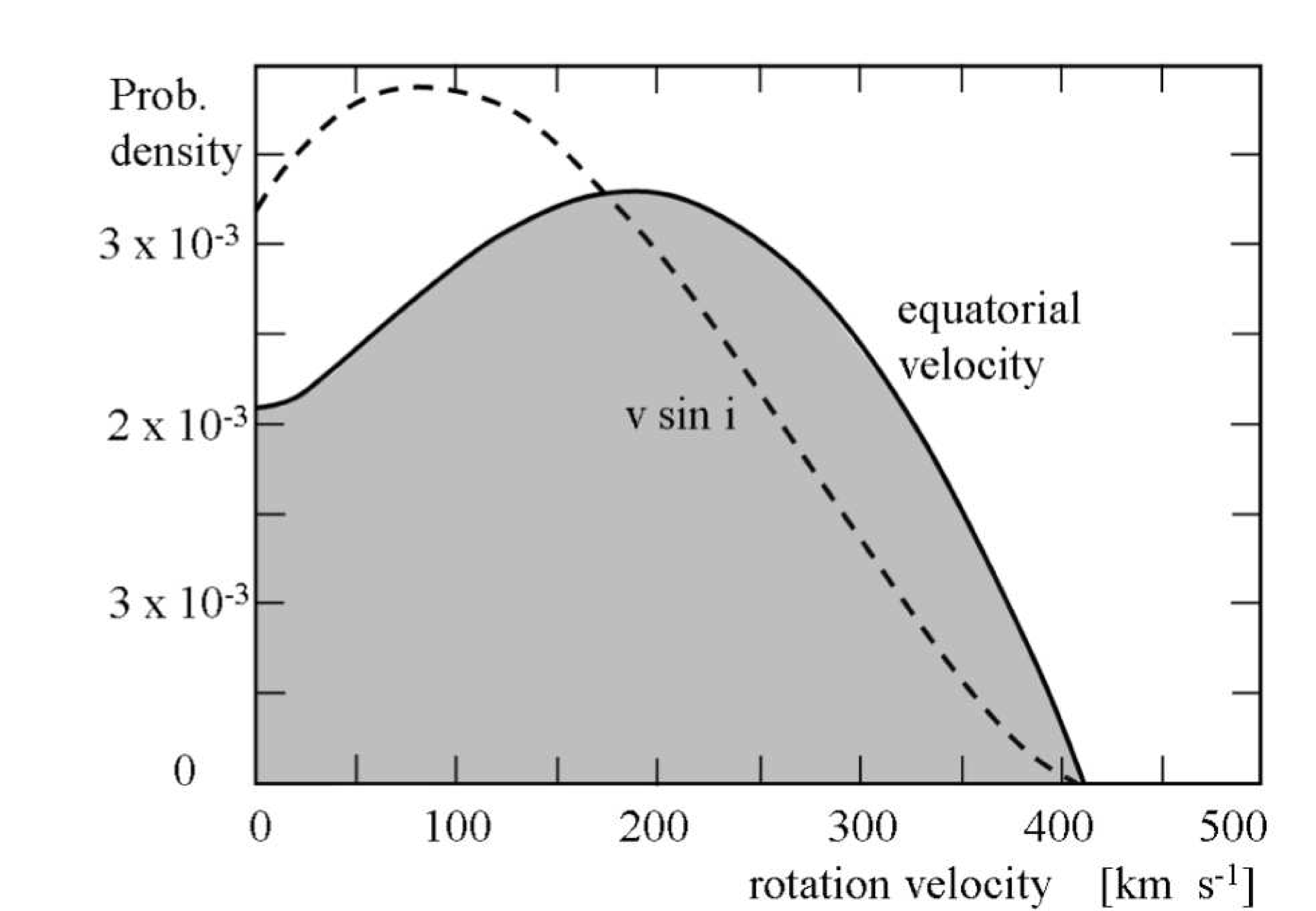
Dominiciano de Souza et al. 2003



Domiciano de Souza et al. 2005

← Altair: pole brighter than equator: Effect compatible with von-Zeipel theorem ()

Rotation velocity Distribution



Data from Huang & Gies 2006, Figure from Maeder 2009

Geneva Stellar Evolution Code

1.5D hydrostatic code (Eggenberger et al 2008)

Rotation: (Maeder & Meynet 2008)

Centrifugal force: KEY FOR GRB prog.

$$\vec{g}_{\rm eff} = \vec{g}_{\rm eff}(\Omega,\theta) = \left(-\frac{GM}{r^2} + \Omega^2 r\,\sin^2\theta\right)\vec{e}_r + \Omega^2 r\,\sin\theta\,\cos\theta\,\vec{e}_\theta$$

Shellular rotation → still 1D: (Zahn 1992)

- Energy conservation:

$$\frac{\partial L_P}{\partial M_P} = \epsilon_{nucl} - \epsilon_\nu + \epsilon_{grav} = \epsilon_{nucl} - \epsilon_\nu - c_P\frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \tag{2.9}$$

- Momentum equation:

$$\frac{\partial P}{\partial M_P} = -\frac{GM_P}{4\pi r_P^4} f_P \tag{2.10}$$

- Mass conservation (or continuity equation):

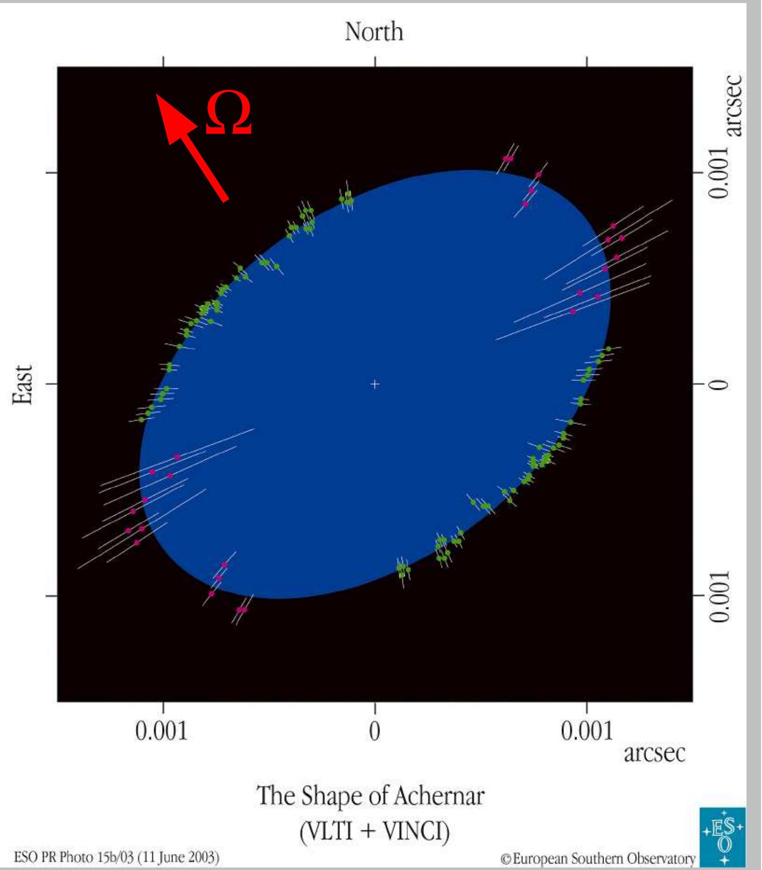
$$\frac{\partial r_P}{\partial M_P} = \frac{1}{4\pi r_P^2\overline{\rho}} \tag{2.11}$$

- Energy transport equation:

$$\frac{\partial \ln \overline{T}}{\partial M_P} = -\frac{GM_P}{4\pi r_P^4} f_P \min[\nabla_{\rm ad}, \nabla_{\rm rad}\frac{f_T}{f_P}] \tag{2.12}$$

where

$$\nabla_{\rm ad} = \frac{P\delta}{\overline{T}\overline{\rho}c_P} \quad \text{(convective zones)},$$
$$\nabla_{\rm rad} = \frac{3}{16\pi acG}\frac{\kappa l P}{m\overline{T}^4} \quad \text{(radiative zones)},$$

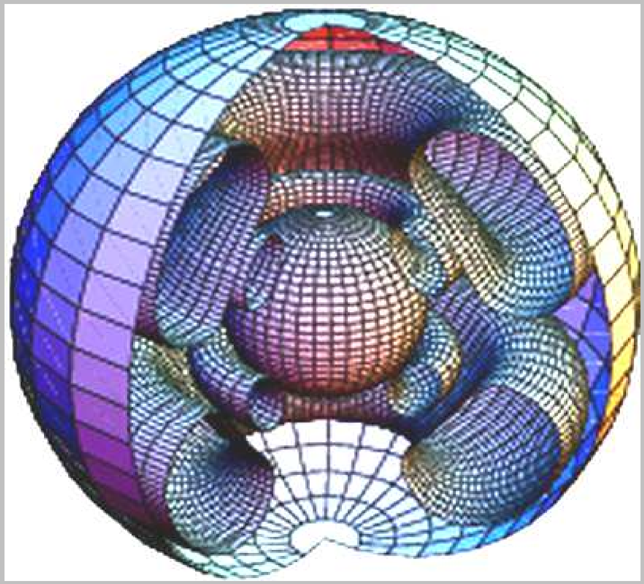


$$f_P = \frac{4\pi r_P^4}{GM_P S_P}\frac{1}{<g^{-1}>},$$
$$f_T = \left(\frac{4\pi r_P^2}{S_P}\right)^2 \frac{1}{<g><g^{-1}>},$$

(Meynet and Meynet 97)

Rotation Induced Transport

Zahn 1992: strong horizontal turbulence



Meynet & Maeder 2000

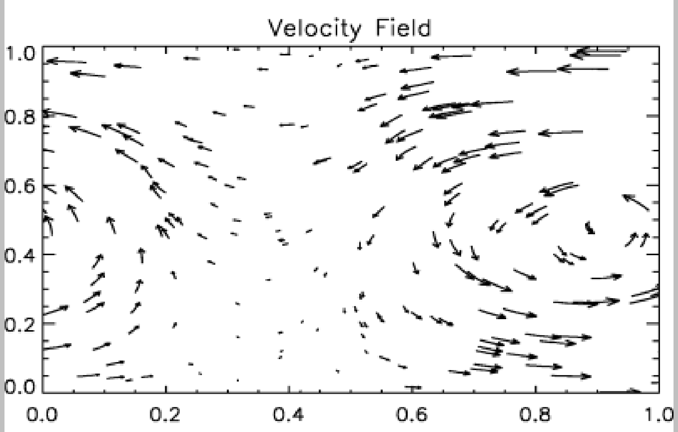
Transport of angular momentum:

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\bar{\Omega}\right)_{M_r} = \underbrace{\frac{1}{5r^2}\frac{\partial}{\partial r}\left(\rho r^4\bar{\Omega}U(r)\right)}_{\text{advection term}} + \underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho D r^4\frac{\partial\bar{\Omega}}{\partial r}\right)}_{\text{diffusion term}}$$

Transport of chemical elements:

$$\rho\frac{\mathrm{d}X_i}{\mathrm{d}t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho r^2\left[D + D_{eff}\right]\frac{\partial X_i}{\partial r}\right) + \left(\frac{\mathrm{d}X_i}{\mathrm{d}t}\right)_{\mathrm{nucl}}$$

Shear instabilities



Brueggen & Hillebrandt 2000

D: diffusion coeff. due to various transport mechanisms (convection, shear)

D_{eff}: diffusion coeff. due to meridional circulation + horizontal turbulence

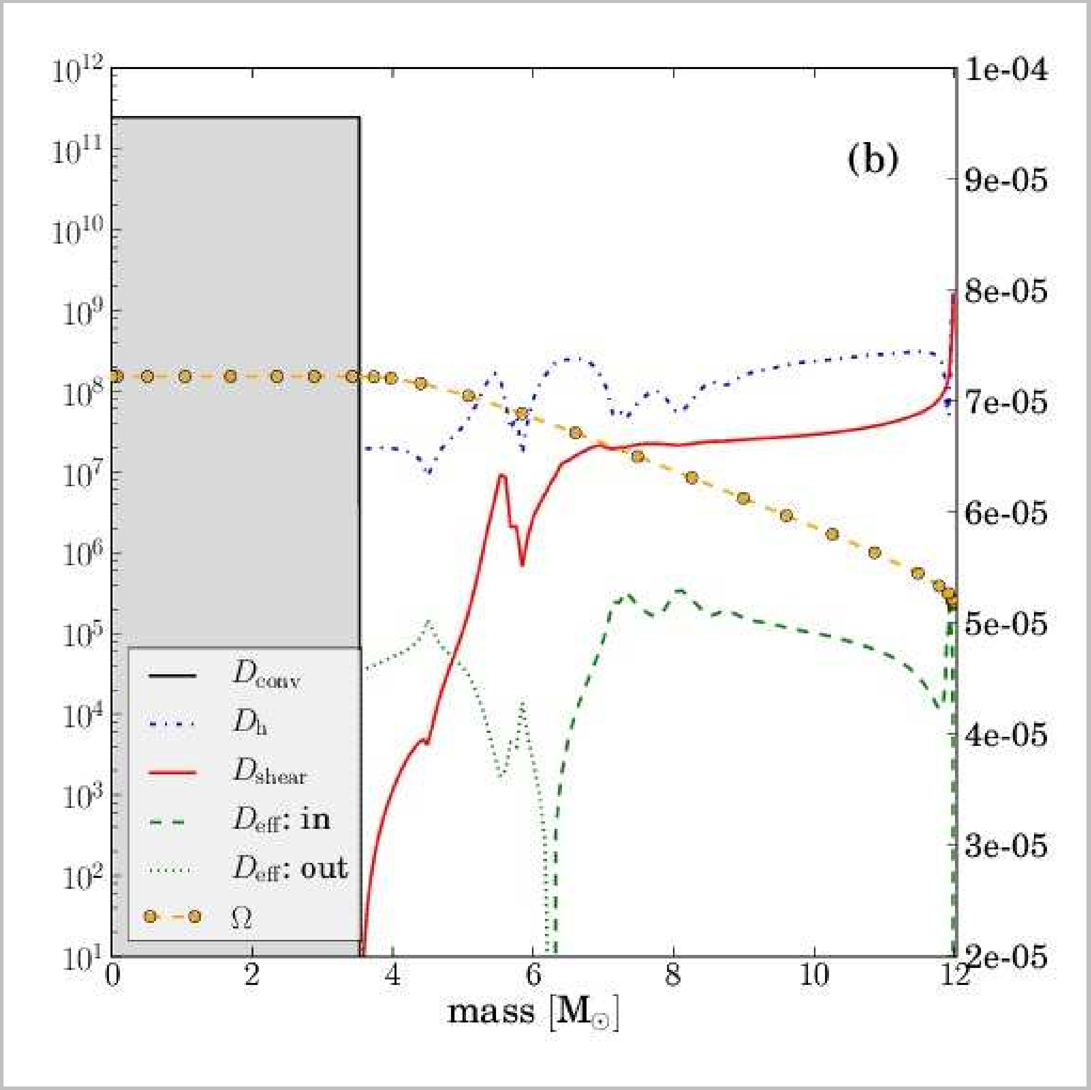
Rotation Induced Transport: Prescriptions

D_h: three prescriptions
Zahn (1992); Maeder (2003); Mathis et al. (2004)

D_{shear}: two prescriptions
Talon & Zahn (1997); Maeder (1997)

Different zones concerned inside the star

mixing ± strong (4%,12%)



Rotation Induced Transport: Prescriptions

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Zahn (1992); Maeder (2003);
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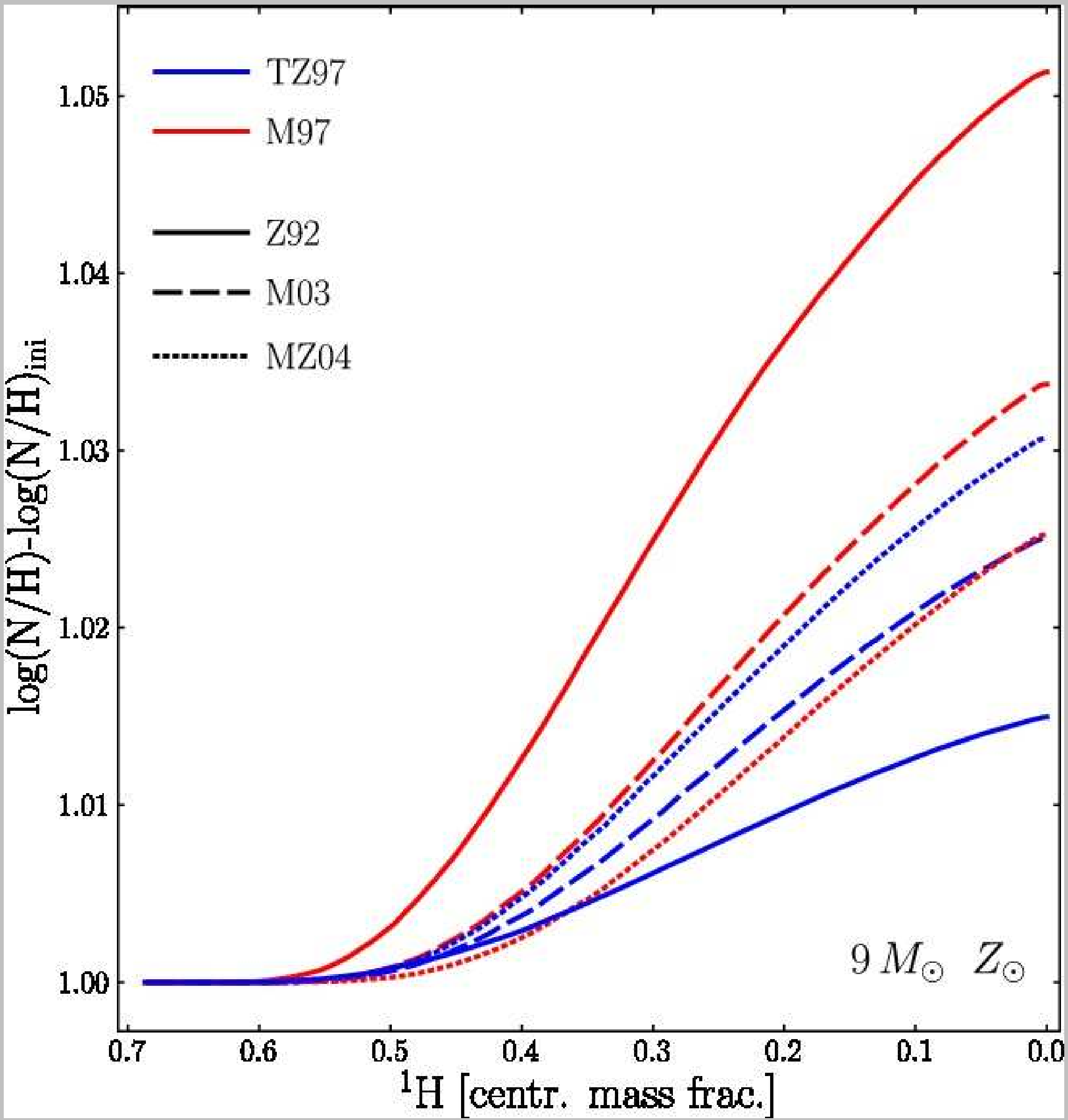


See also Chieffi & Limongi 13

Mass Loss

Mass loss prescription without rotation:

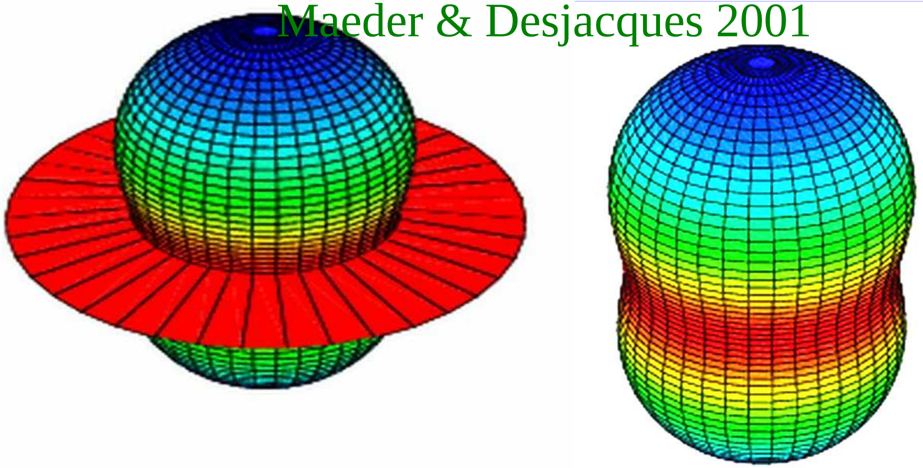
- O-type & LBV stars (bi-stab.): Vink et al 2000, 2001 and RSG de Jager et al 1988
- WR stars (clumping effect): Nugis & Lamers 2000

Effects of rotation:

- Enhancement: Maeder & Meynet 2000

$$\boxed{\frac{\dot{M}(\Omega)}{\dot{M}(0)} \approx \frac{(1-\Gamma)^{\frac{1}{\alpha}-1}}{\left(1-\frac{4}{9}\frac{v^2}{v^2_{crit,1}}-\Gamma\right)^{\frac{1}{\alpha}-1}}}$$

Maeder & Desjacques 2001



T_{eff} =25 000 K T_{eff} =30 000 K

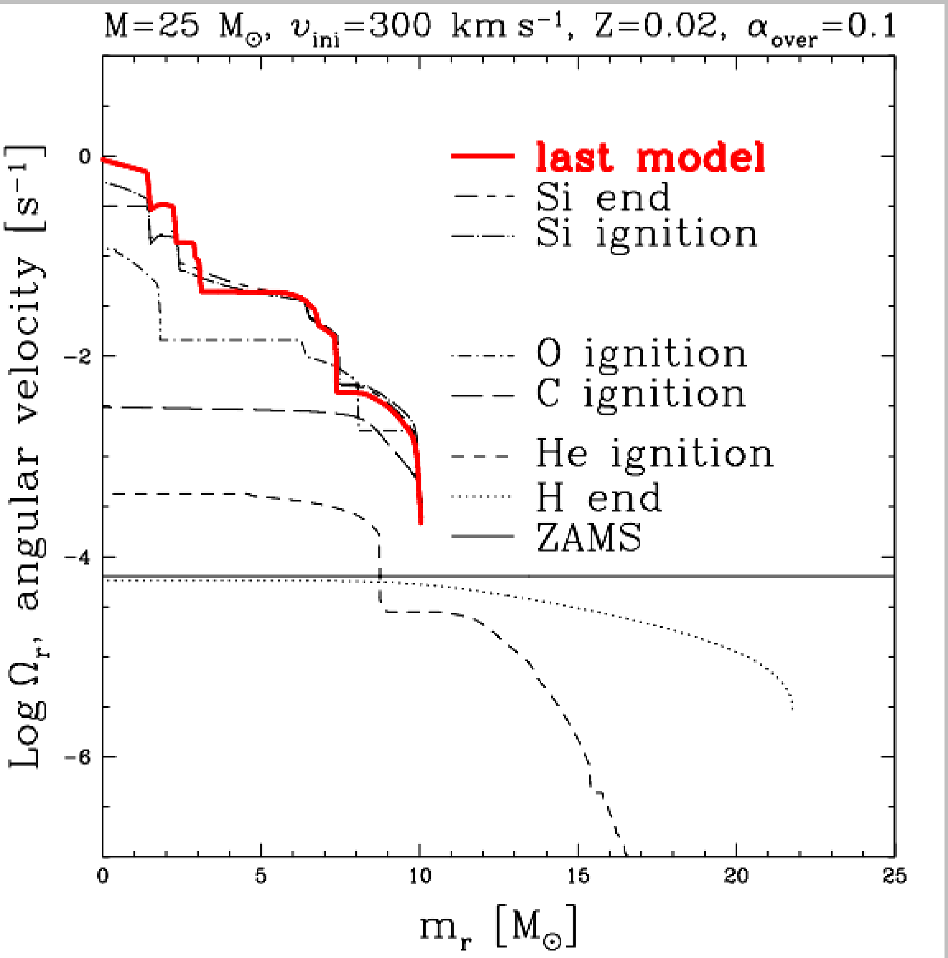
- & anisotropy:

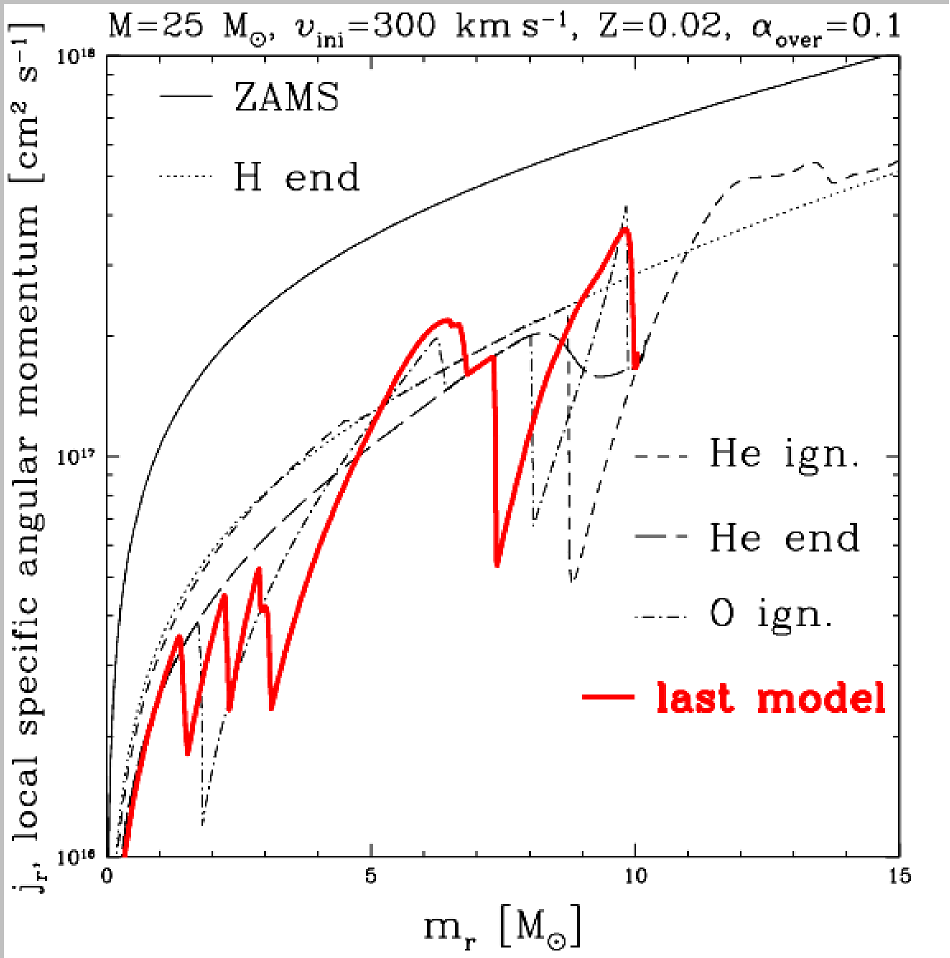
$F_{rad} \sim g_{eff}$: Von Zeipel, 1924 → affects angular momentum loss

Evolution of Rotation

Angular velocity: Ω↑
end Si: Ω~ 1 s^{-1}

Angular momentum, j↓
j(end Si) ~ j(end He)

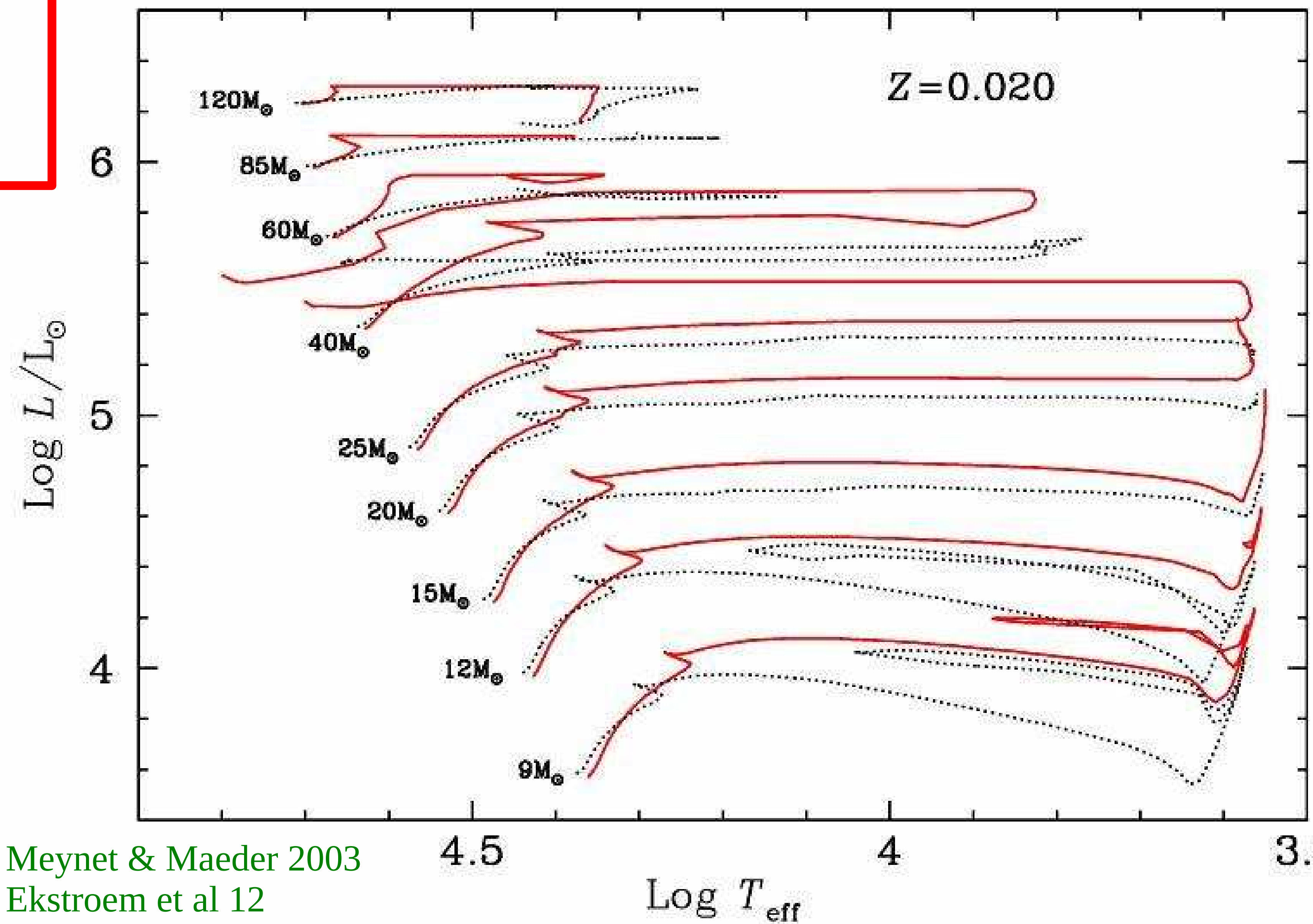




Hirschi et al 2004, A&A

Impact of Rotation (@ solar Z)

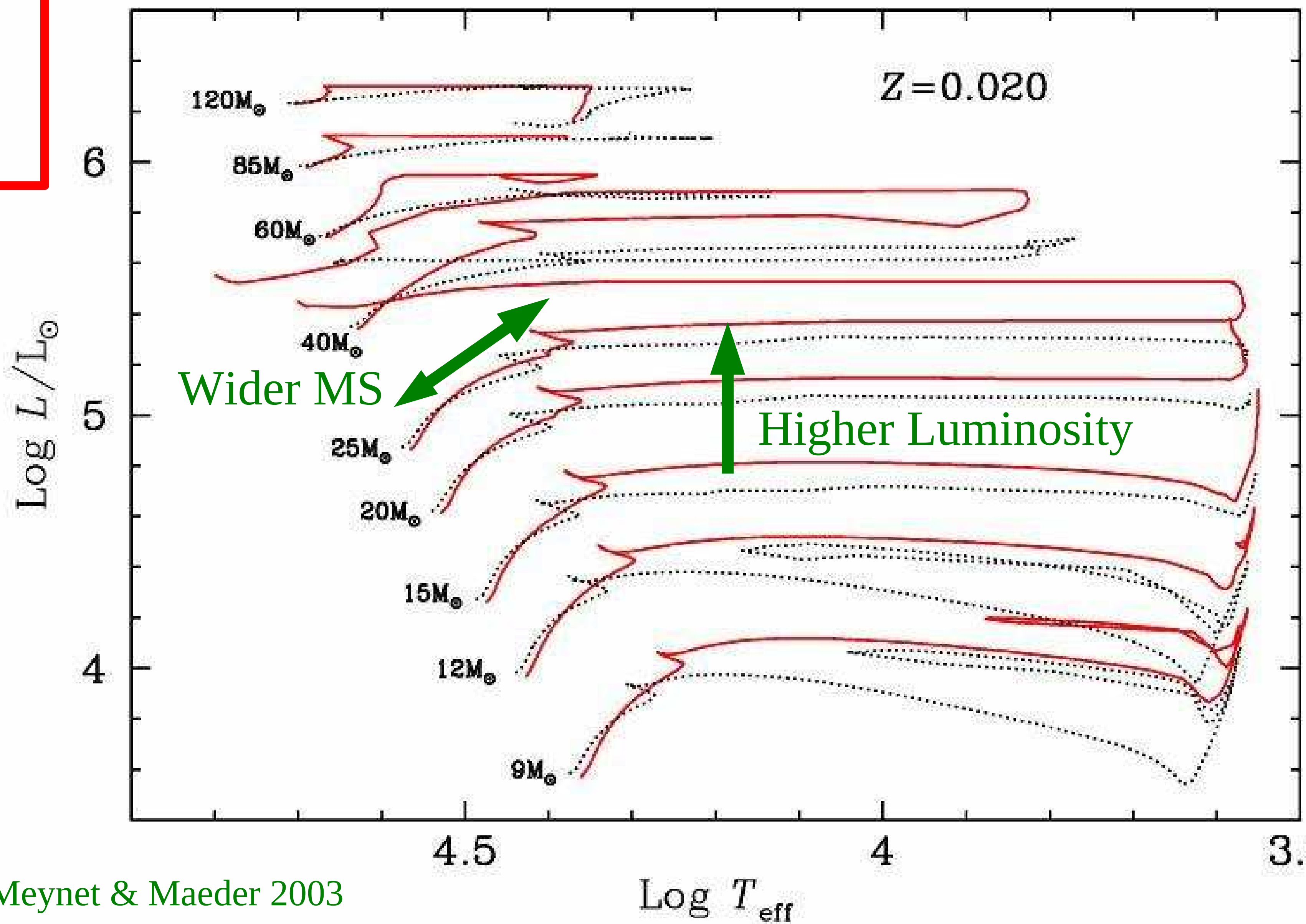
υ_{ini} =
300 km/s
0 km/s



Meynet & Maeder 2003
Ekstroem et al 12

Impact of rotation (@ solar Z)

υ_{ini} =
300 km/s
0 km/s



Meynet & Maeder 2003

Impact of Rotation (@ solar Z): T_{eff}

Roche model: $R_{\rm eq,crit} = \frac{3}{2} R_{\rm pol,crit}$

Modification of the gravity:

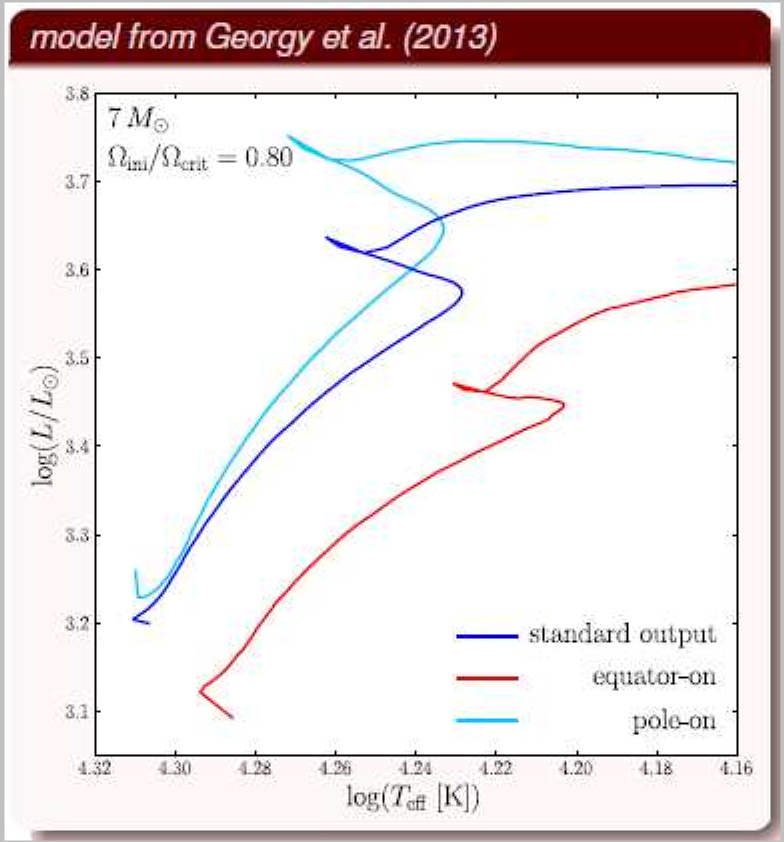
$$\vec{g}_{\rm eff} = \vec{g}_{\rm eff}(\Omega, \theta) =$$

$$\left(-\frac{G\,M}{r^2} + \Omega^2 r\ \sin^2\theta\right)\ \vec{e}_r + \left(\Omega^2 r\ \sin\theta\ \cos\theta\ \right)\vec{e}_\theta$$

and thus of the $T_{\rm eff}$:

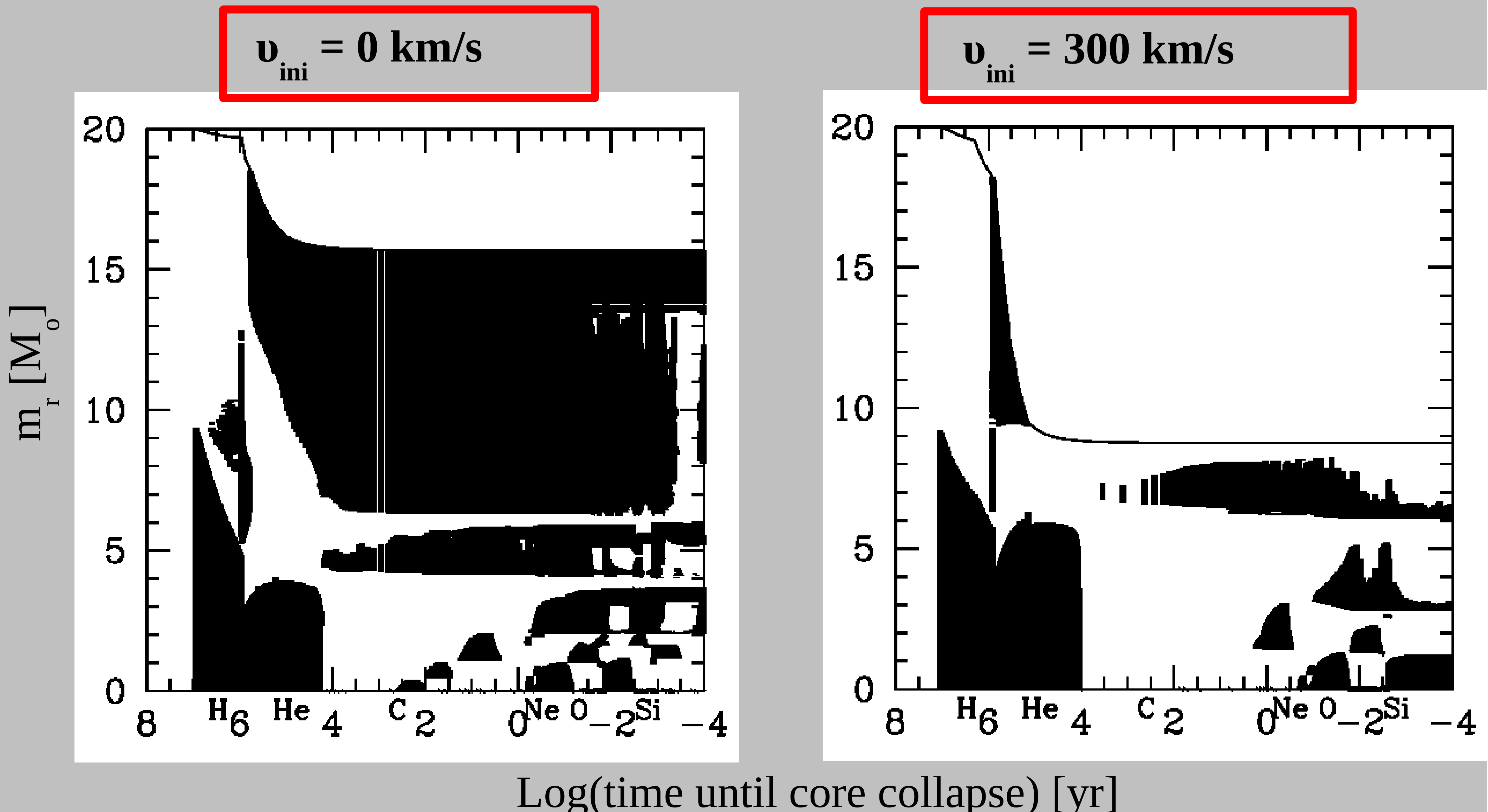
$$T_{\rm eff} = T_{\rm eff}(\Omega,\theta) = \left[\frac{L}{4\pi\sigma\,G\,M^\star}\ g_{\rm eff}(\Omega,\theta)\right]^{1/4}$$

Standard outputs of the models: $= (L/\sigma S_P)^{1/4}$
S_P: true deformed surface

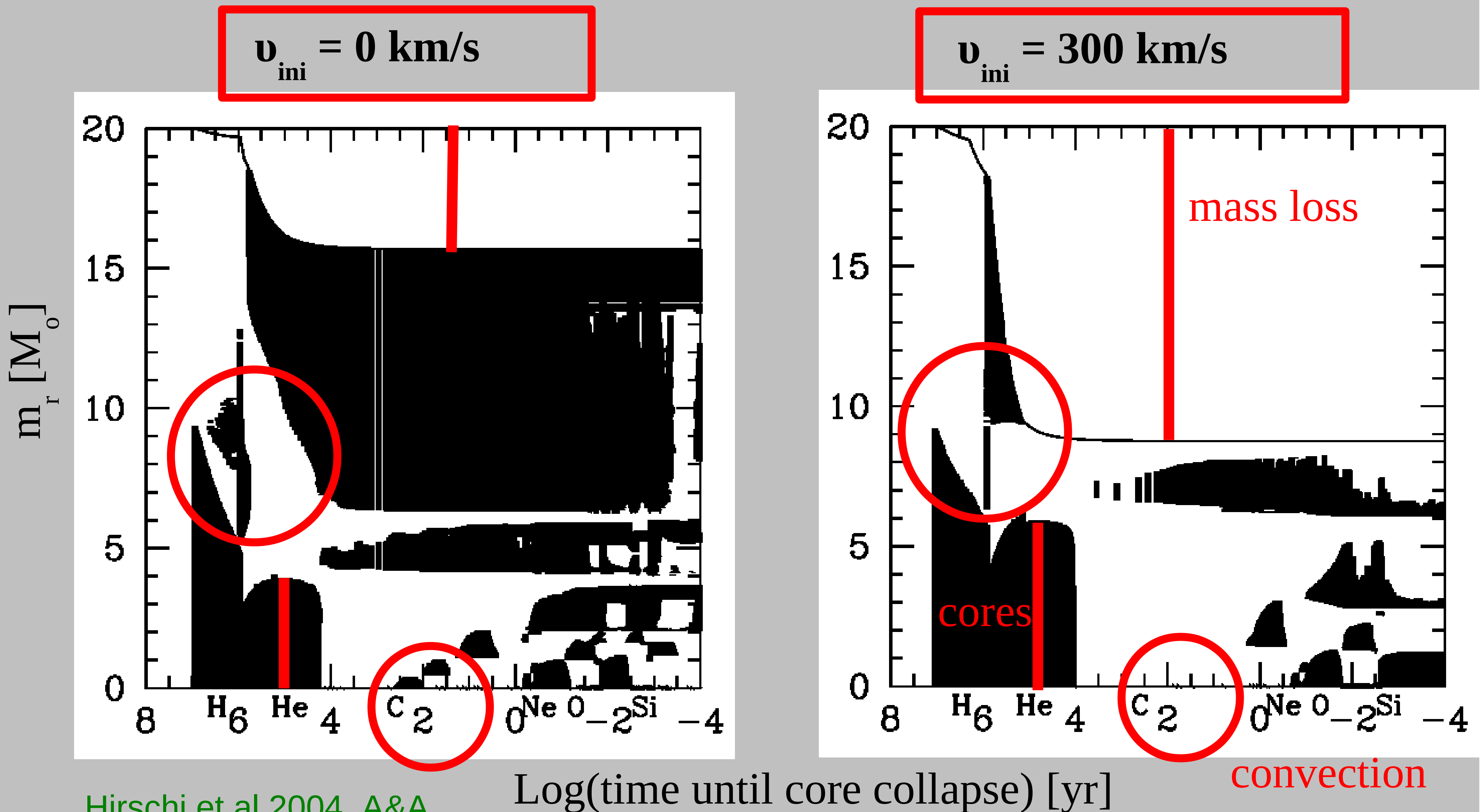


Correction for limb darkening according to Claret 2000

Impact of Rotation, $\mathcal{M}_{ini} = 20\ \mathcal{M}_{\odot}$



Impact of Rotation, $\mathcal{M}_{ini} = 20\ \mathcal{M}_\odot$

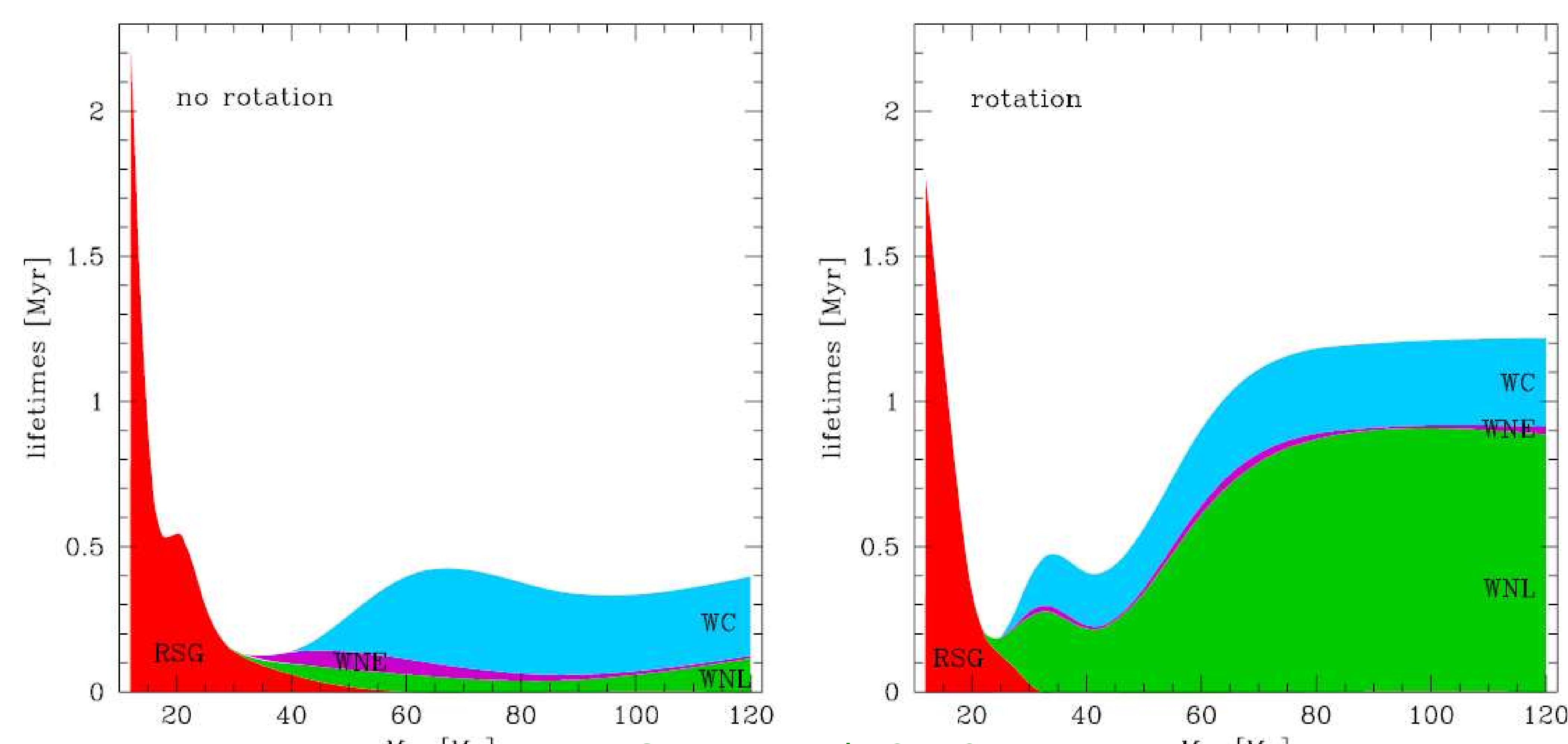


Hirschi et al 2004, A&A

WR Lifetimes @ solar Z

Rotation: decrease of M_{min} for WR formation & increase in WR lifetimes

Meynet & Maeder 03



Georgy et al 12, A&A

NO ROT: $M_{min} \approx$ 25-30 M_o

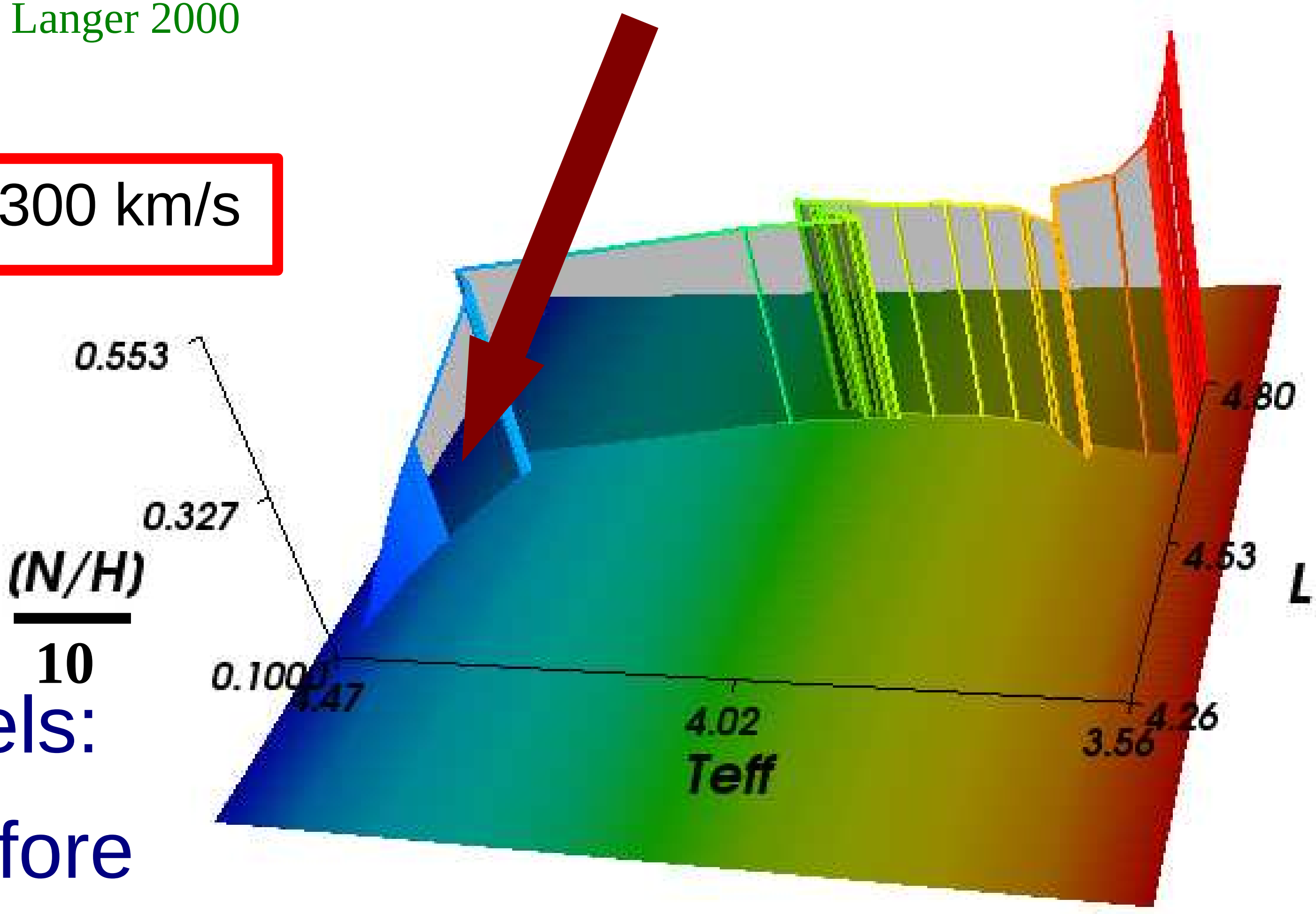
ROT: $M_{min} \approx$ 20 M_o

Nitrogen Surface Enrichment

Rotating models: enrichment starts during MS

Meynet & Maeder 2000, Heger & Langer 2000

15 M_o model with υ_{ini}=300 km/s



Non-rotating models:

no enrichment before

1st dredge up (RSG)

Mixing questioned by FLAMES survey (Hunter et al 08,09)

Nitrogen Surface Enrichment

Flames survey: many stars explained BUT

Explanations:

Single stars:

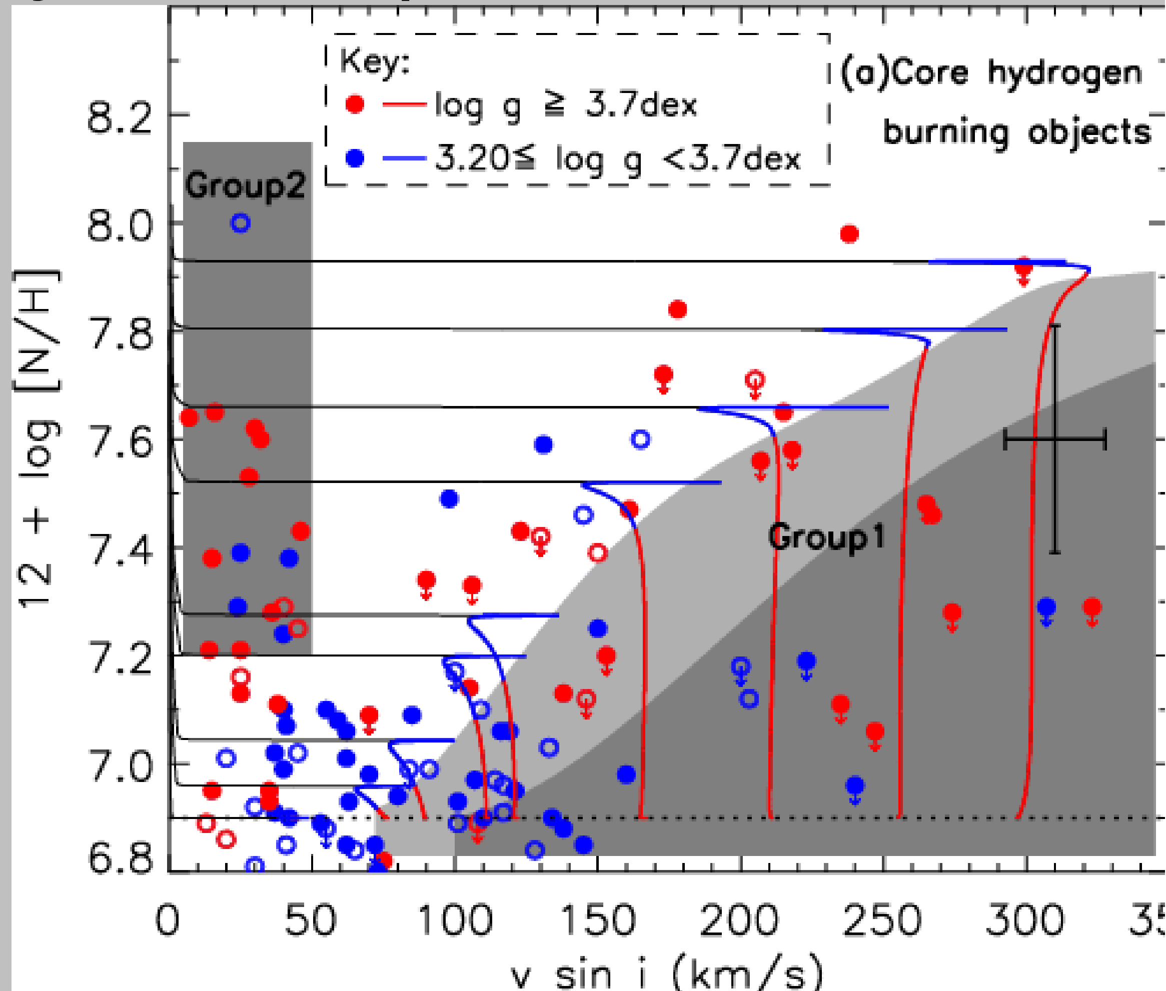
G1: less evolved/ lower mass

G2: pole-on / B-f?

Binary stars: (Langer etal 08)

G1: N-poor matter accr.

G2: * slowed down / B-f?



Other issues: Initial composition, overshooting, enriched blue supergiants

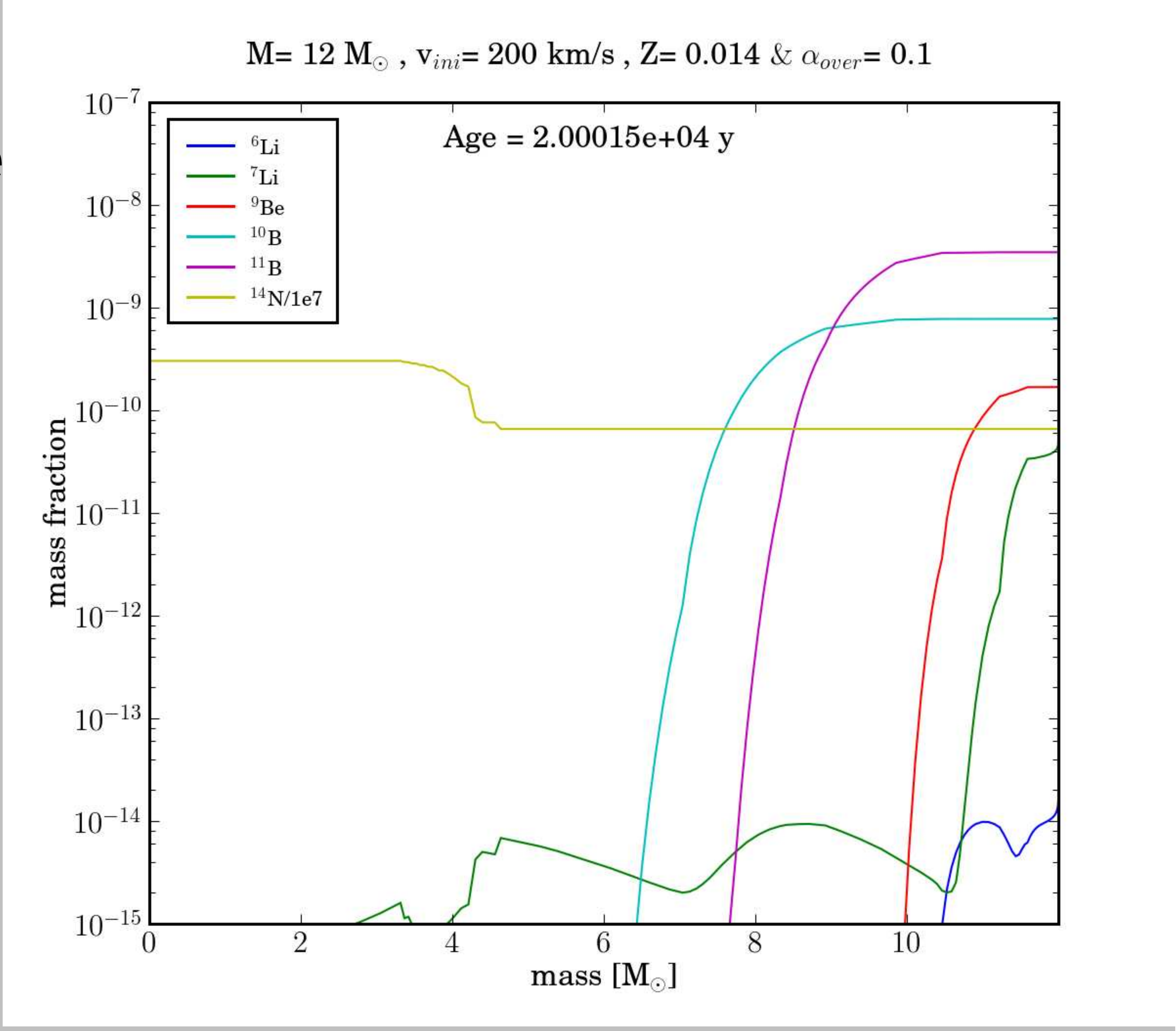
VLT FLAMES survey of massive stars: Hunter etal 07,08,09, ...

Boron can help distinguish between rotation and binarity

(Brott etal 08, 11a,b, Langer etal)

Boron Surface Depletion: Models

- Boron is depleted in the stellar interior.

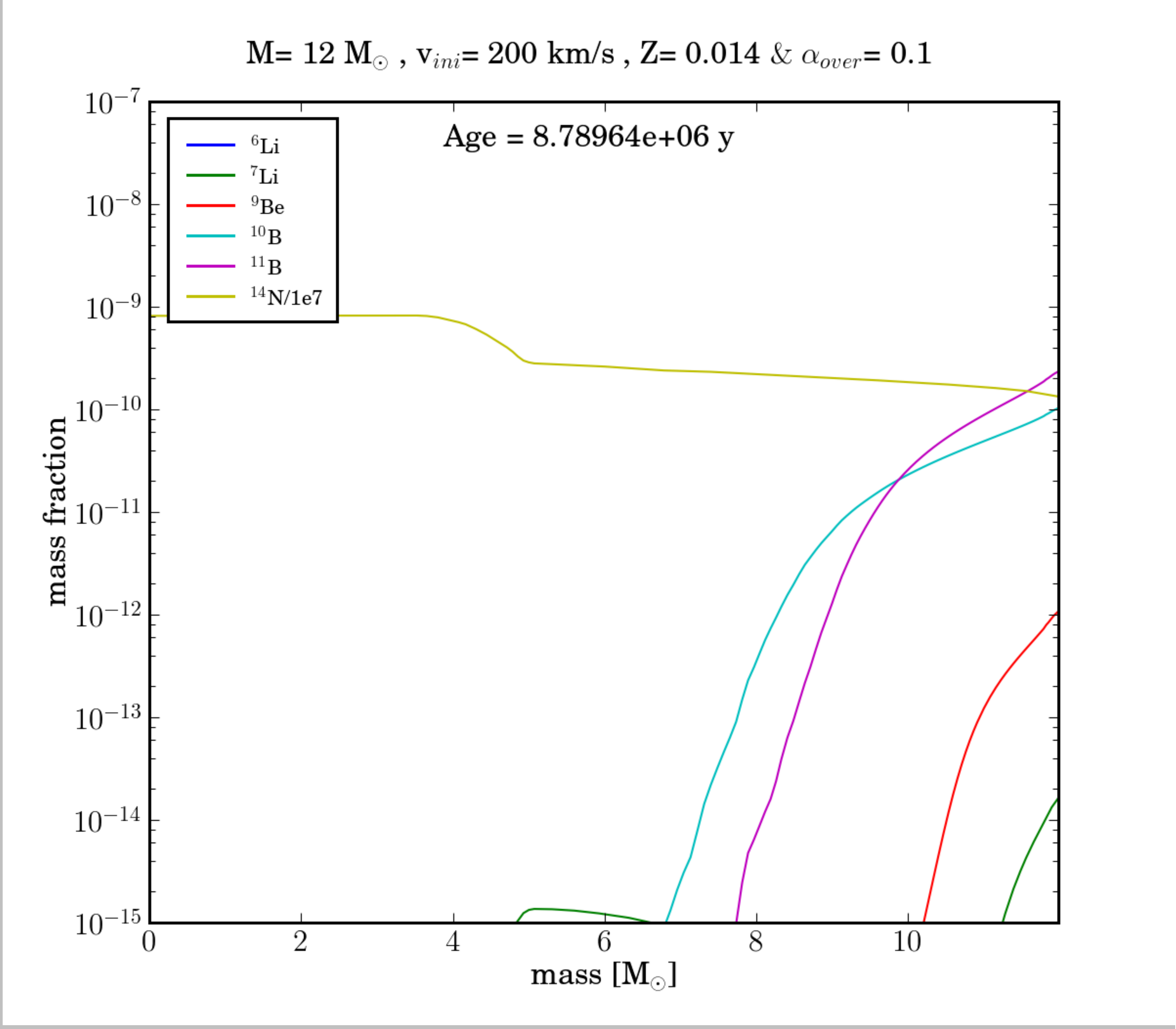


Boron Surface Depletion: Models

Rotational mixing

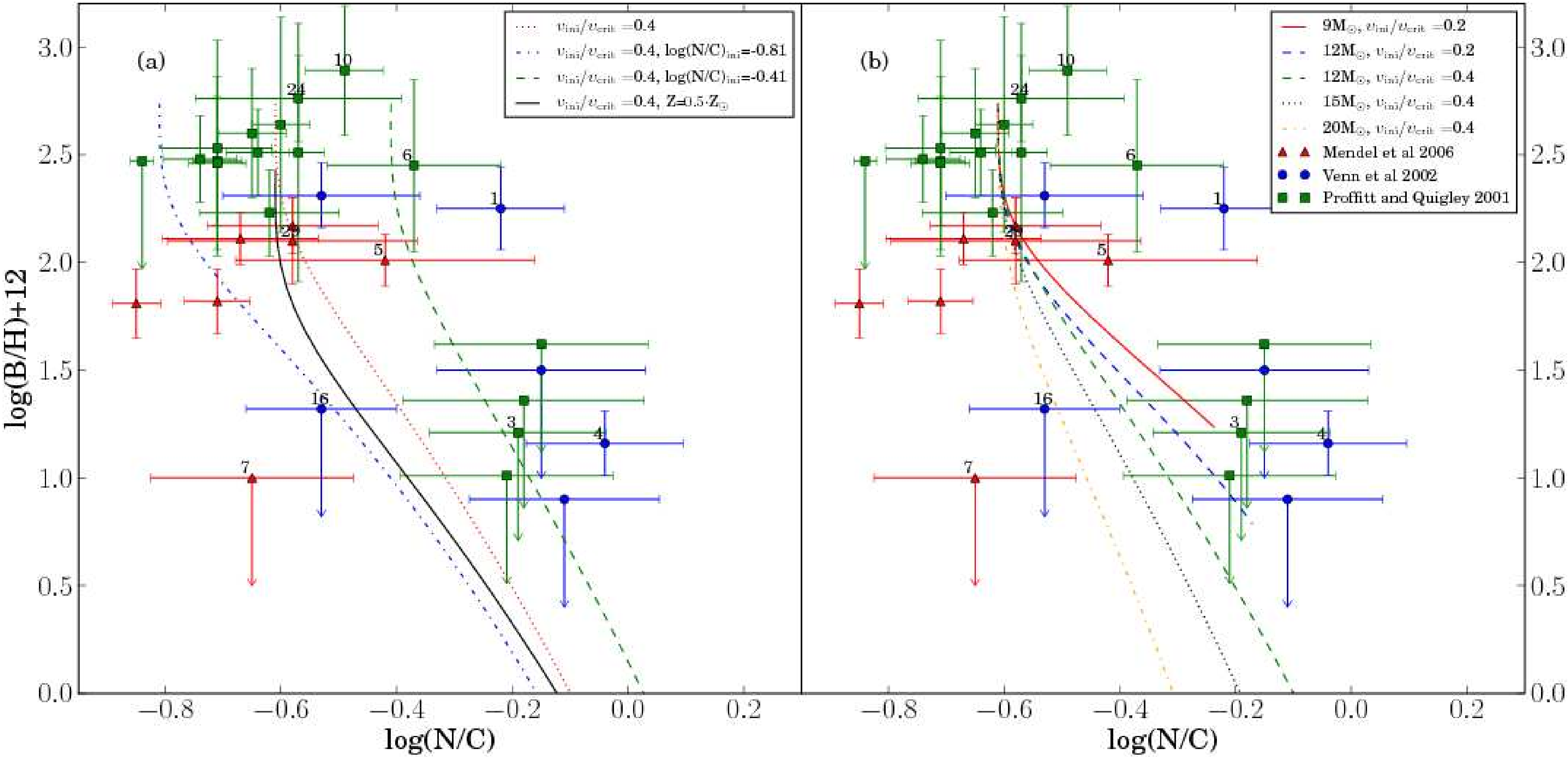
-> surface

boron depletion



Boron Surface Depletion

Rotational mixing -> surface boron depletion



Frischknecht et al A&A 2010

Binaries cannot explain B depletion without N enrichment (Langer et al 2010)

Magnetic Fields

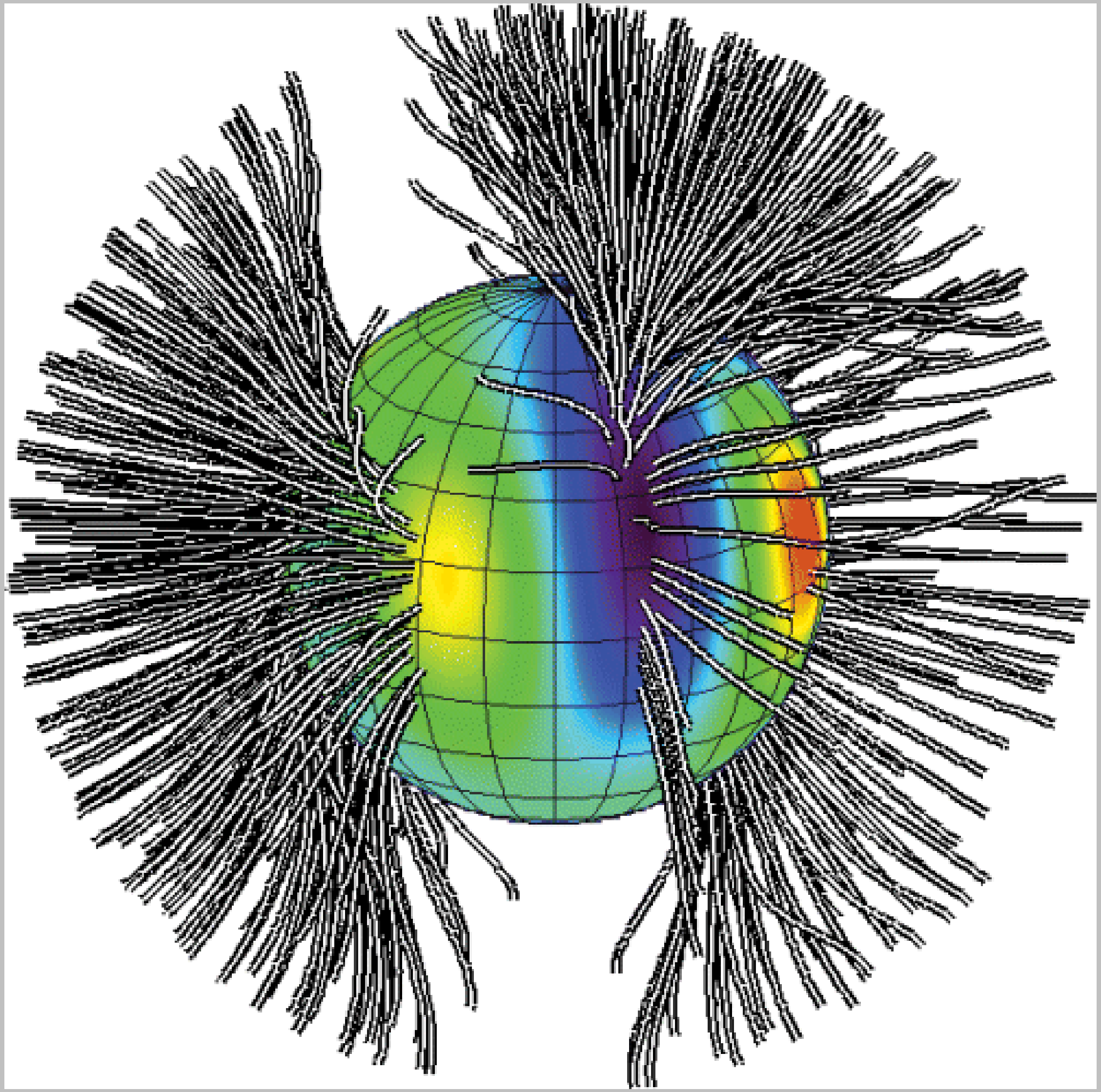
Importance:

- Guides charged-particle
- Shapes stellar winds
- Couples rotation of different parts of the star

Importance debated

Surface Magnetic Fields

τ Sco



Donati et al. 2006

Surface Magnetic Fields

A few dozen He-peculiar stars

7 magnetic OB stars

	Ref	Sp. T.	Vsini Km/s	Prot days	M Msol	Incl. Deg.	β Deg.	Bpol G
HD191612	(6)			538			45	~1500
Θ Ori C	(1)	O4-6V	20	15.4	45	45	42+-6	1100+-100
βCep	(2)	B1IVe	27	12.00	12	60+-10	85+-10	360+-40
τ Sco	(7)	B0.2V		41				~500
V2052 Oph	(3)	B1V	63	3.64	10	71+-10	35+-17	250+-190
ζCas	(4)	B2IV	17	5.37	9	18+-4	80+-4	340+-90
ωOri	(5)	B2IVe	172	1.29	8	42+-7	50+-25	530+-200
He-peculiar		B1-B8p		0.9-22	<10			1000-10000

Only 2 magnetic O star known

(1) Donati et al. 2003 (2) Henrichs et al. 2000 (3,4,5) Neiner et al. 2003abc, (6,7) Donati et al. 2006ab

β Angle between the magnetic axis and the rotation axis

Large ongoing surveys: e.g. MiMes

Most magnetic stars show abundance anomalies: Bp, Ap stars

Magnetic Fields

Question: are these values compatible with magnetic fields observed in pulsars?

Pulsars→ 10^{12} G

$$Br^2 = const.$$

$(10 \text{ km}/5\ R_{sol})^2 \times 10^{12}$ G ~ 10 G.

$$B_+ / B_- = (r_- / r_+)^2$$

Answer: observed magnetic are one-two orders of magnitude higher→More compatible with progenitors of magnetars 10^{15} G

Question: may the observed values have an impact on the wind?

$$\eta(r) \equiv \frac{B^2/8\pi}{\rho v^2/2}$$

if η > 1 → wind behavior

ud-Doula & Owocki (2002)

Answer: YES. For early-type stars, η >1 for B~ 50-100 G

Magnetic Fields: Theory

Taylor Instability (1973)

Small initial horizontal field:
instability of the field lines
→Small vertical component
→Differential rotation winds up
→New horizontal field lines closer and denser:
DYNAMO (Spruit 2002)

More general expressions (Maeder 04, Maeder & Meynet 2005)

Criteria for field amplification:

$$\Omega > \omega_A = \frac{B}{r\sqrt{4\pi\,\bar{\varrho}}}$$

$$q = -\frac{\partial \ln \Omega}{\partial \ln r} > q_{min} = \left(\frac{N}{\Omega}\right)^{7/4} \left(\frac{\eta}{N\,r^2}\right)^{1/4}$$

$$\text{where } N^2 = \frac{\eta/K}{\eta/K+2} N_T^2 + N_\mu^2$$

Transport coefficients:

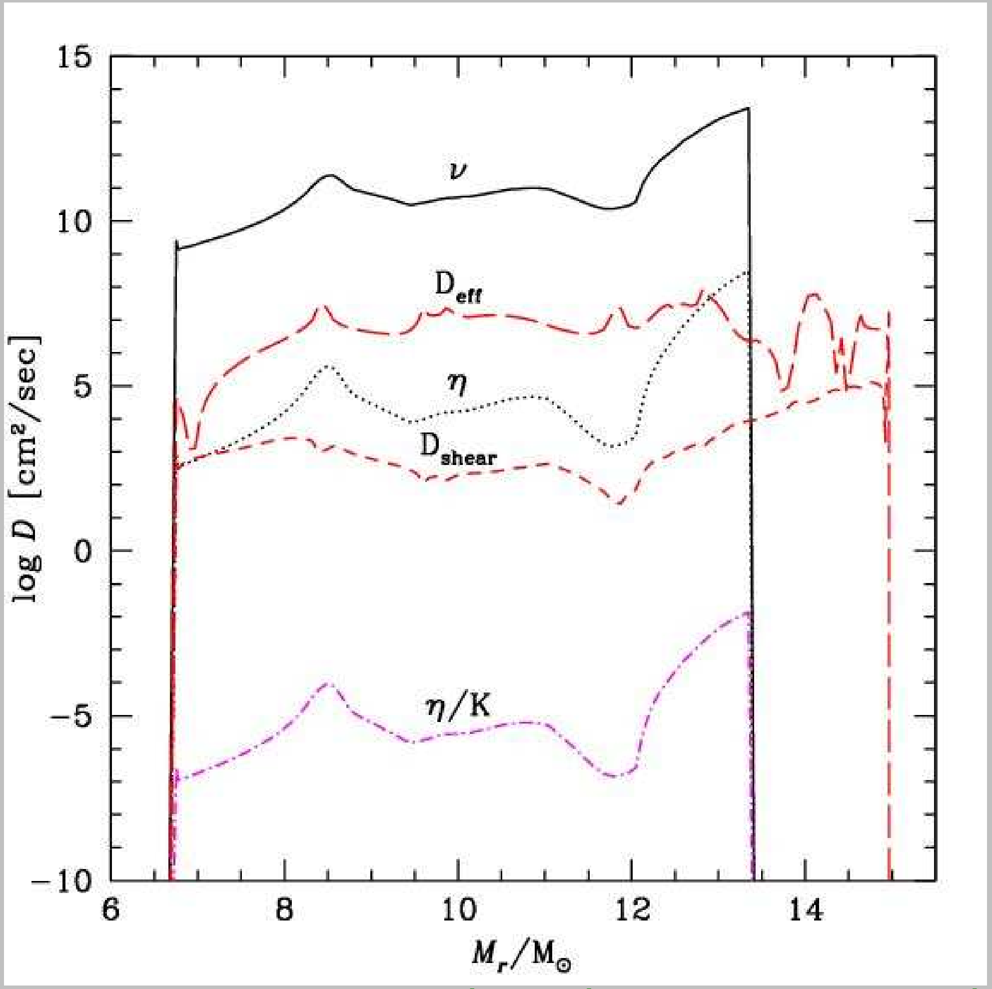
$$D_{\text{chem}} = \frac{r^2\,\Omega}{q^2} \left(\frac{\omega_A}{\Omega}\right)^6$$

$$D_\Omega = \frac{r^2\,\Omega}{q^2} \left(\frac{\omega_A}{\Omega}\right)^3 \left(\frac{\Omega}{N}\right)$$

but Taylor-Spruit dynamo debated (Zahn et al 07)

Magnetic Fields: Models

15 M_o, Z=0.02 & v_{ini} =300 km/s



(Maeder & Meynet 2005)

Transport of Ω (ν):

dominated by B-fields (ν)

Flatter Ω profiles

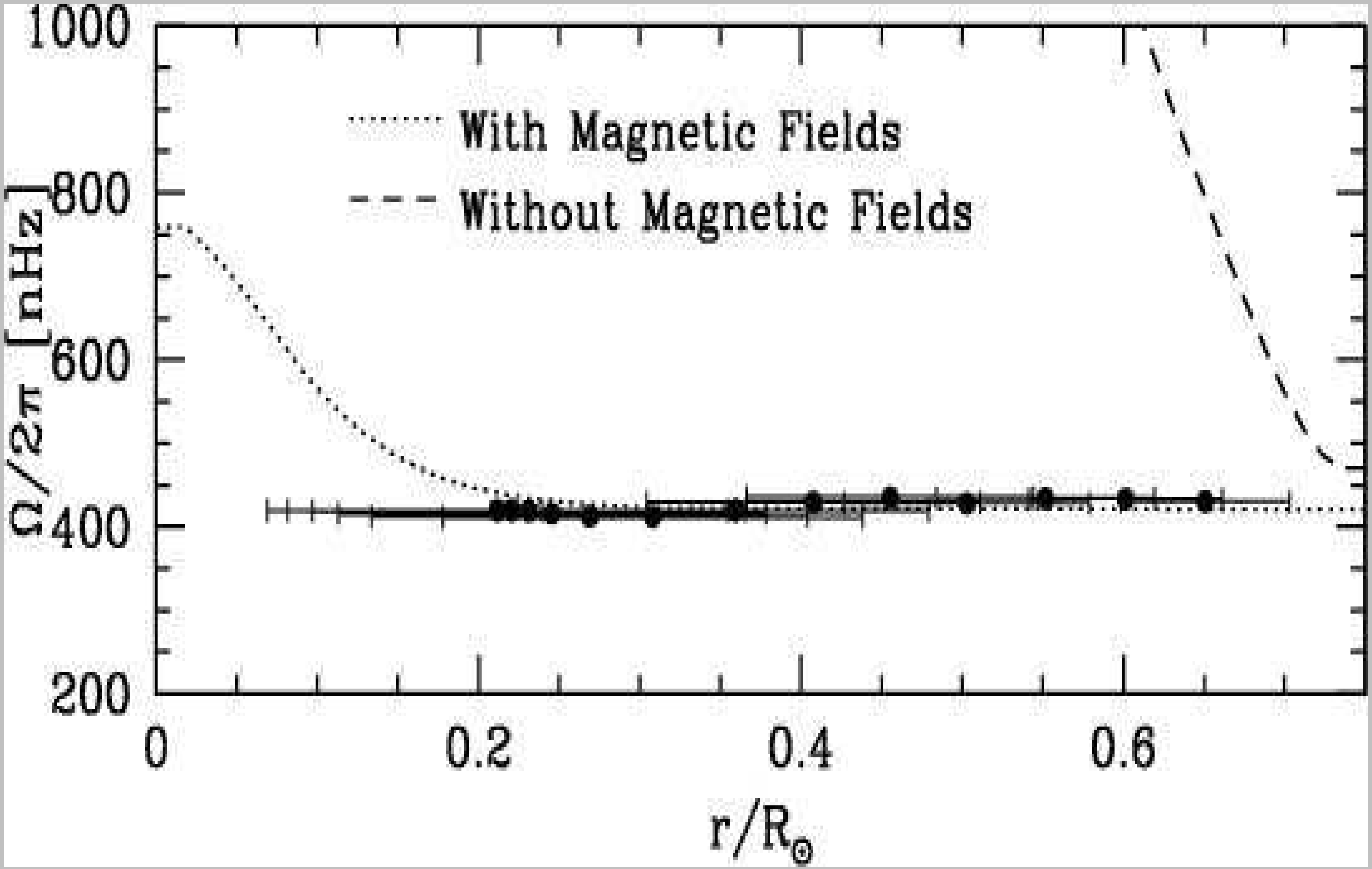
Transport of X_i (η):

Dominated by meridional circulation (D_{eff})

Stronger mixing

Magnetic Fields: Rotation of the Sun

Sun rotation profile compatible with helioseismology
(Eggenberger et al 2005)



Taylor-Spruit dynamo debated
Brun & Zahn 2009

Gravity waves can also help
(Charbonnel & Talon 2005, Arnett & Meakin 2006)

Magnetic Fields: Massive Stars

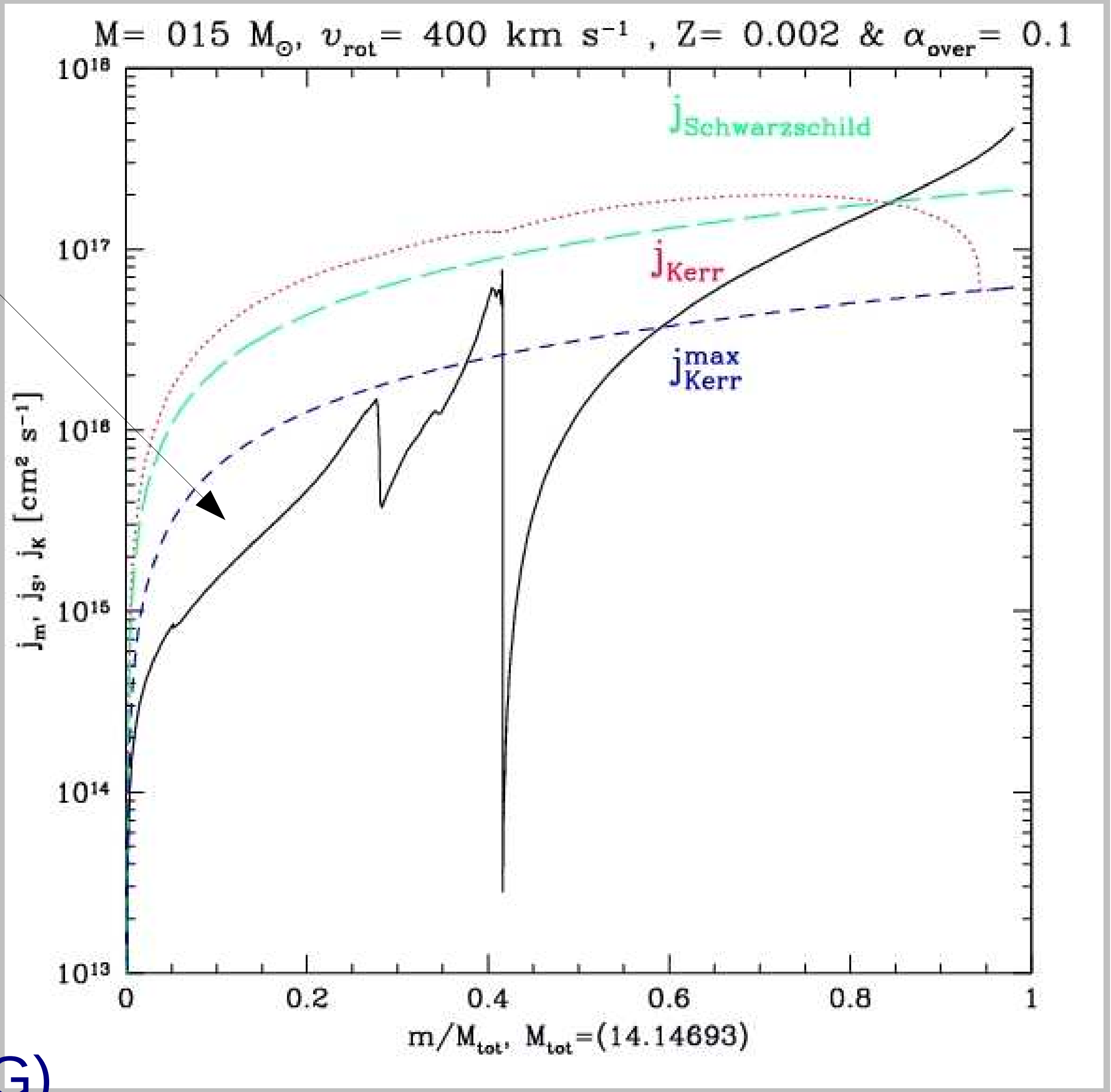
Taylor-Spruit dynamo (Spruit 02, Maeder & Meynet 05) :

Better for pulsar periods

(see also Heger et al 2005)

Not enough for WDs

(Suijs et al 08)



Other mechanisms?

- Dynamo in conv. env.
- During/after explosions

(see discussion in Meynet et al 11,13)

GRBs/MHD explosions?

← Quasi chemically-homog. evol. of fast rot. stars (avoid RSG)

(Yoon et al 06,07, Woosley & Heger 2006)

Magnetic Fields: Massive Stars

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Better for pulsar periods

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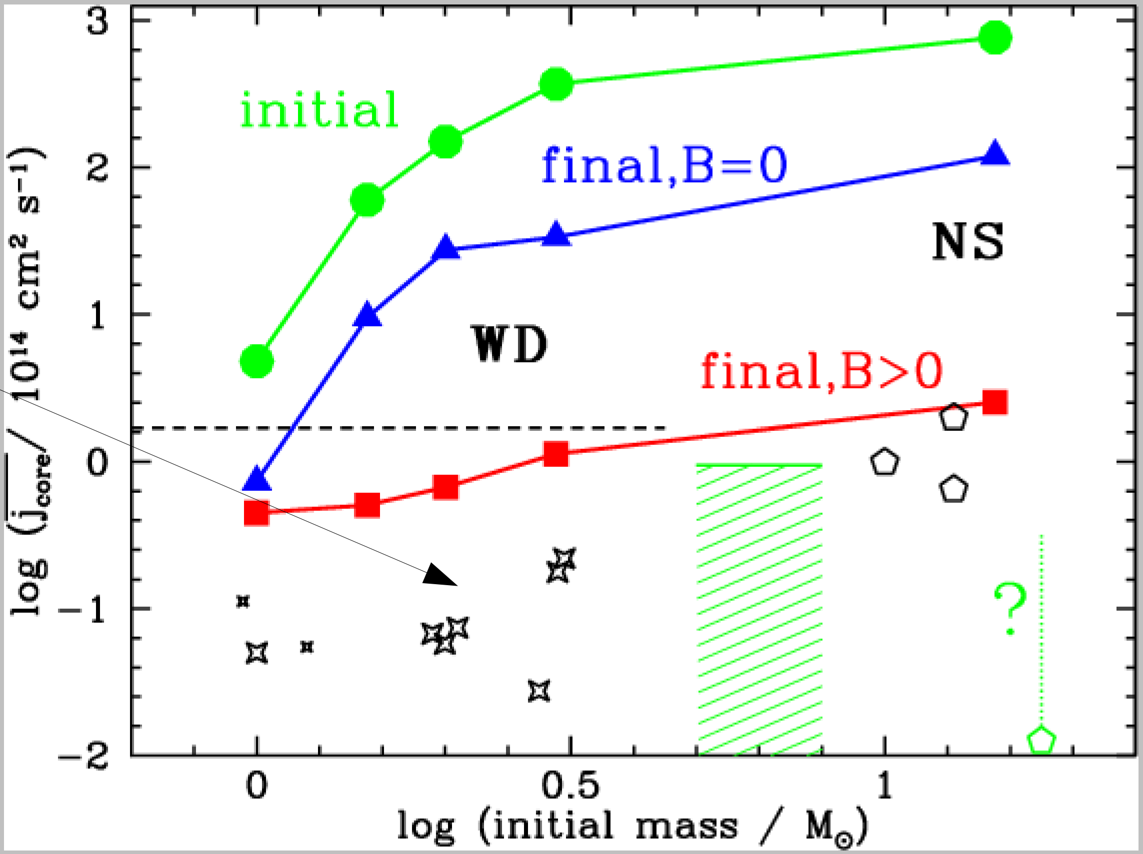

Other mechanisms?

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(Meynet et al 13)

GRBs/MHD explosions?

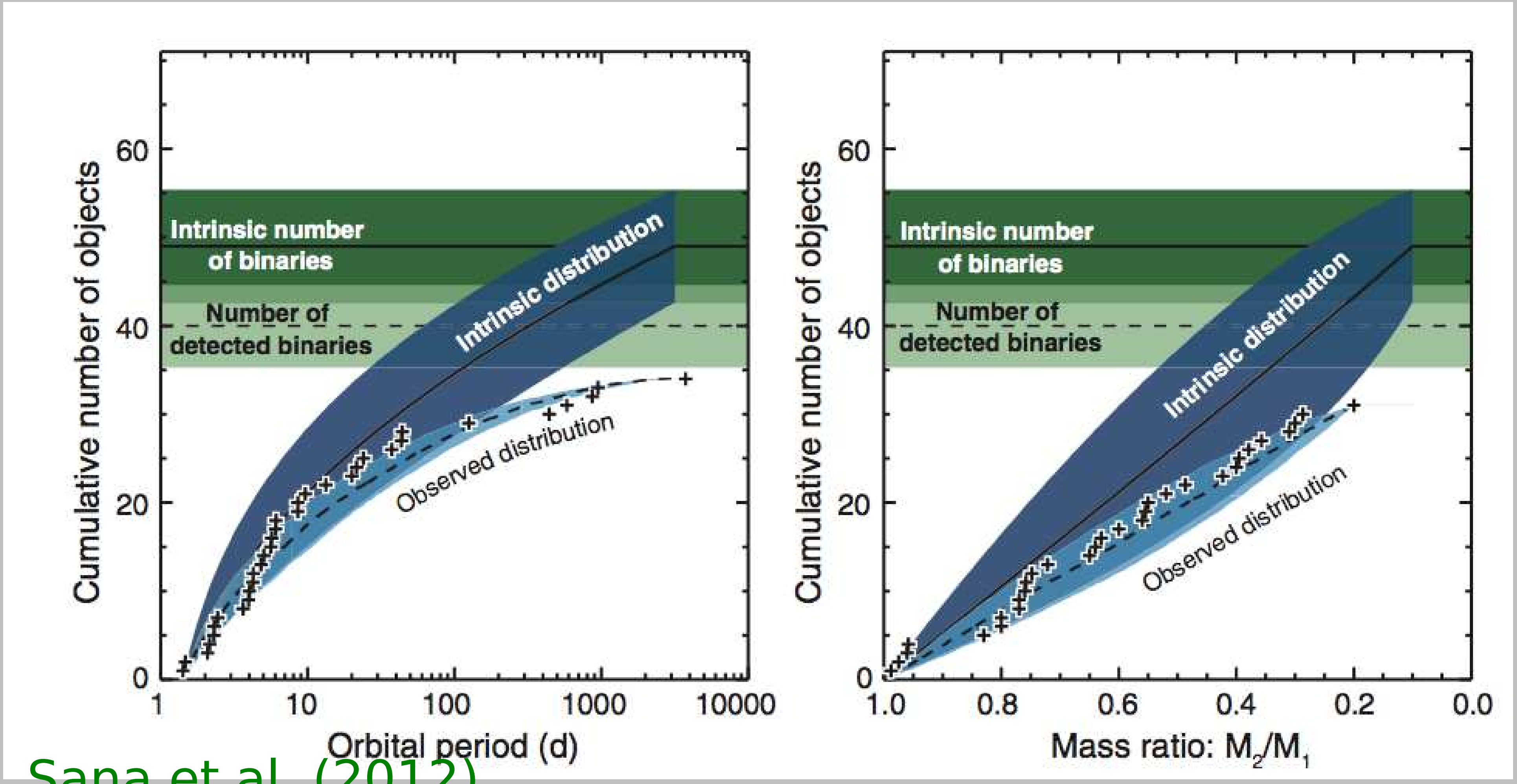
← Quasi chemically-homog.

evol. of fast rot. stars (avoid RSG)

(Yoon et al 06,07, Woosley & Heger 2006)

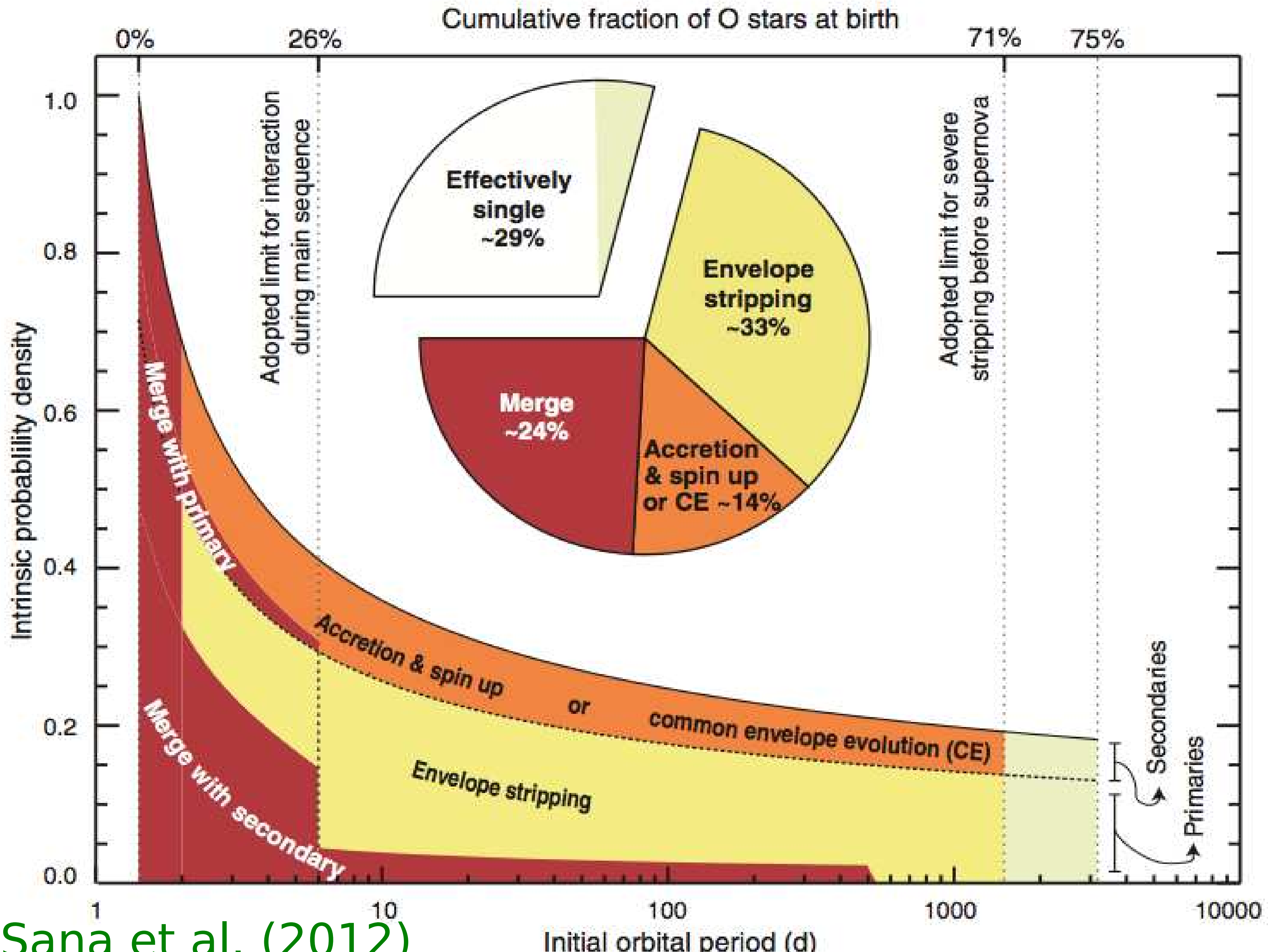
Binarity

- Stars in six nearby **galactic** open clusters→
- 71 single and multiple O-type objects
- 40 detected binaries



Sana et al. (2012)

Binarity



Sana et al. (2012)

Binarity

The VLT-FLAMES Tarantula Survey* (LMC)

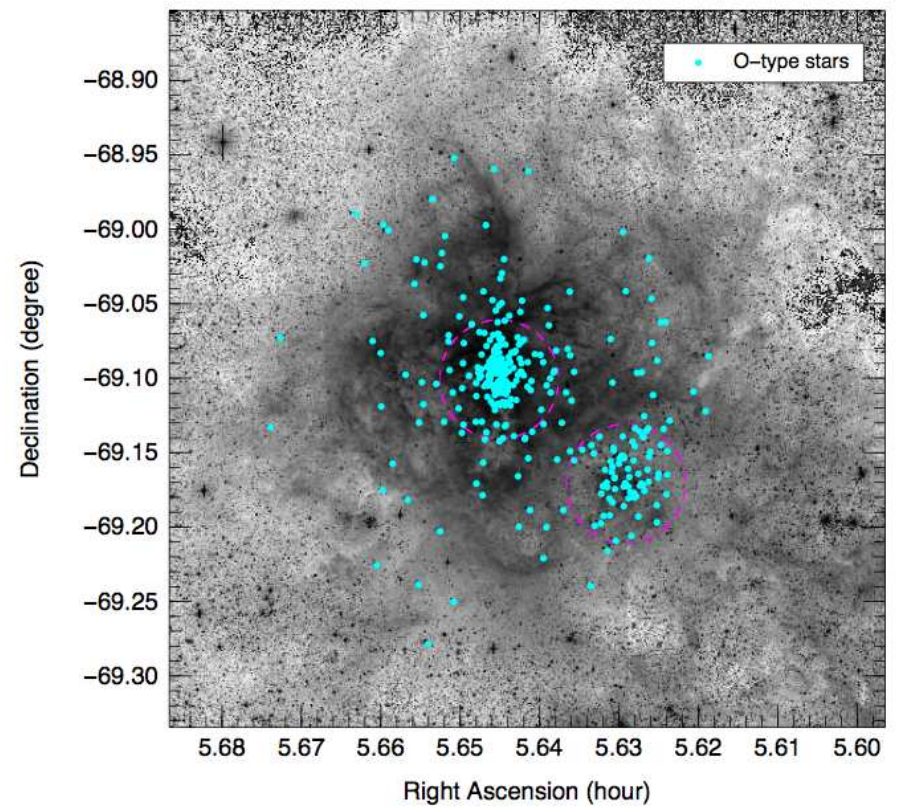
VIII. Multiplicity properties of the O-type star population

H. Sana[1], A. de Koter[1,2], S.E. de Mink[3,4**], P.R. Dunstall[5], C.J. Evans[6], V. Hénault-Brunet[7], J. Maíz Apellániz[8], O.H. Ramírez-Agudelo[1], W.D. Taylor[7], N.R. Walborn[3], J.S. Clark[9], P.A. Crowther[10], A. Herrero[11,12], M. Gieles[13], N. Langer[14], D.J. Lennon[15,3], and J.S. Vink[16]

(Affiliations can be found after the references)

Received May 17, 2012; accepted September 18, 2012

360 O-type stars
Intrinsic binary fraction 51%



Sana et al. (2012)

Equation of State - Ideal gas

$$P = nk_BT = \frac{\mathcal{R}}{\mu}\rho T$$

with $\rho = n\mu m_u$; μ: molecular weight, mass of particle per m_u.
Several components in gas with relative mass fractions $X_i = \frac{\rho_i}{\rho} \rightarrow n_i = \frac{\rho X_i}{m_u \mu_i}$

electrons and ions:

$$P = P_e + \sum_i P_i = (n_e + \sum_i n_i)kT.$$

Ionization – limiting cases

Completely ionized atoms (of mass fraction X_i and charge Z_i):

$$P = nkT = \mathcal{R} \sum_i \frac{X_i(1+Z_i)}{\mu_i} \rho T = \frac{\mathcal{R}}{\mu} \rho T$$

$\mu := \left(\sum_i \frac{X_i(1+Z_i)}{\mu_i} \right)^{-1}$: *mean molecular weight*

For a neutral gas, $\mu = \left(\sum_i \frac{X_i}{\mu_i} \right)^{-1}$.

The mean molecular weight *per free electron* is

$$\mu_e := \left(\sum_i \frac{X_i Z_i}{\mu_i} \right)^{-1} = \frac{2}{(1+X)}$$

Ideal gas with radiation pressure

$$P = P_{\text{gas}} + P_{\text{rad}}$$

$\beta := \frac{P_{\text{gas}}}{P} \Rightarrow \left(\frac{\partial\beta}{\partial T}\right)_P = -\frac{4(1-\beta)}{T}$ and $\left(\frac{\partial\beta}{\partial P}\right)_T = \frac{(1-\beta)}{T}$.

Furthermore

$$\begin{aligned}
\alpha &:= \left(\frac{\partial \ln\rho}{\partial \ln P}\right)_T = \frac{1}{\beta} \\
\delta &:= -\left(\frac{\partial \ln\rho}{\partial \ln T}\right)_P = \frac{4-3\beta}{\beta} \\
\varphi &:= \left(\frac{\partial \ln\rho}{\partial \ln \mu}\right)_{T,P} = 1
\end{aligned}$$

- For $\beta \to 1$, $c_P \to \frac{5\mathcal{R}}{2\mu}$, $\nabla_{\text{ad}} \to 2/5$,

Ionization

Saha-equation (see introductory course for derivation):

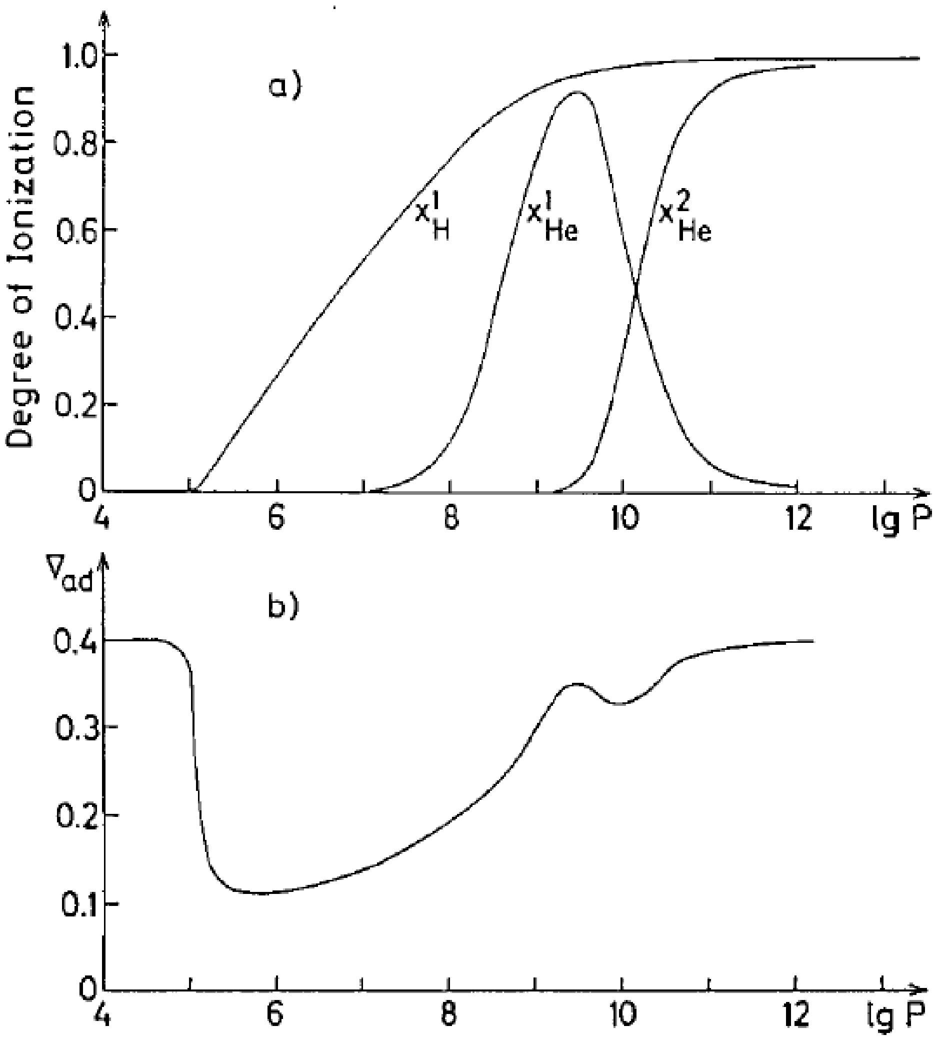


Illustration of ionization of hydrogen and helium within a stellar envelope. In panel (b) the corresponding run of $\nabla_{\rm ad}$ is shown. The depression is due to the increase in c_P due to ionziation. Since $\nabla_{\rm ad}$ is getting smaller, convection will set in.

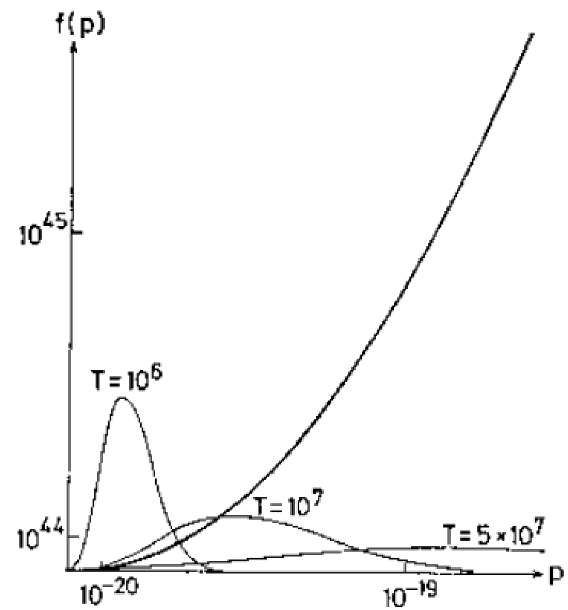
Electron degeneracy

The distribution of electrons in momentum space (Boltzmann equation; p is momentum):

$$f(p)\mathrm{d}p\mathrm{d}V = n_e \frac{4\pi p^2}{(2\pi m_e kT)^{3/2}} \exp\left(-\frac{p^2}{2m_e kT}\right)\mathrm{d}p\mathrm{d}V$$

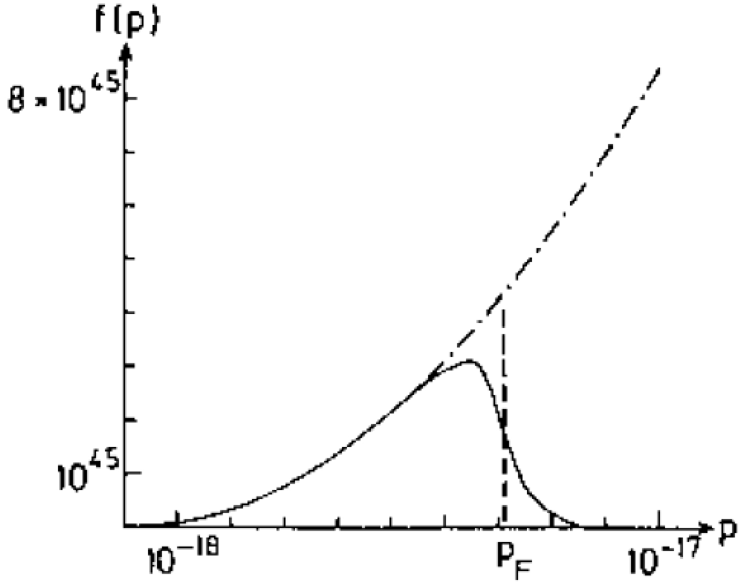
Pauli-principle:

$$f(p)\mathrm{d}p\mathrm{d}V \leq \frac{8\pi p^2}{h^3}\mathrm{d}p\mathrm{d}V$$



Partial degeneracy

$$\ldots U_e = \frac{8\pi}{h^3}\int_0^\infty \frac{Ep^2\,\mathrm{d}p}{1+\exp\left(\frac{E}{kT}-\Psi\right)}$$



$f(p)$ for partially degenerate gas with $n_e = 10^{28}\ \mathrm{cm}^{-3}$ and $T = 1.9{\cdot}10^7$ K corresponding to $\Psi = 10$.

The equation of state for normal stellar matter:

$$P = P_{\rm ion} + P_e + P_{\rm rad} = \frac{\mathcal{R}}{\mu_0}\rho T + \frac{8\pi}{3h^3}\int_0^\infty \frac{p^3 v(p)\,\mathrm{d}p}{1+\exp\left(\frac{E}{kT}-\Psi\right)} + \frac{a}{3}T^4$$

Non-ideal effects

- finite size of atoms $\rightarrow$ pressure ionization
  important already in Sun and low-mass stars
- Coulomb interaction – low density $\rightarrow$ pressure reduction
  important in many stars (envelopes, but also solar core)
- Coulomb interaction – high density $\rightarrow$ crystallization
  white dwarfs, neutron stars
- configuration effects $\rightarrow$ van der Waals gas; quantum effects (spin–spin–interactions)
- neutronization
  neutron stars

EOS implementation in MESA

Combination of sources for EOS in SE codes:

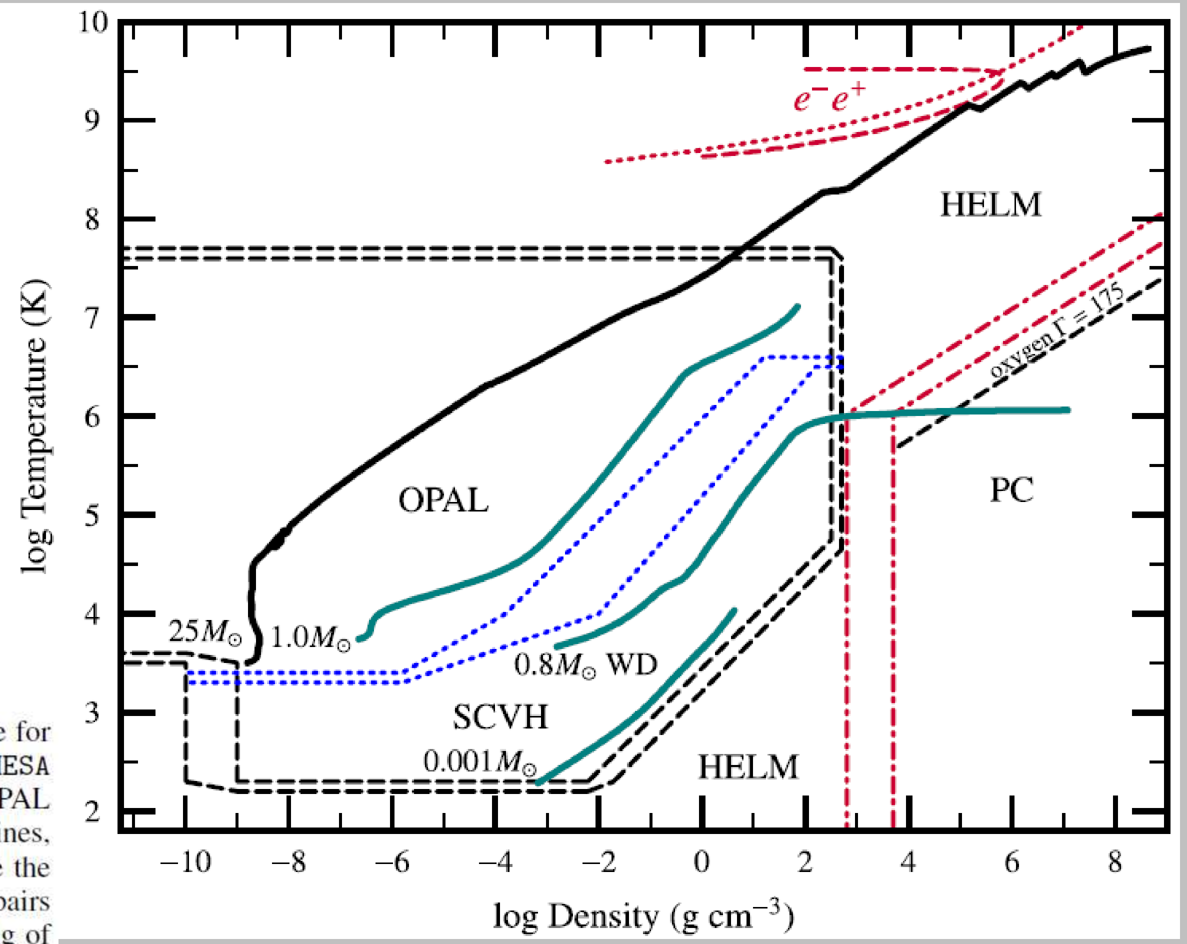


Figure 1. ρ–T coverage of the equations of state used by the eos module for $Z \leqslant 0.04$. Inside the region bounded by the black dashed lines we use MESA EOS tables that were constructed from the OPAL and SCVH tables. The OPAL and SCVH tables were blended in the region shown by the blue dotted lines, as described in the text. Regions outside of the black dashed lines utilize the HELM and PC EOSs, which, respectively, incorporate electron–positron pairs at high temperatures and crystallization at low temperatures. The blending of the MESA table and the HELM/PC results occurs between the black dashed lines and is described in the text. The dotted red line shows where the number of electrons per baryon has doubled due to pair production, and the region to the left of the dashed red line has $\Gamma_1 < 4/3$. The very low density cold region in the leftmost part of the figure is treated as an ideal, neutral gas. The region below the black dashed line labeled as $\Gamma = 175$ would be in a crystalline state for a plasma of pure oxygen and is fully handled by the PC EOS. The red dot-dashed line shows where MESA blends the PC and HELM EOSs. The green lines show stellar profiles for a main-sequence star ($M = 1.0\ M_\odot$), a contracting object of $M = 0.001\ M_\odot$, and a cooling white dwarf of $M = 0.8\ M_\odot$. The heavy dark line is an evolved $25\ M_\odot$ star that has a maximum infalling speed of 1000 km s^{-1}. The jagged behavior reflects the distinct burning shells.

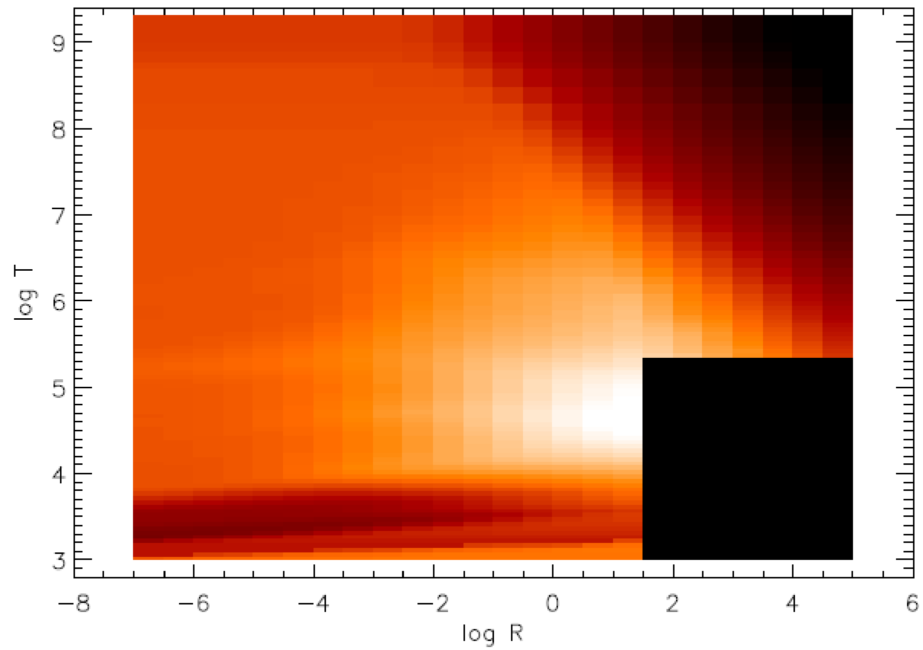
Opacity

- **Electron scattering:** Thomson-scattering:
  $\kappa_{\mathrm{sc}} = \frac{8\pi}{3}\frac{r_e^2}{m_e m_u} = 0.20(1+X)\ \mathrm{cm^2 g^{-1}}$
- **Compton-scattering:** $T > 10^8$: momentum exchange $\rightarrow$ $\kappa < \kappa_{\mathrm{sc}}$
- **free-free transitions:**
  $\kappa_{\mathrm{ff}} \propto \rho T^{-7/2}$ (*Kramers* formula)
- **bound-free transitions:**
  $\kappa_{\mathrm{bf}} \propto Z(1+X)\rho T^{-7/2}$
- b-f for H^--ion below 10^4 K (major source)
- **bound-bound transitions:** below 10^6 K. No simple formula.
- e^-**-conduction:** $\kappa_{\mathrm{c}} \propto \rho^{-2} T^2$

Opacities – practical use

- complications: complex line structures, many elements, molecules, underlying EOS, transition probabilities . . .

- no on-line calculation accurate enough $\rightarrow$ treat separately
- use of pre-calculated tables for many mixtures
- Opacity Project (Sun, atomic data); OPAL (Sun and stars); Ferguson & Alexander (low T; molecules)

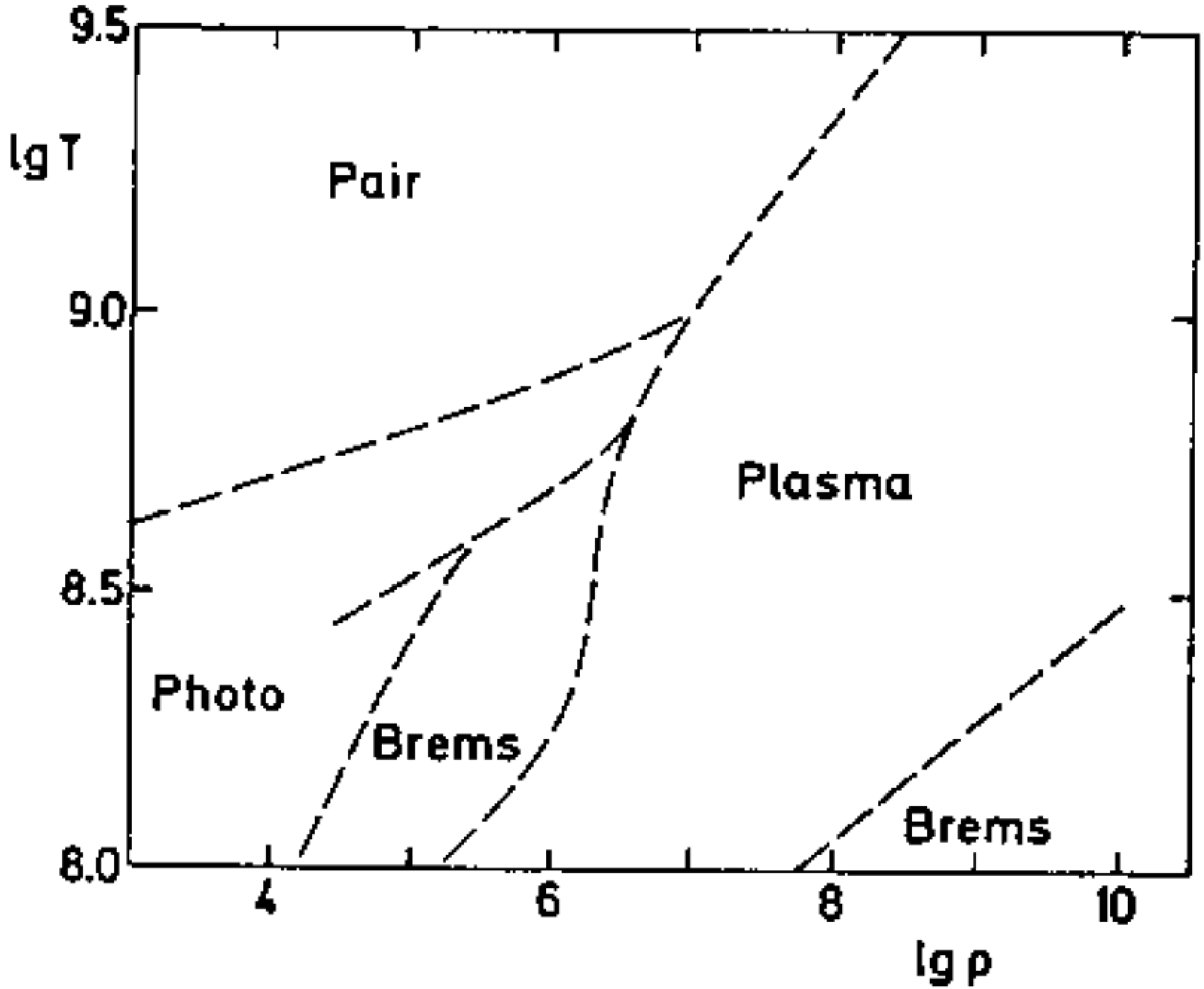


Plasma neutrino emission

Stellar plasma emits neutrinos, which leave star without interaction and lead to energy loss L_ν.
Processes are:

1. Pair annihilation: $e^- + e^+ \to \nu + \bar{\nu}$ at $T > 10^9$ K.
2. Photoneutrinos: $\gamma + e^- \to e^- + \nu + \bar{\nu}$ (as Compton scattering, but with ν-pair instead of γ).
3. Plasmaneutrinos: $\gamma_{\rm pl} \to \nu + \bar{\nu}$; decay of plasma state $\gamma_{\rm pl}$
4. Bremsstrahlung: inelastic nucleus–e^- scattering, but emitted photon replaced by a ν-pair.
5. Synchroton neutrinos: as synchroton radiation, but again a photon replaced by a ν-pair.

Regions of plasma-neutrino processes



Non-Degenerate Conditions

Let us first consider a uniform contraction of a mass M. In that case a variation in radius ΔR corresponds to a variation in pressure ΔP and to a variation in density $\Delta\rho$ so that we have the following relations:

$$\frac{\Delta P}{P} = -4\frac{\Delta R}{R}, \quad \text{and} \quad \frac{\Delta\rho}{\rho} = -3\frac{\Delta R}{R}.$$

The first equality is deduced from the hydrostatic equilibrium equation and the second from the continuity equation. From these two relations, we can write

$$\Delta \ln P = \frac{4}{3}\Delta \ln \rho.$$

Let us now write the equation of state as follows

$$\Delta \ln \rho = \alpha \Delta \ln P - \delta \Delta \ln T,$$

where α and δ are defined by $\alpha = \left(\frac{\partial \ln \rho}{\partial \ln P}\right)_{T,\mu}$ and $\delta = -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{P,\mu}$, and where μ, the mean molecular weight, is supposed to remain constant. From these two relations one obtains, by eliminating ΔP the two following relations between a variation in log T and log ρ:

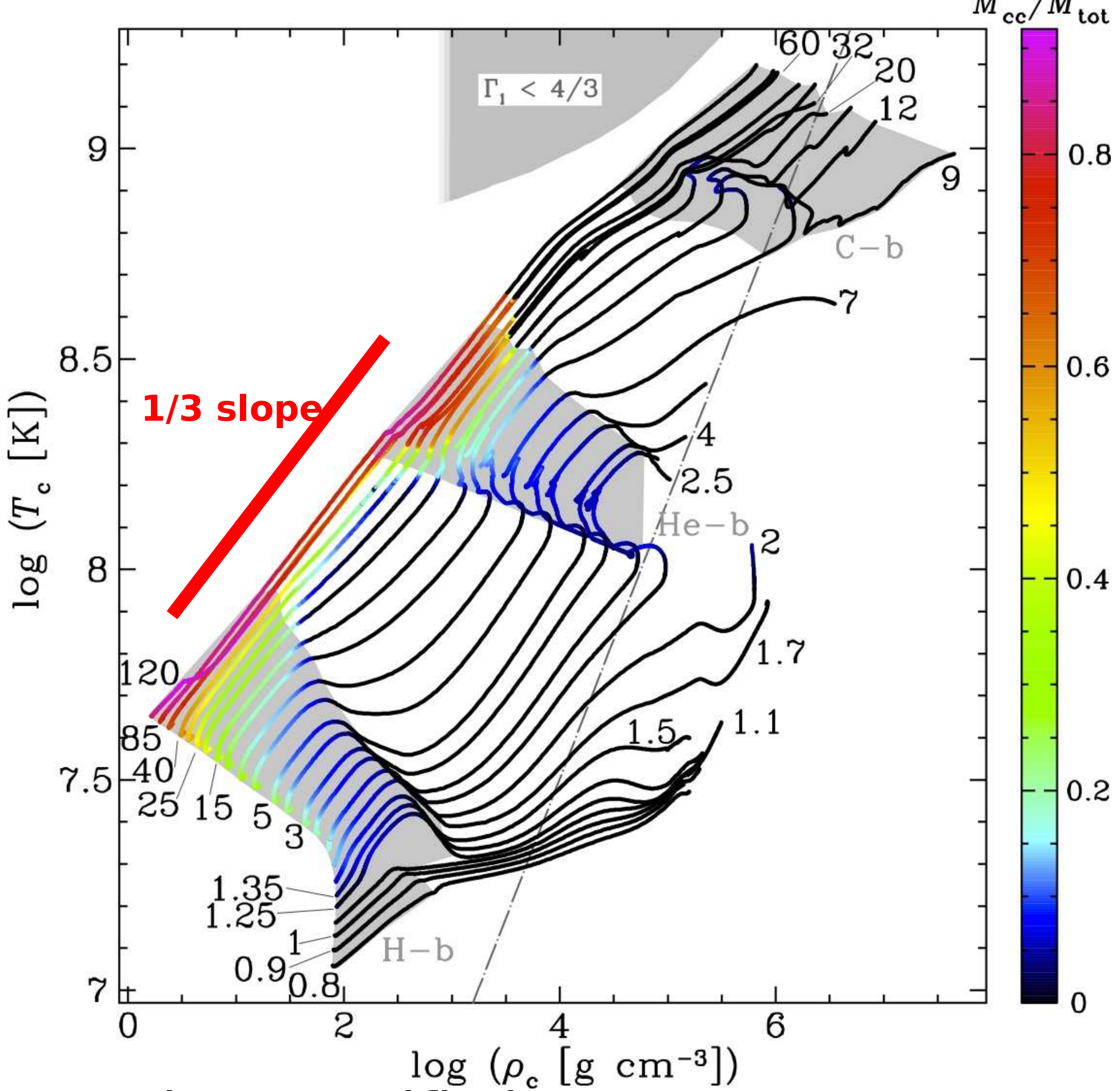
$$\Delta \ln T = \left(\frac{4\alpha - 3}{3\delta}\right)\Delta \ln \rho. \tag{1}$$

For a perfect gas law we have $\alpha = \delta = 1$. Therefore an increase of, for instance, 30% in density implies an increases of 10% in temperature.

Non-Degenerate Conditions

Models by

Ekström et al. (2012)
A&A, 537, A146)



Stars=system with a negative specific heat!

Degenerate Conditions

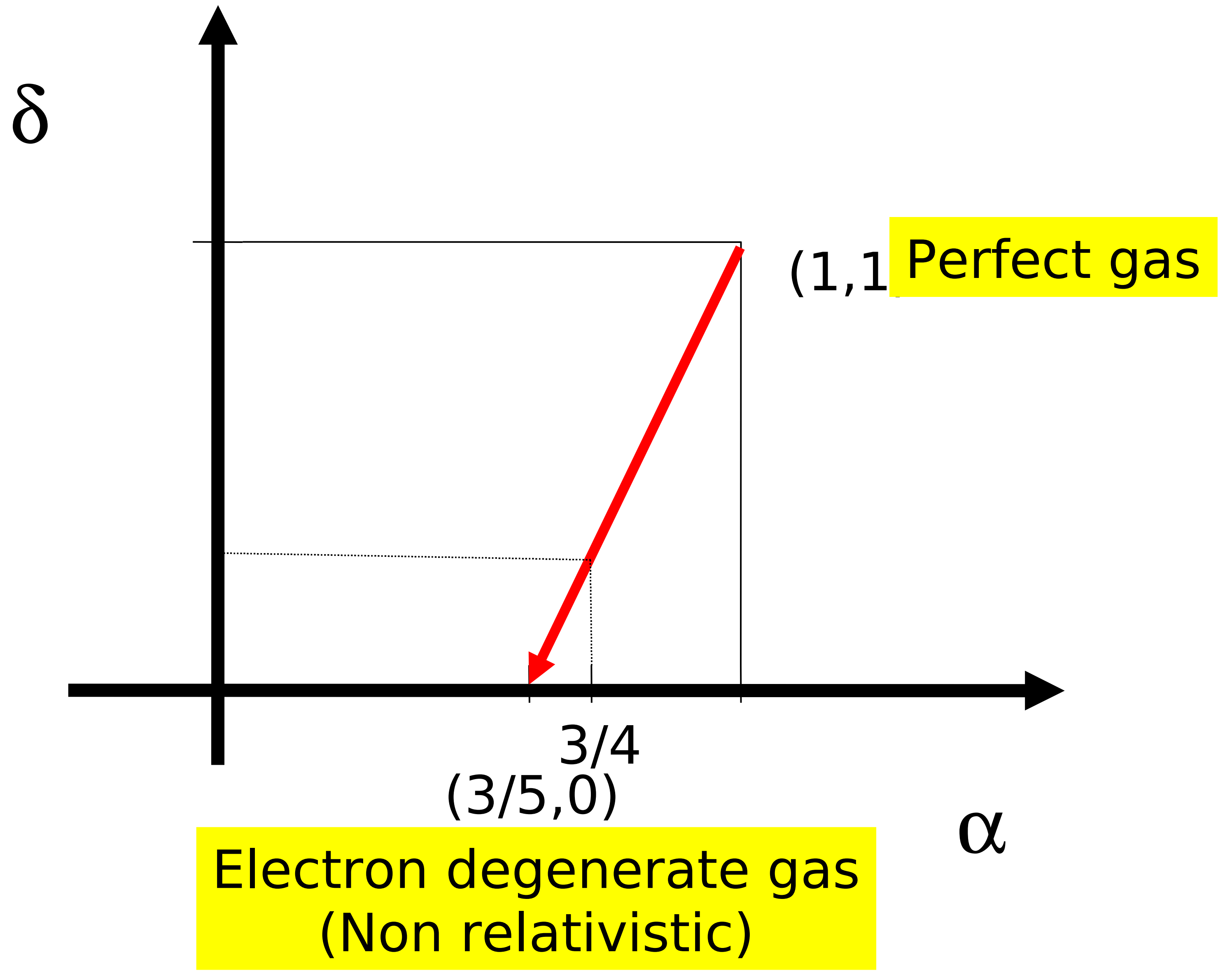
no longer valid, but if during the course of evolution, when the central conditions pass from the non-degenerate region to the degenerate one, α becomes inferior to three quarters before δ is equal to zero, then a contraction can produce a cooling! This can be understood as due to the fact that, in order to allow electrons to occupy still higher energy state, some energy has to be extracted from the non degenerate nuclei which, as a consequence, cool down.

$$\Delta \ln T = \left(\frac{4\alpha - 3}{3\delta} \right) \Delta \ln \rho.$$

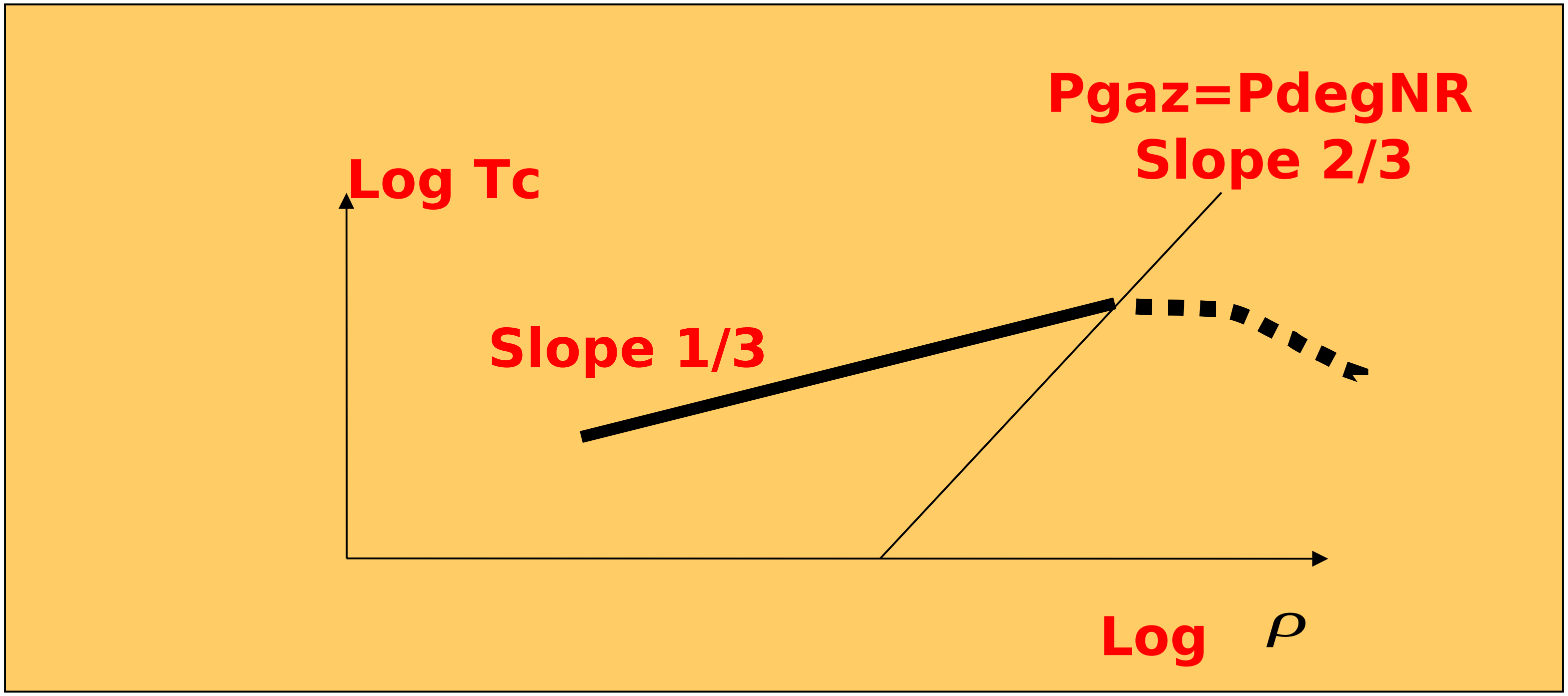
$$P \propto \rho^{5/3}$$

$$\alpha = 3/5 \text{ and } \delta = 0$$

Non → Degenerate Conditions



Evolution of the temperature and density at the centre



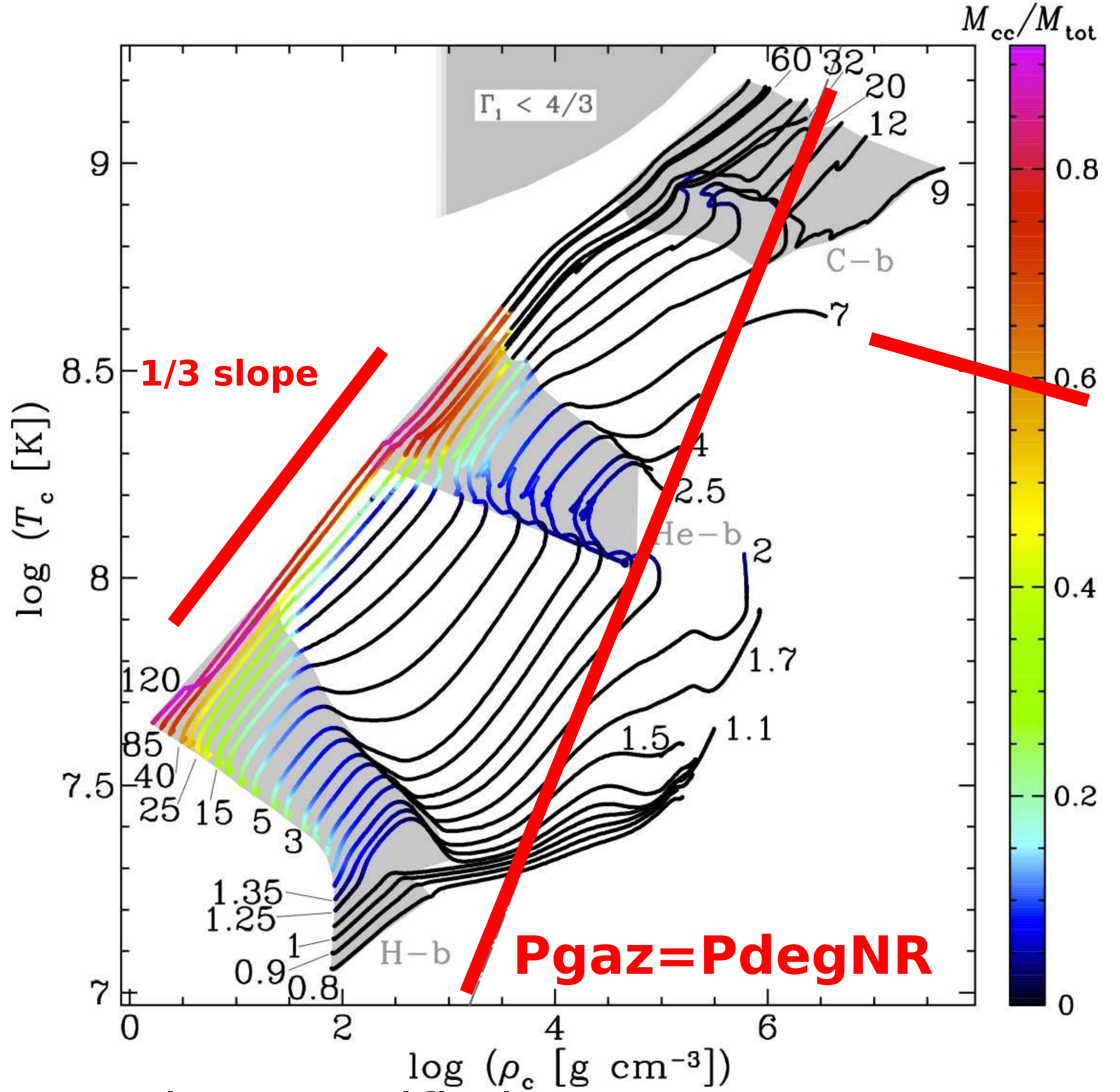
Pgaz=PdegNR

$$\frac{k}{\mu m_H}\rho T = K_1\left(\frac{\rho}{\mu e}\right)^{5/3} \rightarrow T = K_1\frac{\mu m_H}{k}\frac{1}{\mu_e^{5/3}}\rho^{2/3}$$

Non → Degenerate Conditions

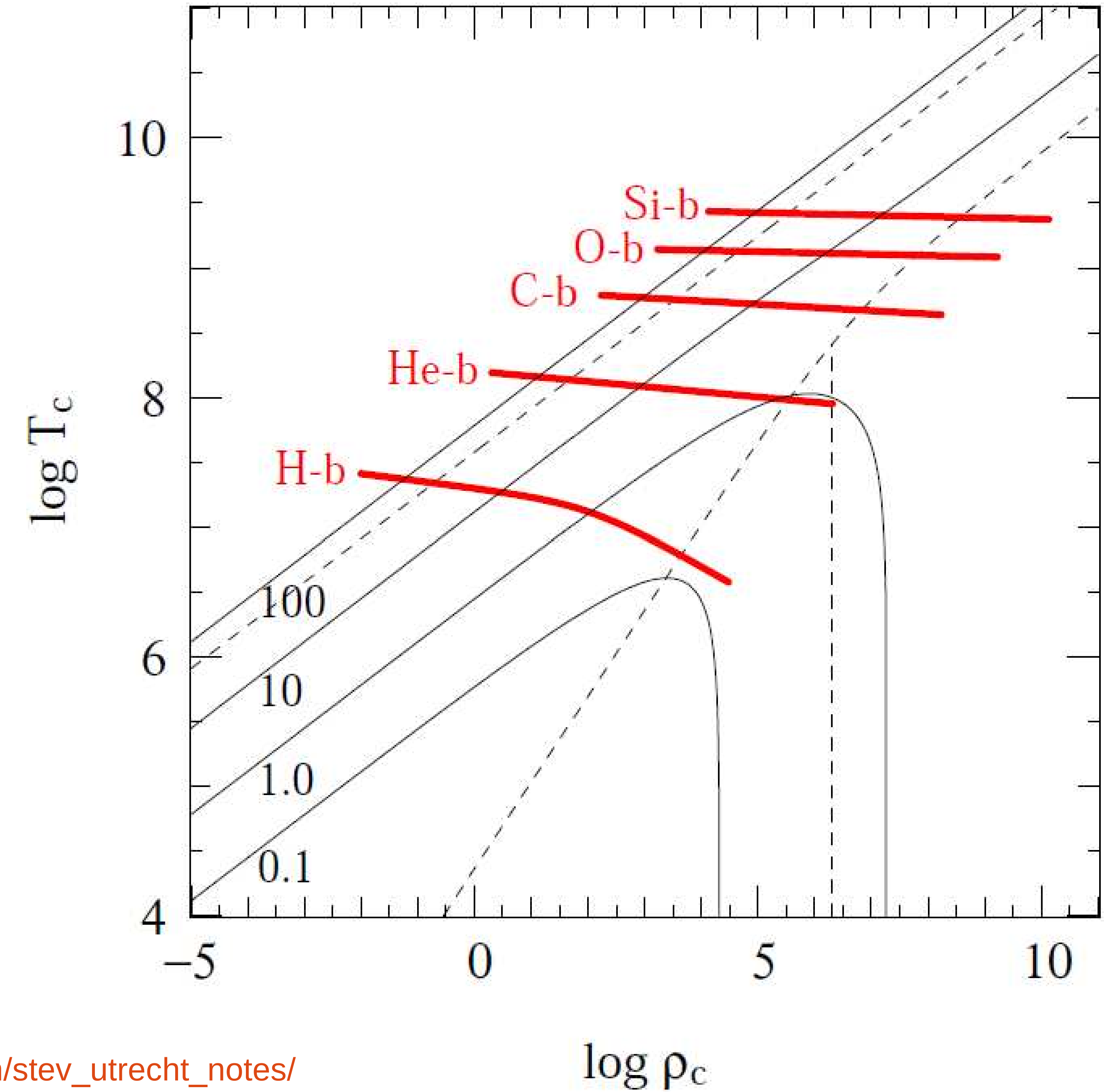
Models by

Ekström et al. (2012) A&A, 537, A146)



Stars=system with a negative specific heat!

Mass Domains



Lecture notes from O. Pols taken from:
http://www.astro.ru.nl/~onnop/education/stev_utrecht_notes/

Mass Domains

- 0.08 M_{sun} inferior mass limit for core H-burning : Brown Dwarfs
- 0.08 M_{sun} – 0.5M_{sun}: H burning OK, degenerate before core He-burning (lifetime > Hubble time→ no He white dwarf from single stars)
- 0.5-7M_{sun}: core H OK, core He OK (He-flash below 1.8 M_{sun}), degenerate CO white dwarf
- 7-9 M_{sun}: Core C burning OK→ WD(?) or Complete destruction (?) or collapse through electron captures (?)

- 9- 150 M_{sun}: core H, He, C, Ne, O, Si-→ Fe cores
- 150-250 M_{sun}: Pair Creation Supernovae

Mass Domains

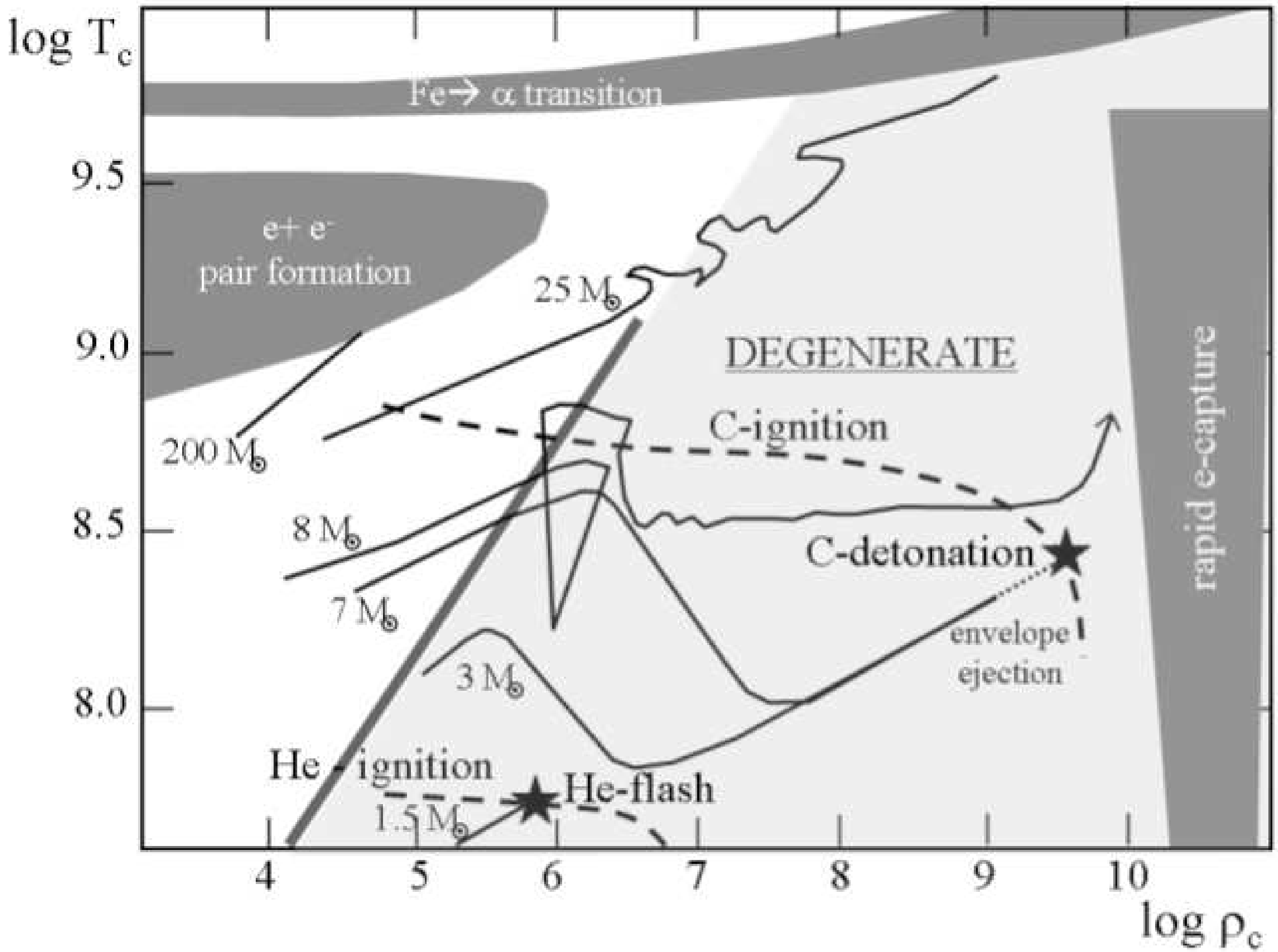


Fig. 26.10. Evolution of central conditions for different masses with indications of instability domains (Sect. 7.8), the Fe–α transition indicates the photodesintegration of Fe nuclei into α particles. The degenerate region is light gray. Dashed lines show the place where nuclear energy generation rates balance neutrino losses. Adapted from T.J. Mazurek and J.C. Wheeler [401]

L2: Physical Ingredients

Importance, basics, effects, uncertainties of:

(- Nuclear reactions)

- Convection in L1

- Mass loss

- Rotation

- Magnetic fields

- Binarity

- Equation of state, opacities & neutrino losses

including metallicity dependence

Recent Papers/Reviews

- Maeder, A. 2009, Physics, Formation and Evolution of Rotating Stars (Springer Verlag)

- Maeder and Meynet, "Rotating massive stars: From first stars to gamma ray bursts", 2012RvMP...84...25M

- Ekstroem, S., Georgy, C., Eggenberger, P., et al. 2012, A&A, 537, A146

- Chieffi, Limongi, "Pre-supernova Evolution of Rotating Solar Metallicity Stars in the Mass Range 13-120 $M_\odot$ and their Explosive Yields", 2012ApJS..199...38L

- Langer, "Pre-Supernova Evolution of Massive Single and Binary Stars", ARAA, 2012, astroph-1206.5443

Pressure and energy

Pressure as momentum transfer $\perp$ area= $n(\epsilon) \cdot \vec{p}(\epsilon) \cdot \vec{v}(\epsilon)$
Mean over incident angle ($1/3 \cdot p \cdot v$) and particle energy ϵ:

$$P = \frac{1}{3} \int_0^\infty n(\epsilon) p(\epsilon) v(\epsilon) \mathrm{d}\epsilon$$

relativistic case:

$\gamma := \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$; $p = \gamma m v$; $\epsilon = (\gamma - 1) m c^2 \Rightarrow$

$$P = \frac{1}{3} \int_0^\infty n\epsilon \left(1 + \frac{2mc^2}{\epsilon}\right) \left(1 + \frac{mc^2}{\epsilon}\right)^{-1} \mathrm{d}\epsilon$$

Pressure and energy

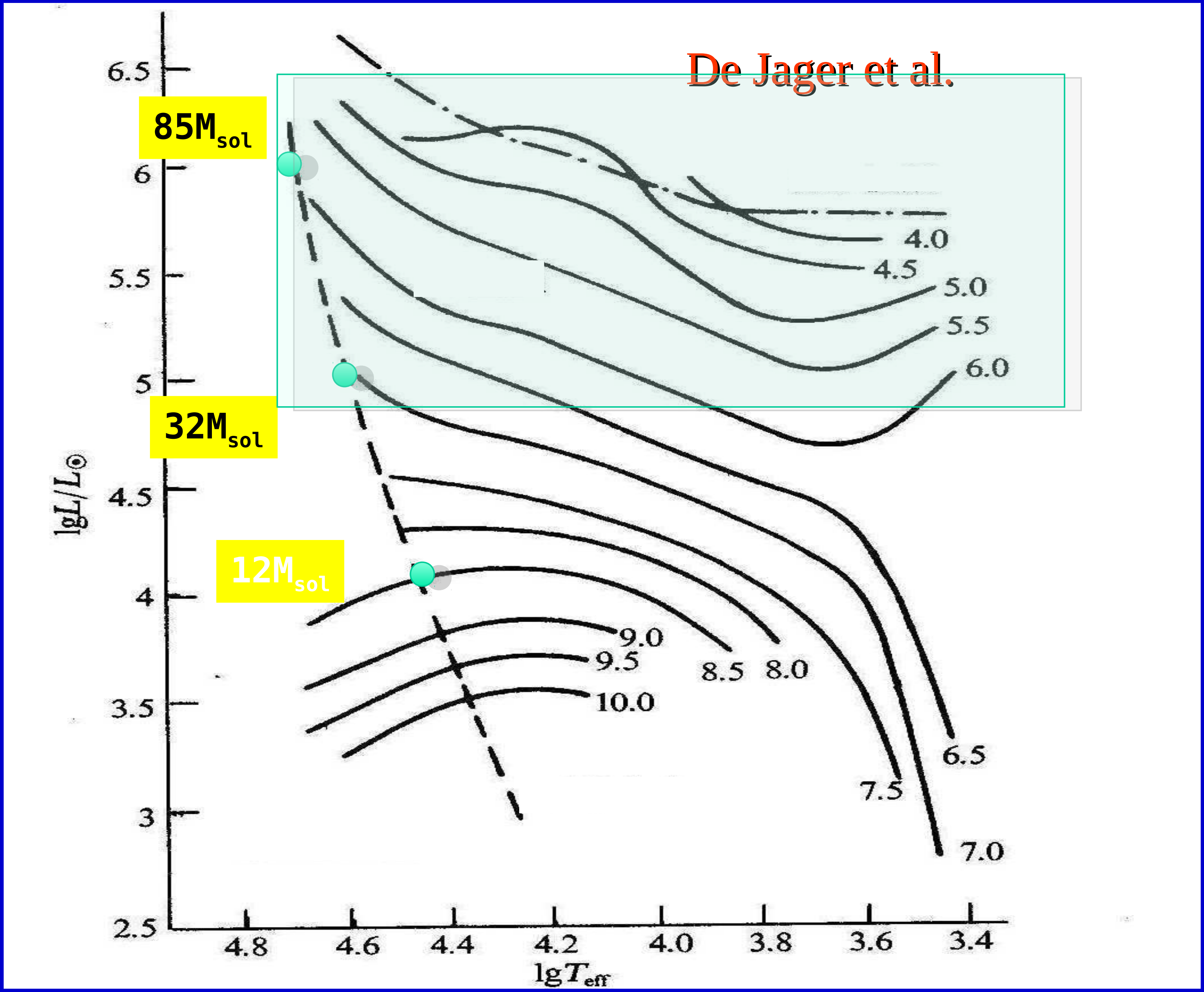
limiting cases

- non-relativistic: $mc^2 \gg \epsilon \rightarrow$
  $P_{\mathrm{NR}} = \frac{2}{3}\int n\epsilon \mathrm{d}\epsilon = \frac{2}{3}\langle n\epsilon\rangle = \frac{2}{3}U_{\mathrm{NR}}$
- extrem-relativistic: $mc^2 \ll \epsilon \rightarrow P_{\mathrm{ER}} = \frac{1}{3}U_{\mathrm{ER}}$

$\Rightarrow$ general relation $P \sim U$ (energy density)

Radiation pressure:

$$P_{\mathrm{rad}} = \frac{1}{3}U = \frac{a}{3}T^4 \; \left(a = 7.56 \cdot 10^{-15}\,\frac{\mathrm{erg}}{\mathrm{cm^3K^4}}\right)$$



De Jager et al.
85M sol
32M sol
12M sol
6.5
6
5.5
5
4.5
4
3.5
3
2.5
lgL/L⊙
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
lgT eff
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0
9.5
10.0

Typical mass-loss rates for galactic O-type stars on the MS

0.5-20 x 10^{-6} M_{sol} year^{-1}

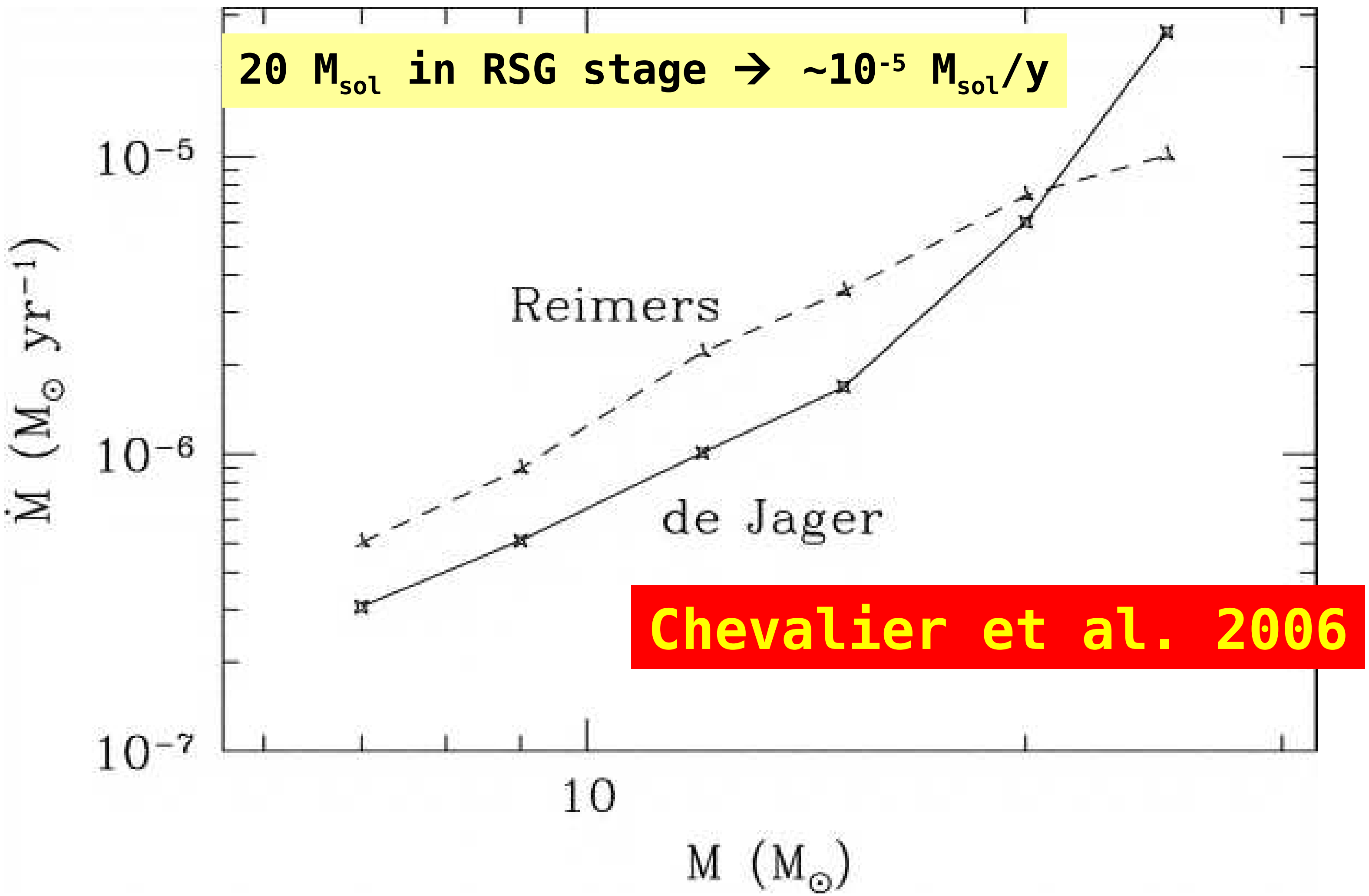
$$\dot{M} \propto L^{1.7}$$

$$\Longrightarrow \quad \dot{M} \propto M^{3.4}$$

$$L \propto M^{2}$$

$$\tau_{MS} \propto M^{-0.6} \quad \Longrightarrow \quad \Delta M \propto M^{2.8}$$

$$\Delta M / M \propto M^{1.8}$$

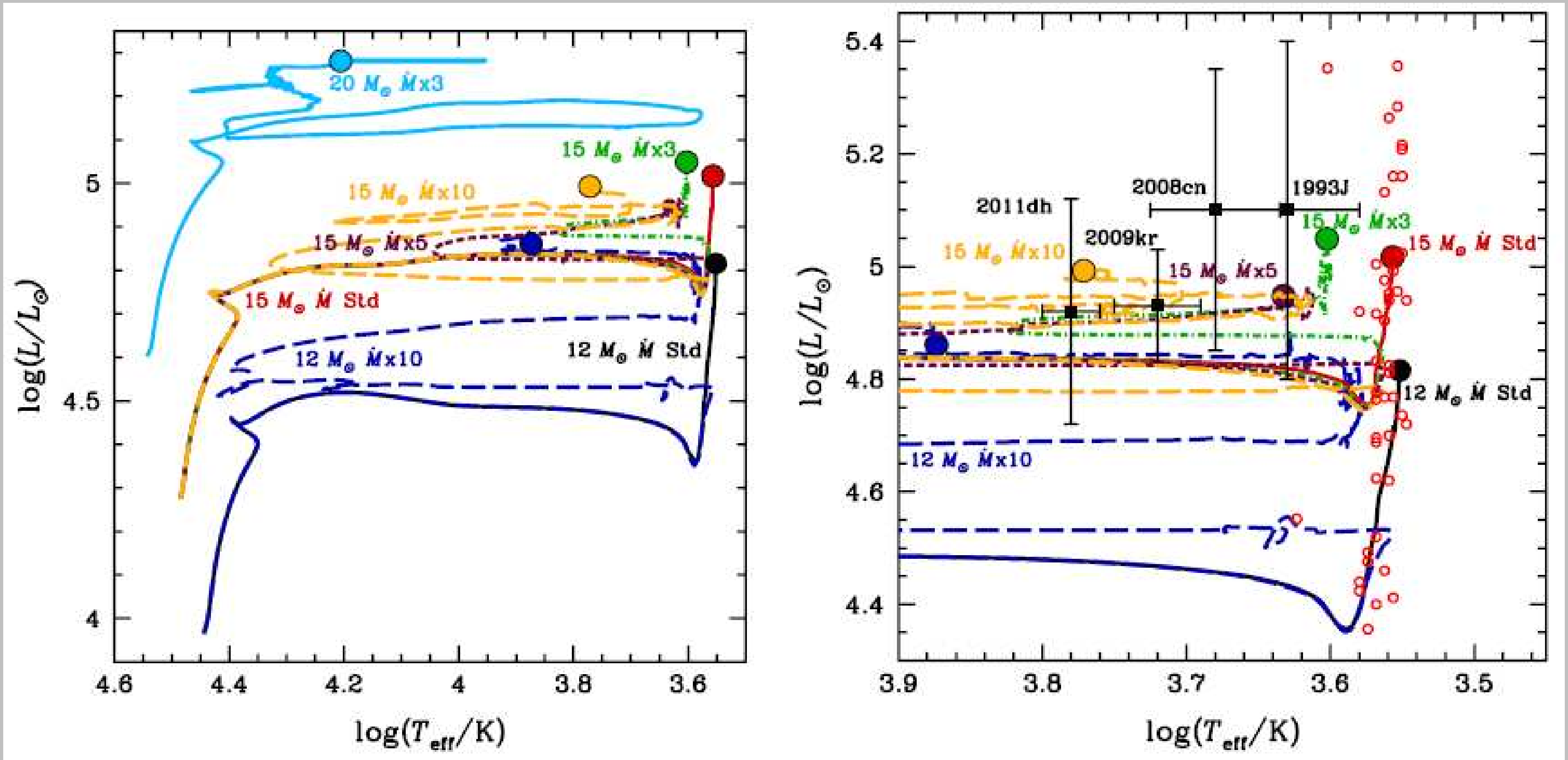


Dust enshrouded red supergiant may have higher mass loss (factor between 3 and 50) van Loon et al. (2005).

RSG/YSG/WR – SN II, IIb, Ib, Ic

RSG Mdot: - log(Teff/K) > 3.7: de Jager et al. (1988)
- log(Teff/K) < 3.7: linear fit from the data of Sylvester et al. (1998) and van Loon et al. (1999) (see Crowther 2001)

$$\dot{M} = -1.479 \times 10^{-14} \times \left(\frac{L}{L_\odot}\right)^{1.7}$$

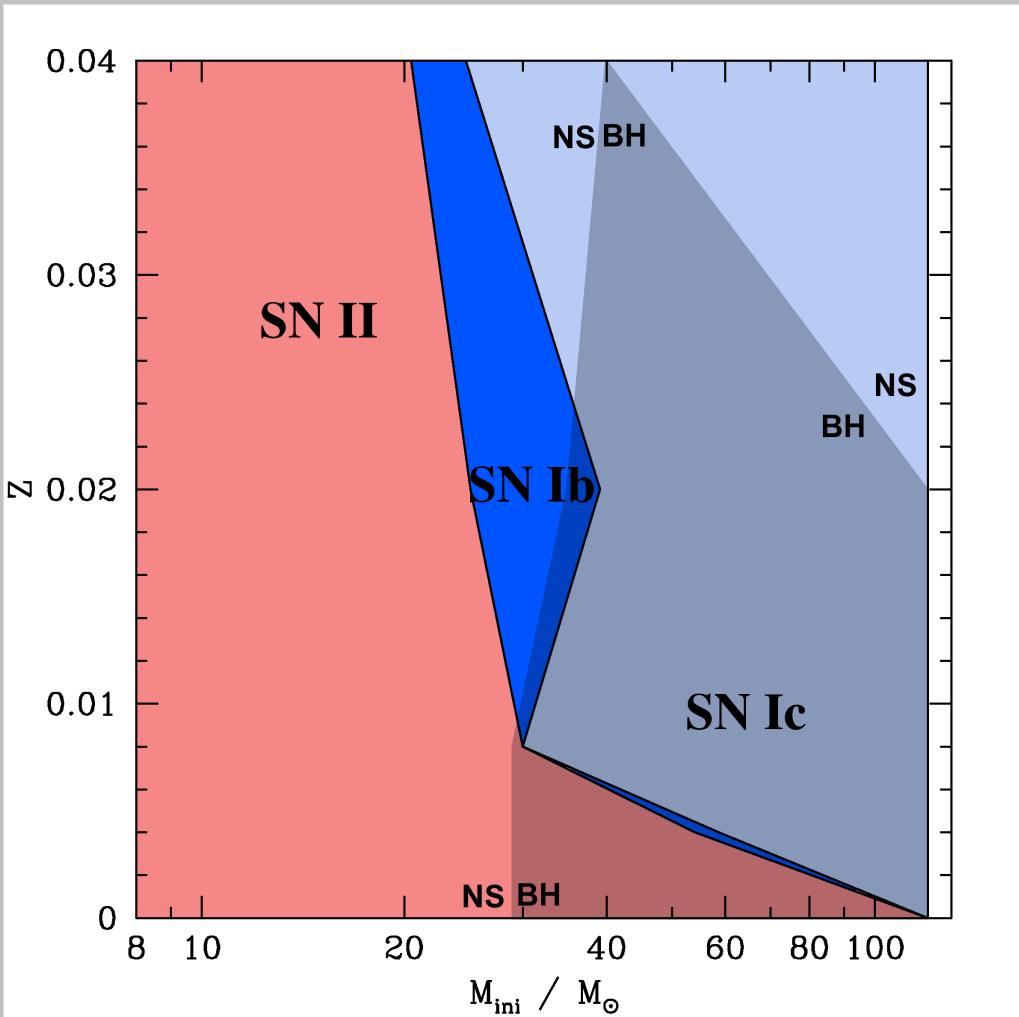


Models: Georgy 12 (see also Eldridge et al 13)

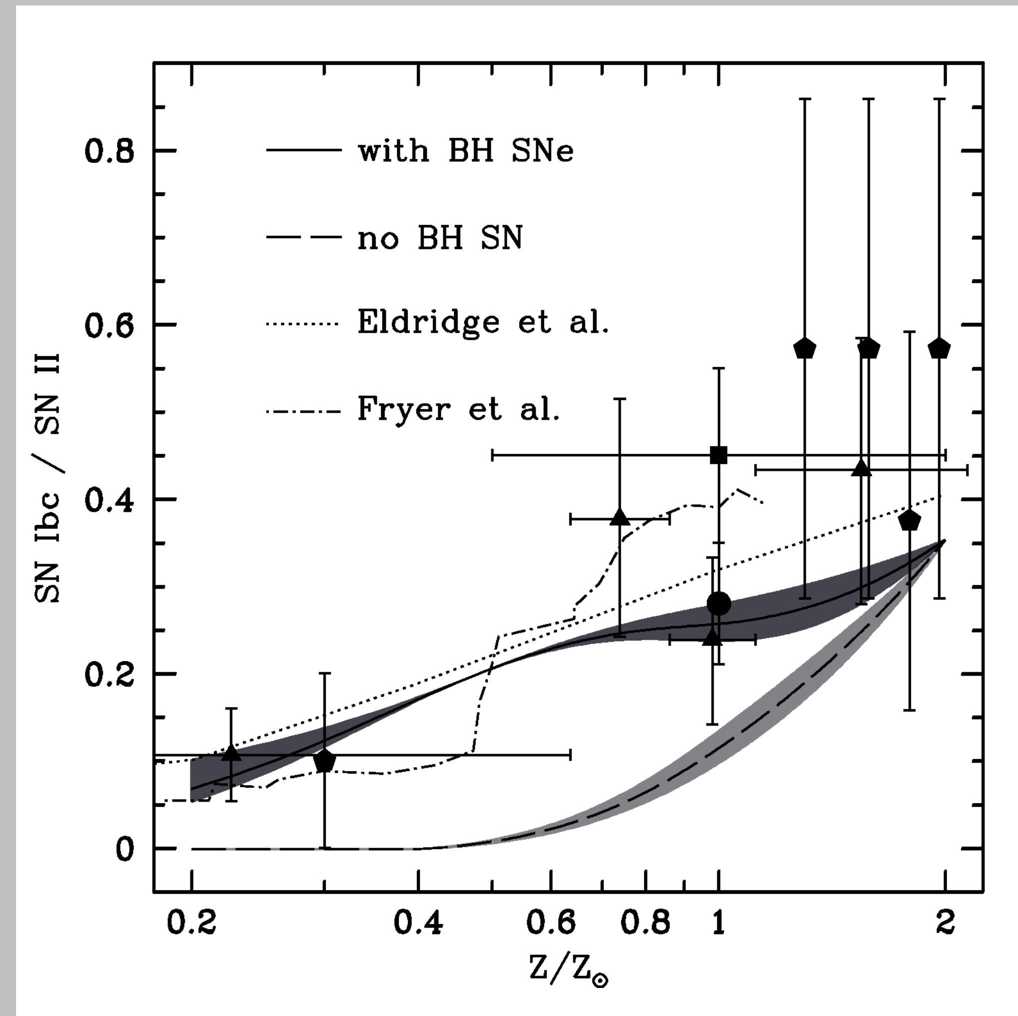
Super-Eddington layers → increased Mdot (see Ekstroem et al 13)

Final stages & SN type

Ratio SNIbc/SNII: tests final type



Georgy et al 09



- THEORY: Georgy et al 09 (solid line)
  binaries: Eldridge etal 08 (dotted)
- OBS: Prantzos & Boissier 03 (triangles)
  Prieto etal 08 (pentagons)

Long soft GRB-SN Ic connection: GRB060218/SN2006aj

Cusumano et al 2006, ...

GRB 031203-SN 2003lw / GRB 030329-SN 2003dh / GRB 980425-SN 1998bw, ...

Tagliaferri, G et al 2004 / Matheson 2003, ... / Iwamoto, K. 1999, ...

Collapsar progenitors must: (Woosley 1993, A. Mc Fadyen)

Form a BH

Lose their H-rich envelope → WR star

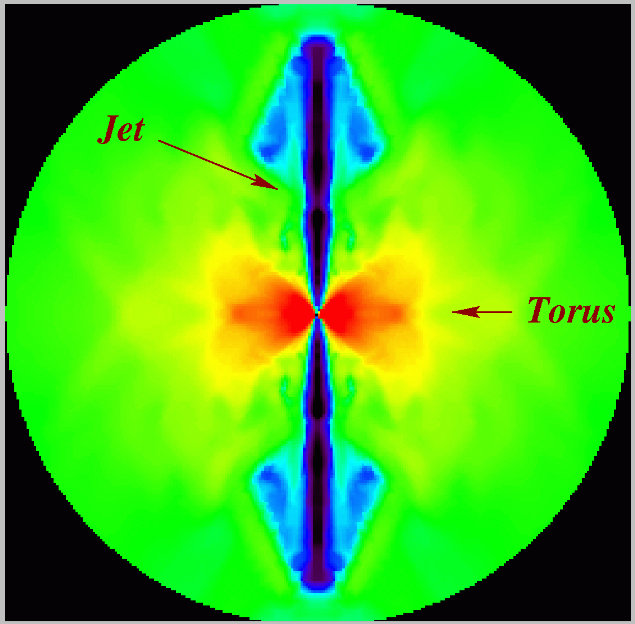
Core w. enough angular momentum

Observational info:

Z of close-by GRBs is lower than solar

~ Z (Magellanic clouds)

(Stanek et al 06, Le Floc'h er al 2003, Fruchter et al 2006)



(simulation by Mc Fadyen)

Theoretical GRB rates (without B-fields)

Obs: $R(GRB)=3x10^{-6}$ to $6x10^{-4}$ & $R(SNII,Ib,c)\sim 7x10^{-3}$ $[yr^{-1}]$

Podsiadlowski et al 04

GRB from all WR types:

Too many

GRB from WO (SN Ic):

OK with obs.

Hirschi et al A&A, 443, 581, 2005

	Z_{SMC}	Z_{LMC}	Z_{o}	Z_{GC}
M_{GRB}^{min}(WR)	32	25	22	21
M_{GRB}^{max}(WR)	95	95	75	55
R_{GRB}^{max}(WR)	1.15E-03	1.74E-03	2.01E-03	1.92E-03
M_{GRB}^{min}(WO)	50	45	-	-
M_{GRB}^{max}(WO)	95	95	-	-
R_{GRB}^{max}(WO)	4.74E-04	5.99E-04	-	-

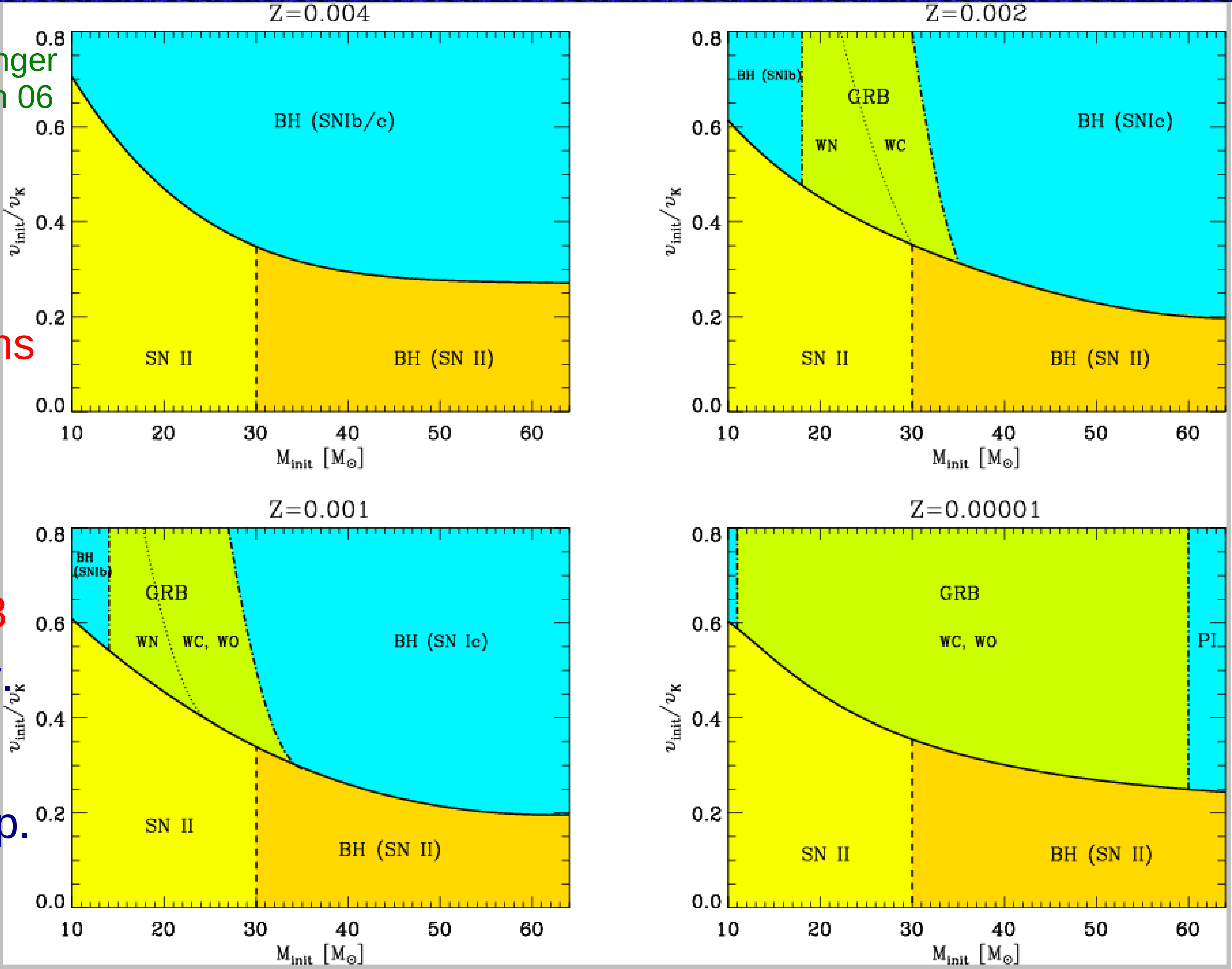
GRB favoured at low Z, maybe also very low Z (85 Mo)

GRB progenitors with B-Fields

Yoon, Langer & Norman 06

Rates compatible with observations

Z_{max}~0.003 is a bit low. Dep. on Mdot & Solar comp.



GRB progenitors with B-Fields

Taylor-Spruit dynamo (Spruit 2002) : better for NS (Heger et al 2005, Suijs etal 08)

No A_{BH}>1 in Fe-core @ pre-SN stage with B-fields (Petrovic et al 2005, ...)

- A_{BH}~1 ← Quasi chemically-homog. evol. of fast rot. stars (avoid RSG)

  (Yoon & Langer 06, Woosley & Heger 2006)

40 M_o models

V_{ini} [km/s]	Z_o	Z(SMC)	$Z=10^{-3}$	$Z=10^{-5}$	$Z=10^{-8}$
~230	-	-	-	No	-
~300	-	WR	-	-	-
400-500	WR	WR	WR	WR	No
700	-	-	-	-	WR

This study

- WR (SNIb,c) & GRBs predicted down to Z=~0 (Yoon et al 06)

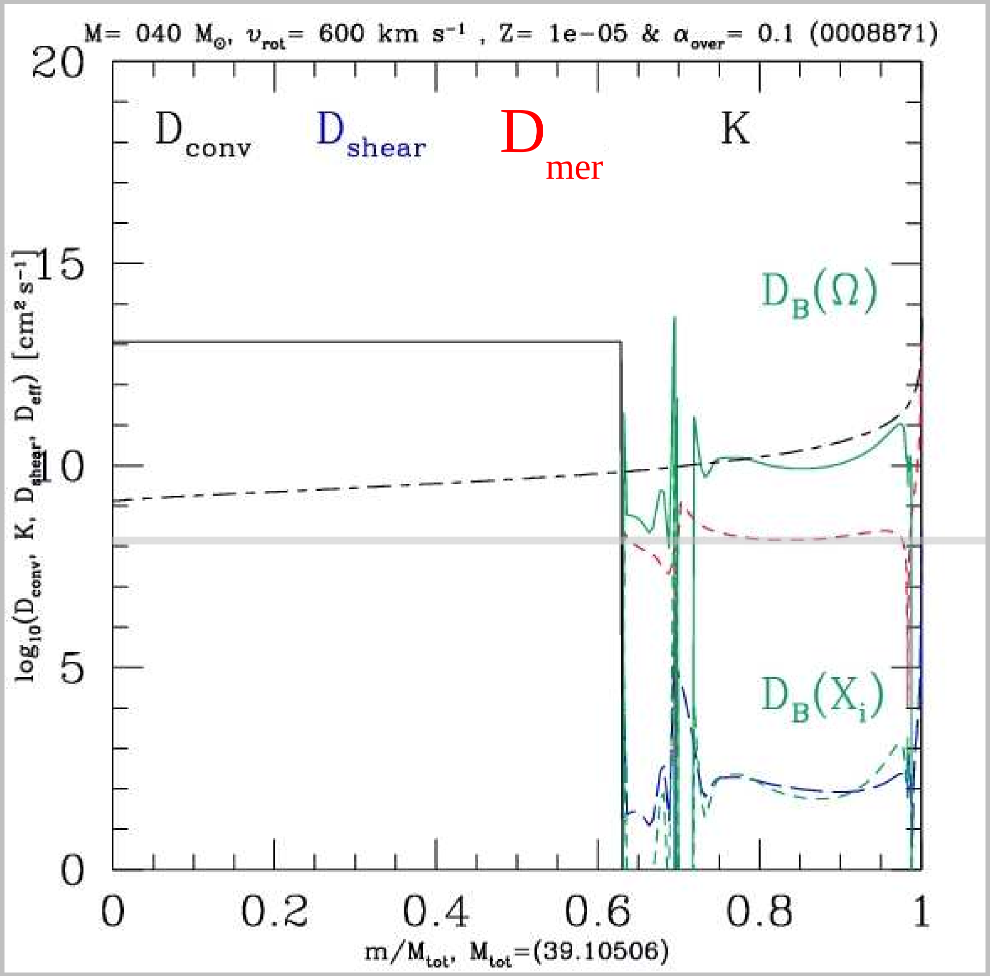
Question:

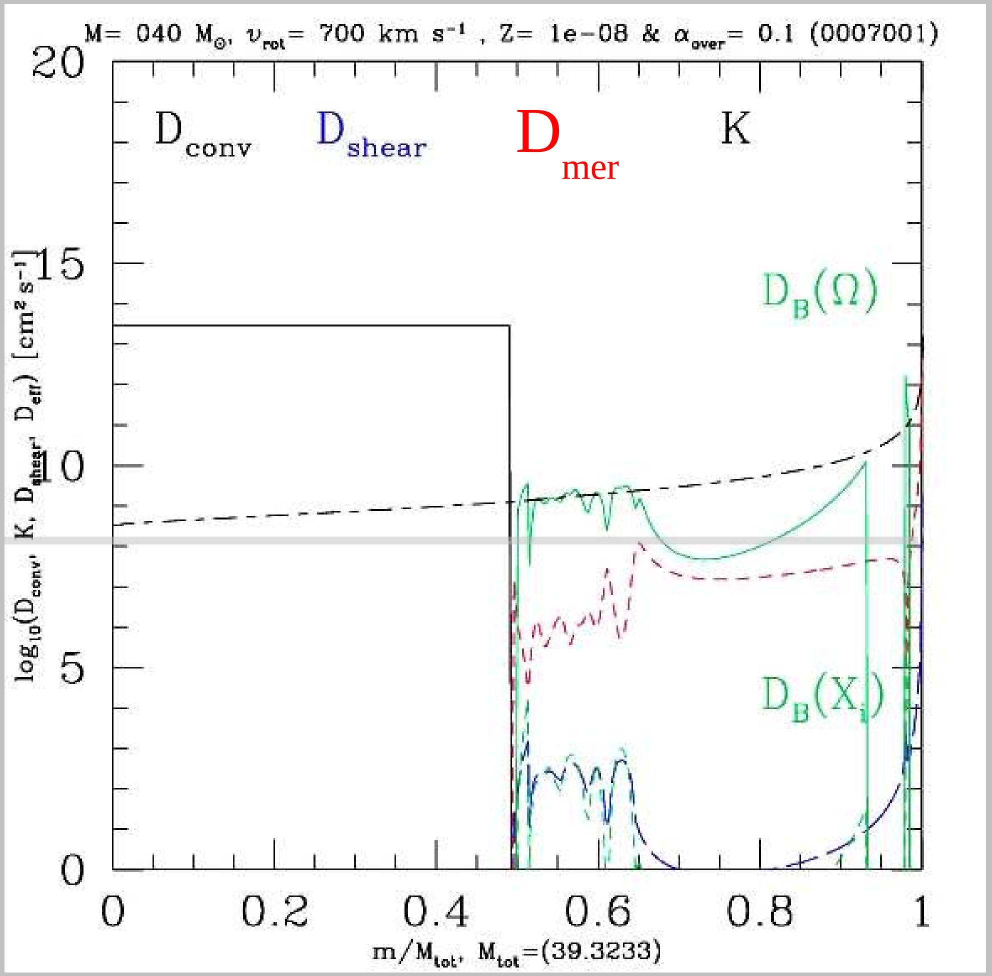
- GRBs around Z(LMC) & Z(SMC)? Dep. On mass loss / NO GRB @ Z_o

  (Meynet & Maeder 2007)

Quasi-Chem. Evol. @ very low Z? 40M_o models

Z=1e-5, v_{ini}=600 km/s (v_{ini}/v_{crit} =0.59) **Z=1e-8, v_{ini}=700 km/s (v_{ini}/v_{crit} =0.55)**





Diff. Coeff. Smaller --> Quasi-Chem. Evol. harder for the first stellar generations

Stellar Evolution: From the Most to the Least massive Stars

Acknowledgements & Bibliography

- Slides in white background (with blue title) were taken from Achim Weiss' lecture slides, which you can find here: http://www.mpa-garching.mpg.de/~weiss/lectures.html

- Some graphs were taken from Onno Pols' lecture notes on stellar evolution, which you can find here:

http://www.astro.ru.nl/~onnop/education/stev_utrecht_notes/

- Some slides (colourful ones) and content was taken from George Meynet's summer school slides.

Acknowledgements & Bibliography

Recommended further reading:

- R. Kippenhahn & A. Weigert, Stellar Structure and Evolution, 1990, Springer-Verlag, ISBN 3-540-50211-4

- A. Maeder, Physics, Formation and Evolution of Rotating Stars, 2009, Springer-Verlag, ISBN 978-3-540-76948-4

- D. Prialnik, An Introduction to the Theory of Stellar Structure and Evolution, 2000, Cambridge University Press, ISBN 0-521-65937-X

- C.J. Hansen, S.D. Kawaler & V. Trimble, Stellar Interiors, 2004, Springer-Verlag, ISBN 0-387-20089-4

- M. Salaris & S. Cassisi, Evolution of Stars and Stellar Populations, 2005, John Wiley & Sons, ISBN 0-470-09220-3

L2: Physical Ingredients

Importance, basics, effects, uncertainties of:

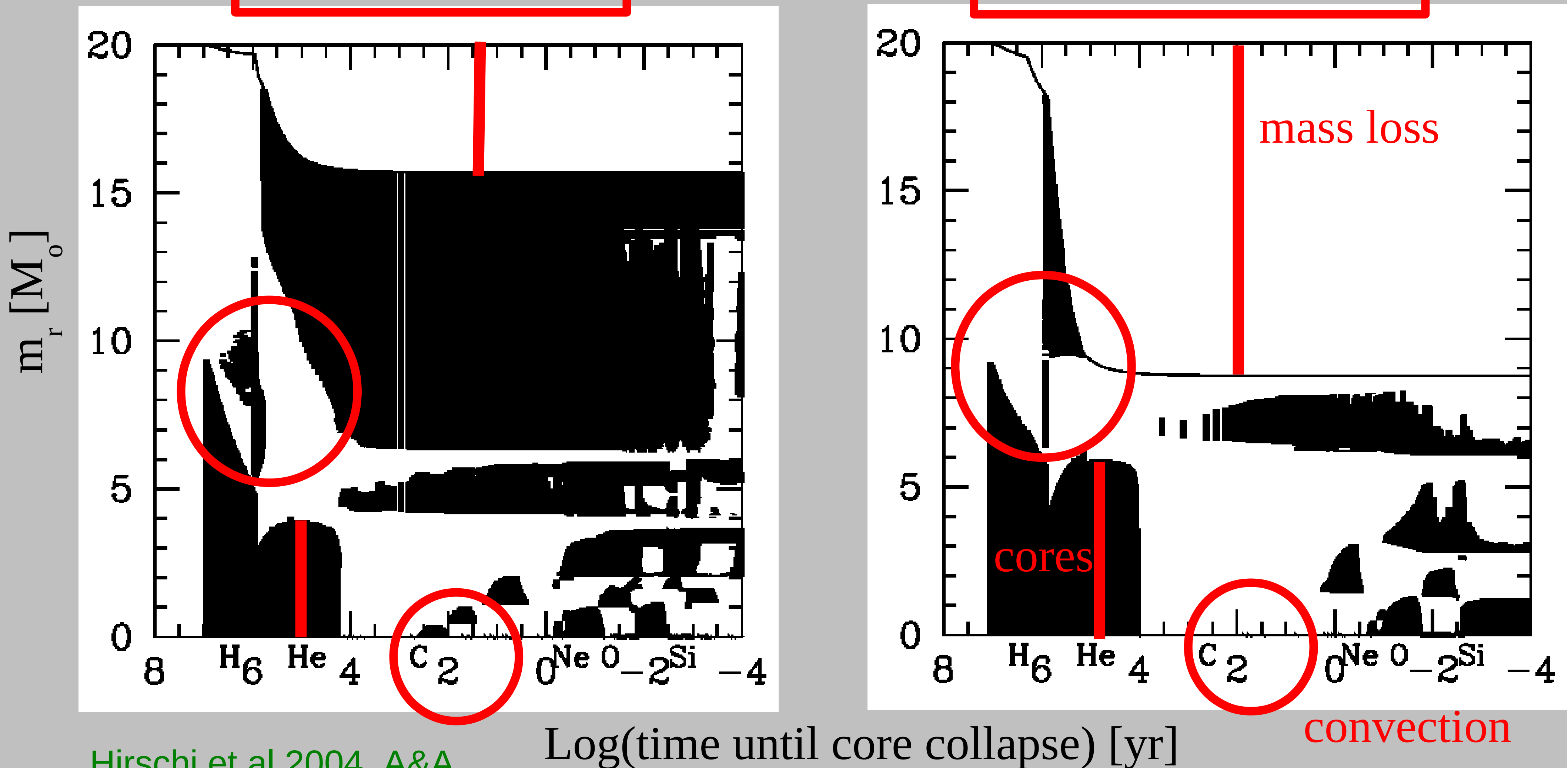
- Nuclear reactions → B. Meyer
- Convection in L1
- Mass loss
- Rotation
- Magnetic fields
- Binarity
- Equation of state, opacities & neutrino losses

  including metallicity dependence

Impact of Rotation, $\mathcal{M}_{ini} = 20\ \mathcal{M}_\odot$

υ_{ini} = 0 km/s

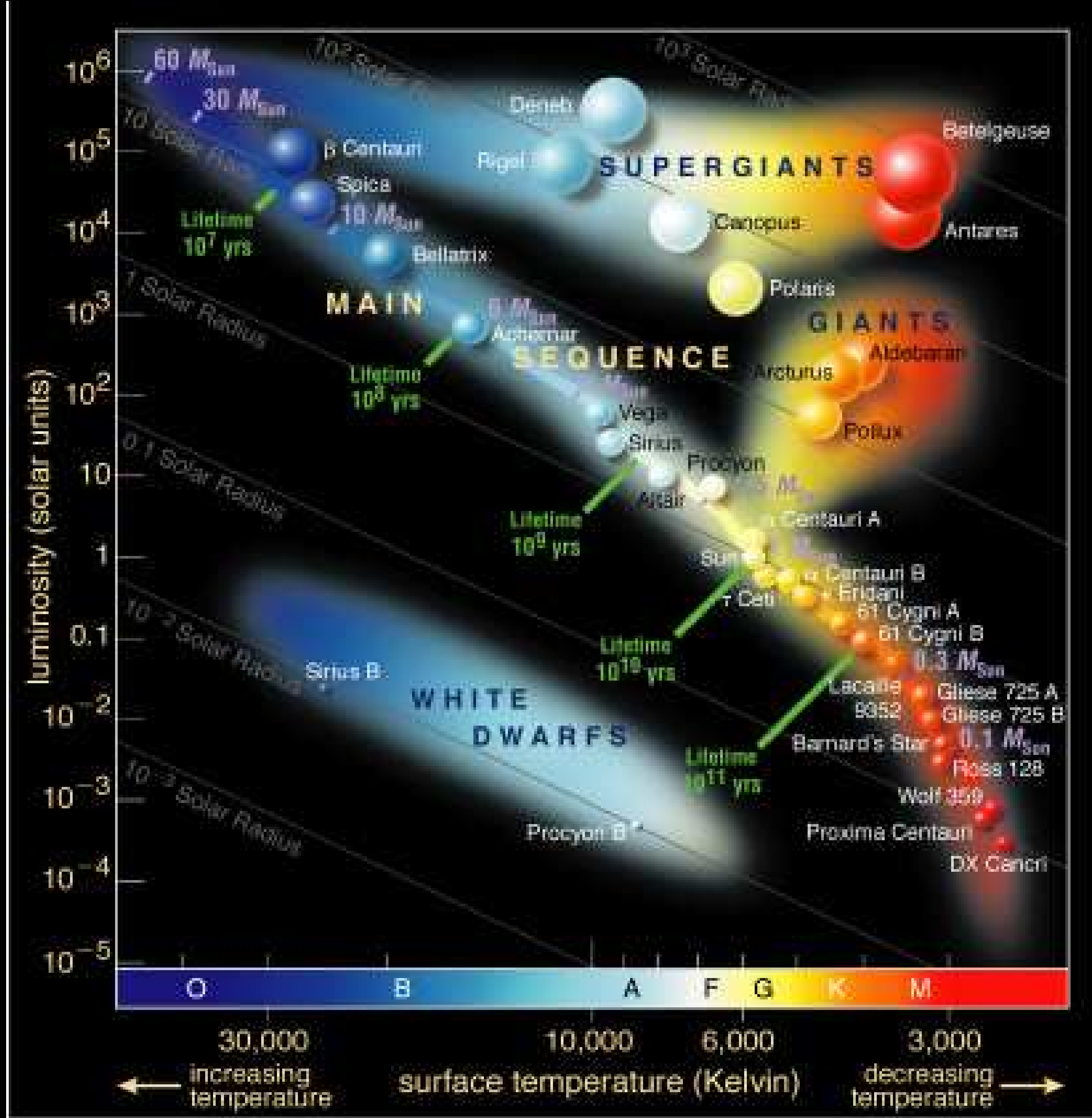
υ_{ini} = 300 km/s



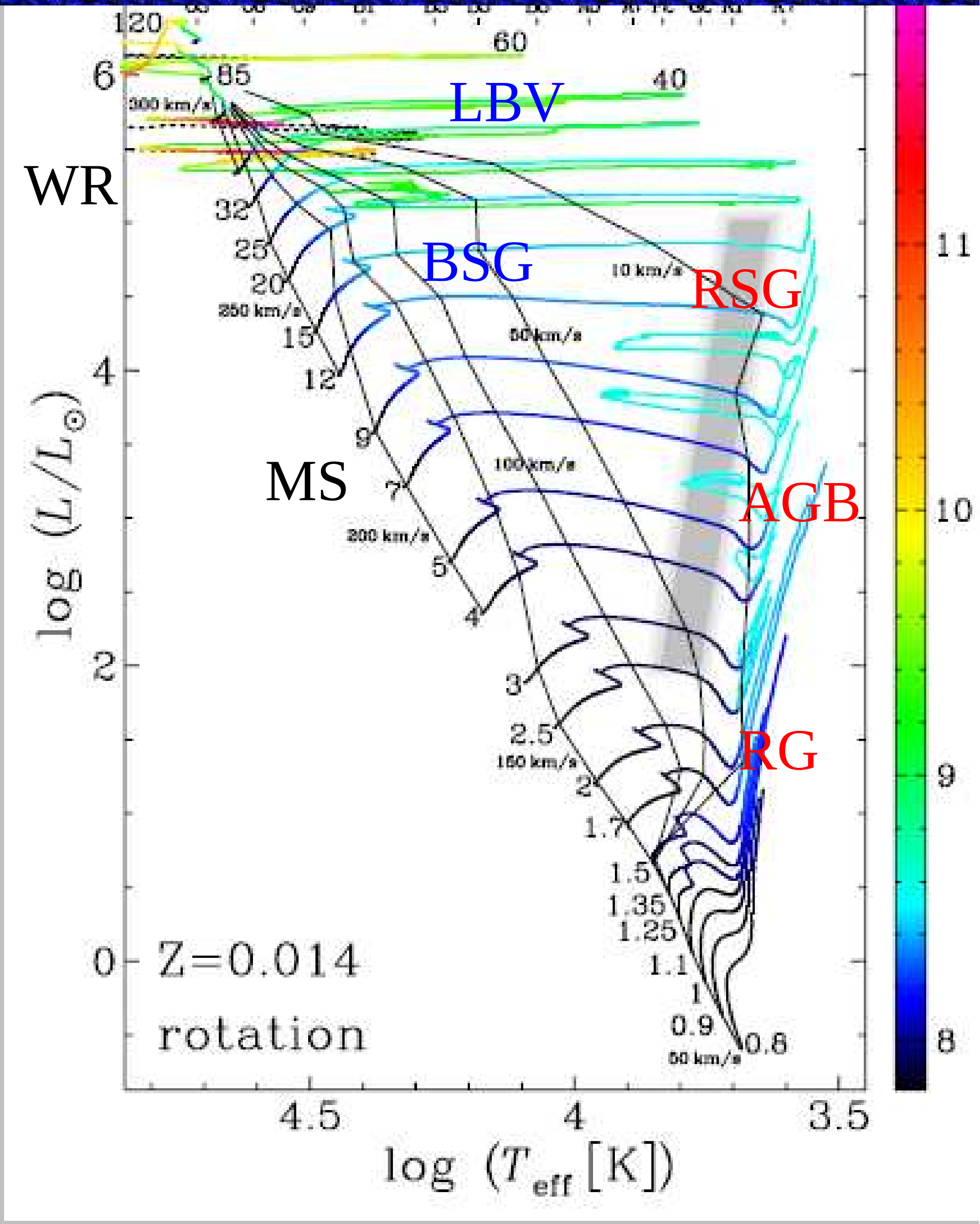
Hirschi et al 2004, A&A

Importance as Stellar Objects

Hertzsrpung-Russell (HR) Diagram:



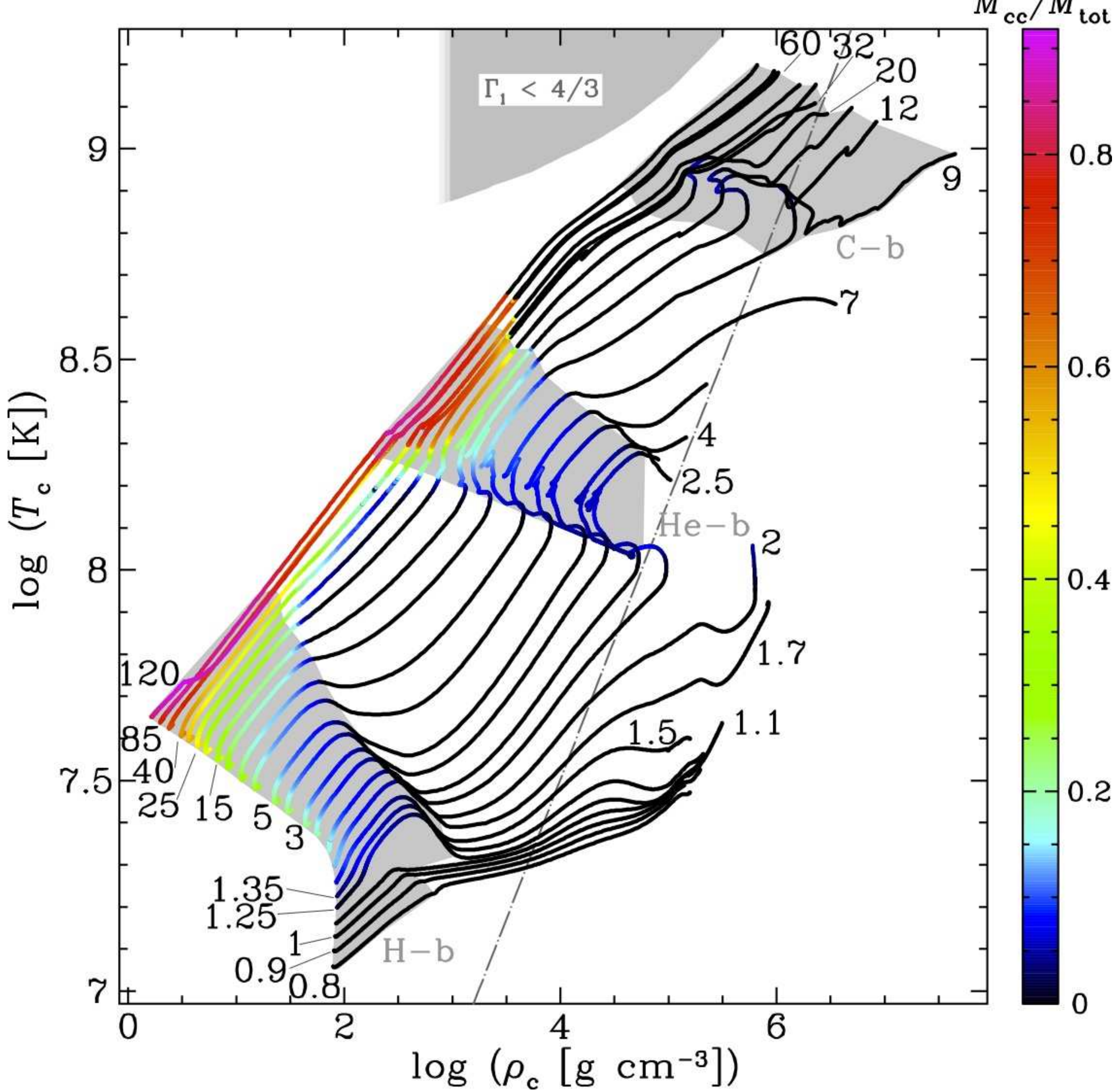
Evolutionary tracks →



Ekstroem et al 12

Central Temperature vs Central Density Diagram

Evolutionary tracks →



Ekstroem et al 12

L3-4

- EOS and partial degeneracy
- Standard massive stars
- The most massive stars
- Weak s-process
- Intermediate- and low-mass stars
- Stars at the boundary between massive and intermediate-mass stars

Equation of State - Ideal gas

$$P = nk_BT = \frac{\mathcal{R}}{\mu}\rho T$$

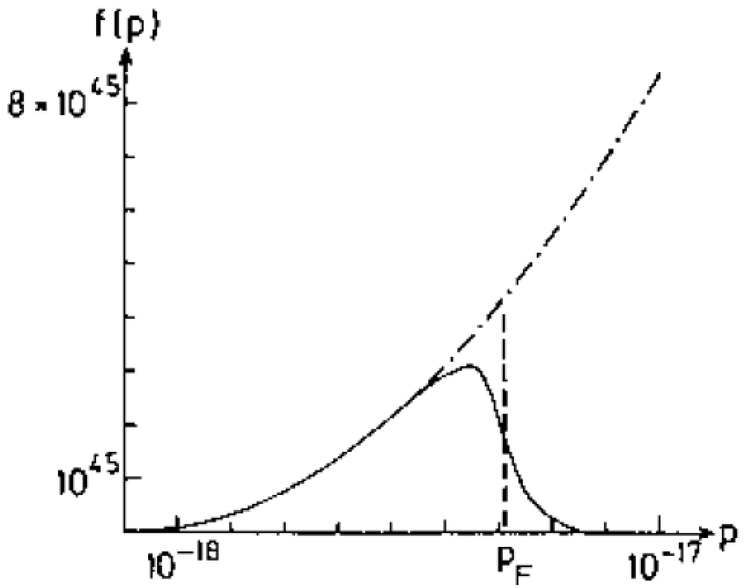
with $\rho = n\mu m_u$; μ: molecular weight, mass of particle per m_u.
Several components in gas with relative mass fractions $X_i = \frac{\rho_i}{\rho} \rightarrow n_i = \frac{\rho X_i}{m_u \mu_i}$

electrons and ions:

$$P = P_e + \sum_i P_i = (n_e + \sum_i n_i)kT.$$

Partial degeneracy

$$\ldots U_e = \frac{8\pi}{h^3}\int_0^\infty \frac{Ep^2\,\mathrm{d}p}{1+\exp\left(\frac{E}{kT}-\Psi\right)}$$



$f(p)$ for partially degenerate gas with $n_e = 10^{28}\ \mathrm{cm}^{-3}$ and $T = 1.9{\cdot}10^7\ \mathrm{K}$ corresponding to $\Psi = 10$.

The equation of state for normal stellar matter:

$$P = P_{\rm ion} + P_e + P_{\rm rad} = \frac{\mathcal{R}}{\mu_0}\rho T + \frac{8\pi}{3h^3}\int_0^\infty \frac{p^3 v(p)\,\mathrm{d}p}{1+\exp\left(\frac{E}{kT}-\Psi\right)} + \frac{a}{3}T^4$$

Non-ideal effects

- finite size of atoms $\rightarrow$ pressure ionization
  important already in Sun and low-mass stars
- Coulomb interaction – low density $\rightarrow$ pressure reduction
  important in many stars (envelopes, but also solar core)
- Coulomb interaction – high density $\rightarrow$ crystallization
  white dwarfs, neutron stars
- configuration effects $\rightarrow$ van der Waals gas; quantum effects (spin–spin–interactions)
- neutronization
  neutron stars

EOS implementation in MESA

Combination of sources for EOS in SE codes:

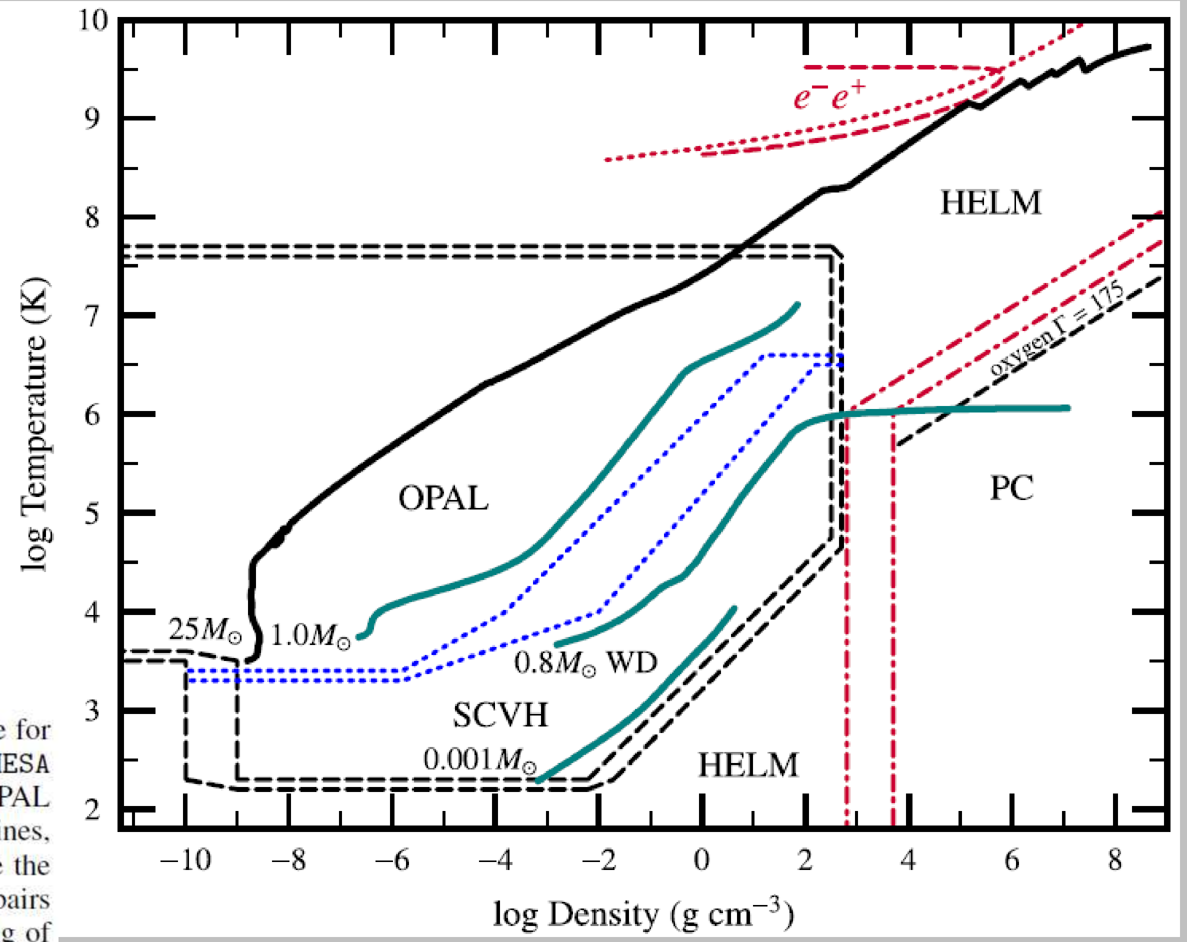
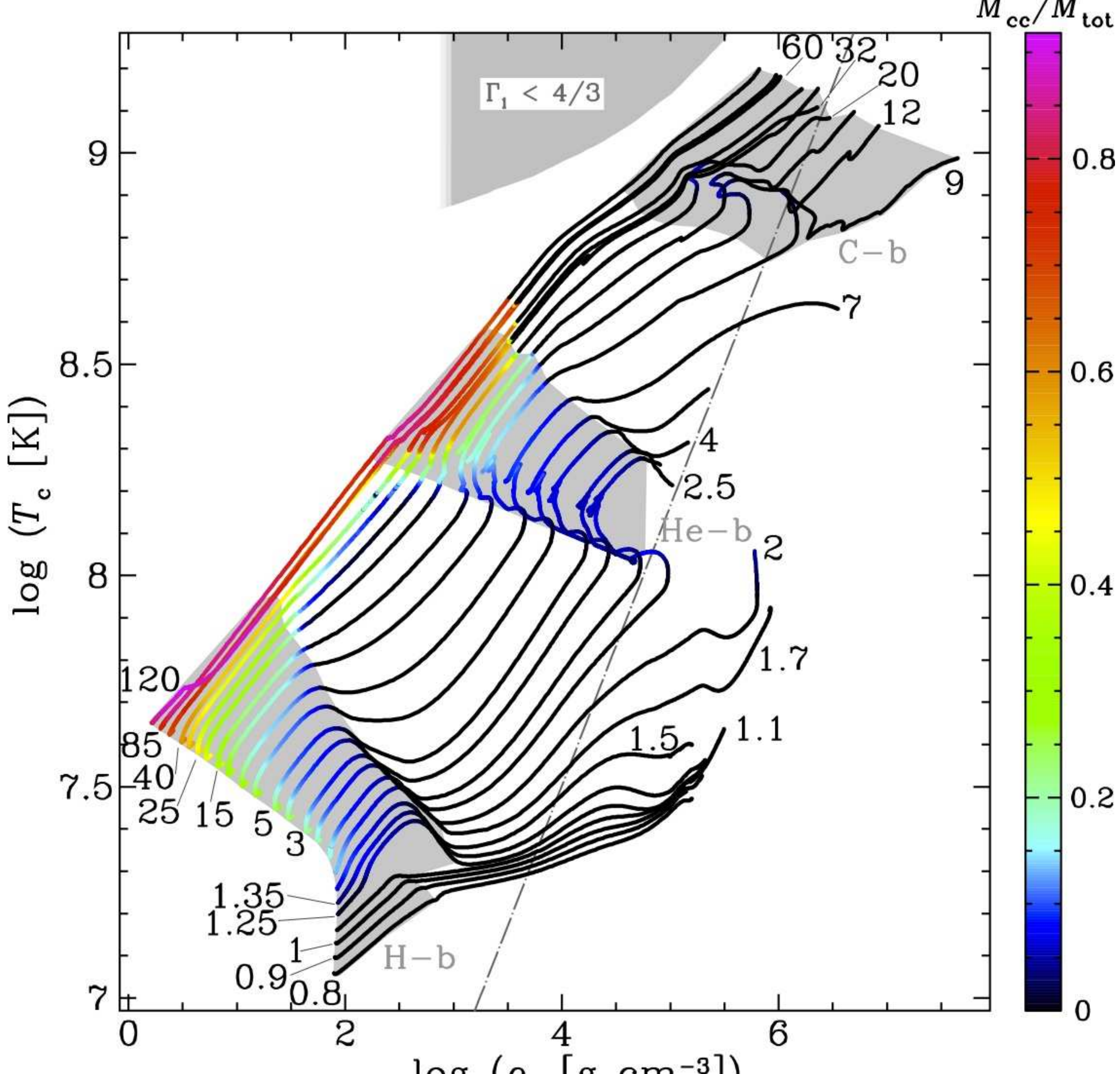


Figure 1. ρ–T coverage of the equations of state used by the eos module for $Z \leqslant 0.04$. Inside the region bounded by the black dashed lines we use MESA EOS tables that were constructed from the OPAL and SCVH tables. The OPAL and SCVH tables were blended in the region shown by the blue dotted lines, as described in the text. Regions outside of the black dashed lines utilize the HELM and PC EOSs, which, respectively, incorporate electron–positron pairs at high temperatures and crystallization at low temperatures. The blending of the MESA table and the HELM/PC results occurs between the black dashed lines and is described in the text. The dotted red line shows where the number of electrons per baryon has doubled due to pair production, and the region to the left of the dashed red line has $\Gamma_1 < 4/3$. The very low density cold region in the leftmost part of the figure is treated as an ideal, neutral gas. The region below the black dashed line labeled as $\Gamma = 175$ would be in a crystalline state for a plasma of pure oxygen and is fully handled by the PC EOS. The red dot-dashed line shows where MESA blends the PC and HELM EOSs. The green lines show stellar profiles for a main-sequence star ($M = 1.0\ M_\odot$), a contracting object of $M = 0.001\ M_\odot$, and a cooling white dwarf of $M = 0.8\ M_\odot$. The heavy dark line is an evolved $25\ M_\odot$ star that has a maximum infalling speed of 1000 km s^{-1}. The jagged behavior reflects the distinct burning shells.

Central Temperature vs Central Density Diagram

Evolutionary tracks →



What is the slope of the evolutionary tracks?

Ekstroem et al 12

Non-Degenerate Conditions

Let us first consider a uniform contraction of a mass M. In that case a variation in radius ΔR corresponds to a variation in pressure ΔP and to a variation in density $\Delta\rho$ so that we have the following relations:

$$\frac{\Delta P}{P} = -4\frac{\Delta R}{R}, \quad \text{and} \quad \frac{\Delta\rho}{\rho} = -3\frac{\Delta R}{R}.$$

The first equality is deduced from the hydrostatic equilibrium equation and the second from the continuity equation. From these two relations, we can write

$$\Delta \ln P = \frac{4}{3}\Delta \ln \rho.$$

Let us now write the equation of state as follows

$$\Delta \ln \rho = \alpha \Delta \ln P - \delta \Delta \ln T,$$

where α and δ are defined by $\alpha = \left(\frac{\partial \ln \rho}{\partial \ln P}\right)_{T,\mu}$ and $\delta = -\left(\frac{\partial \ln \rho}{\partial \ln T}\right)_{P,\mu}$, and where μ, the mean molecular weight, is supposed to remain constant. From these two relations one obtains, by eliminating ΔP the two following relations between a variation in log T and log ρ:

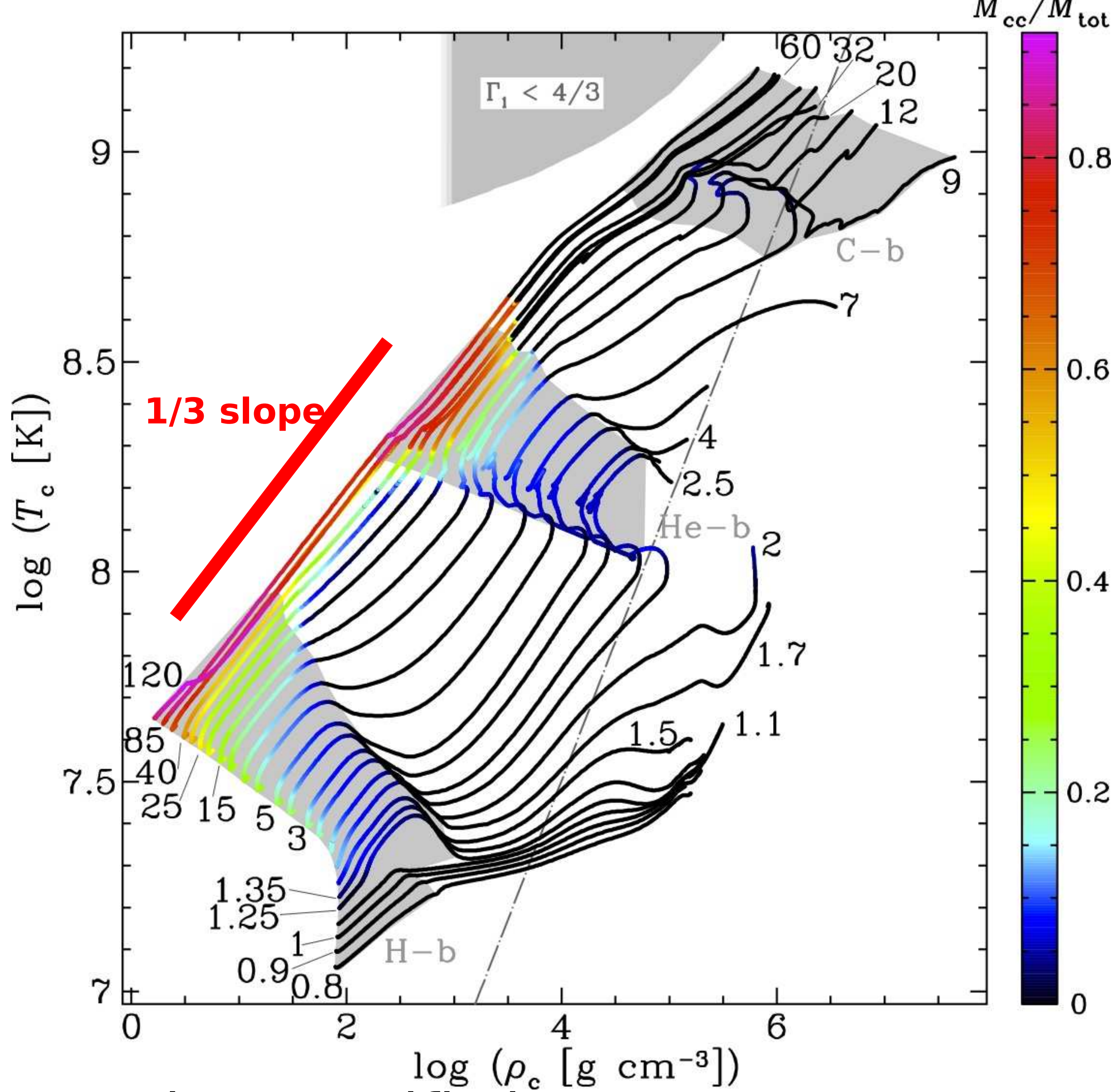
$$\Delta \ln T = \left(\frac{4\alpha - 3}{3\delta}\right)\Delta \ln \rho. \qquad (1)$$

For a perfect gas law we have $\alpha = \delta = 1$. Therefore an increase of, for instance, 30% in density implies an increases of 10% in temperature.

Non-Degenerate Conditions

Models by

Ekström et al. (2012)
A&A, 537, A146)



Stars=system with a negative specific heat!

Degenerate Conditions

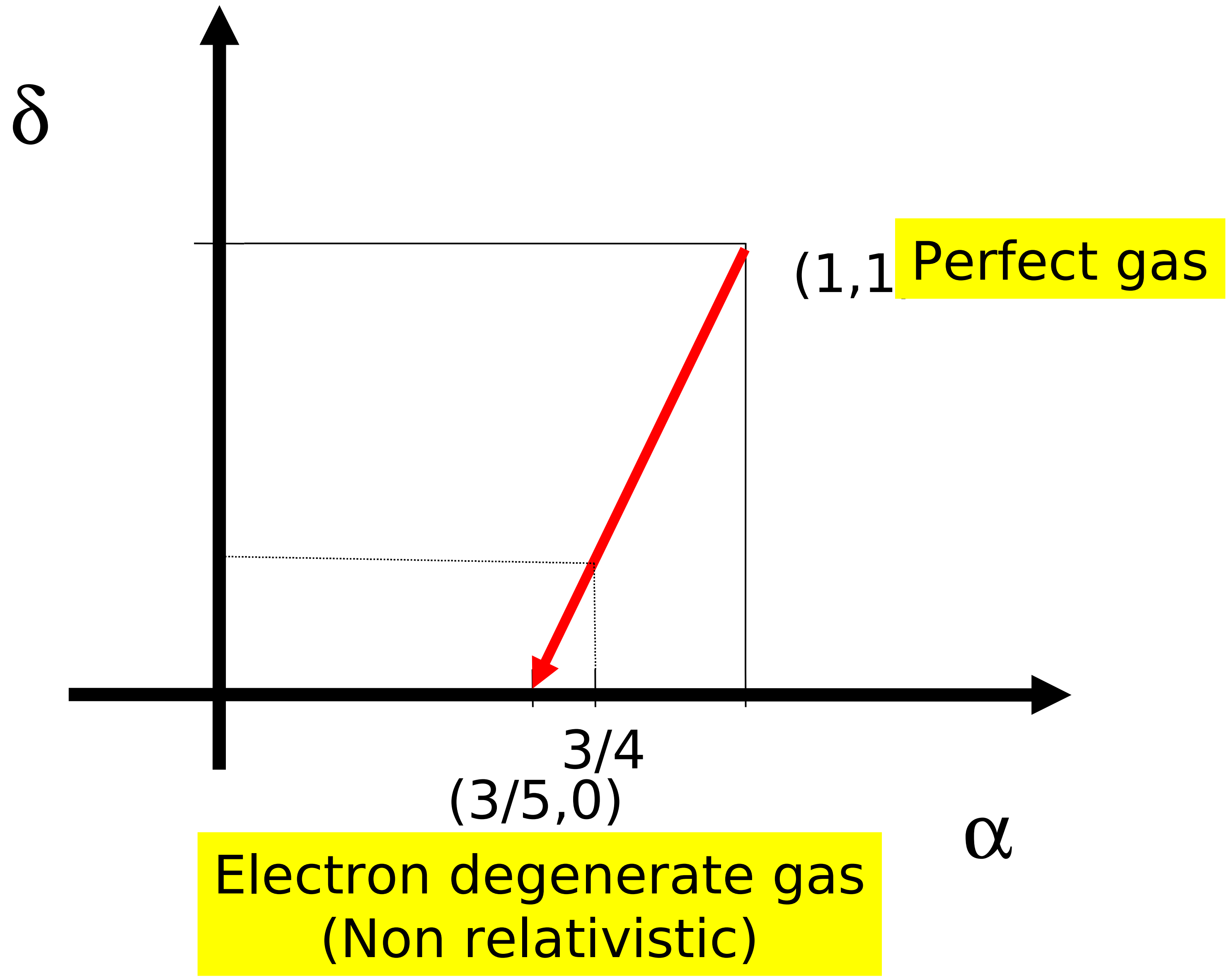
no longer valid, but if during the course of evolution, when the central conditions pass from the non-degenerate region to the degenerate one, α becomes inferior to three quarters before δ is equal to zero, then a contraction can produce a cooling! This can be understood as due to the fact that, in order to allow electrons to occupy still higher energy state, some energy has to be extracted from the non degenerate nuclei which, as a consequence, cool down.

$$\Delta \ln T = \left(\frac{4\alpha - 3}{3\delta} \right) \Delta \ln \rho.$$

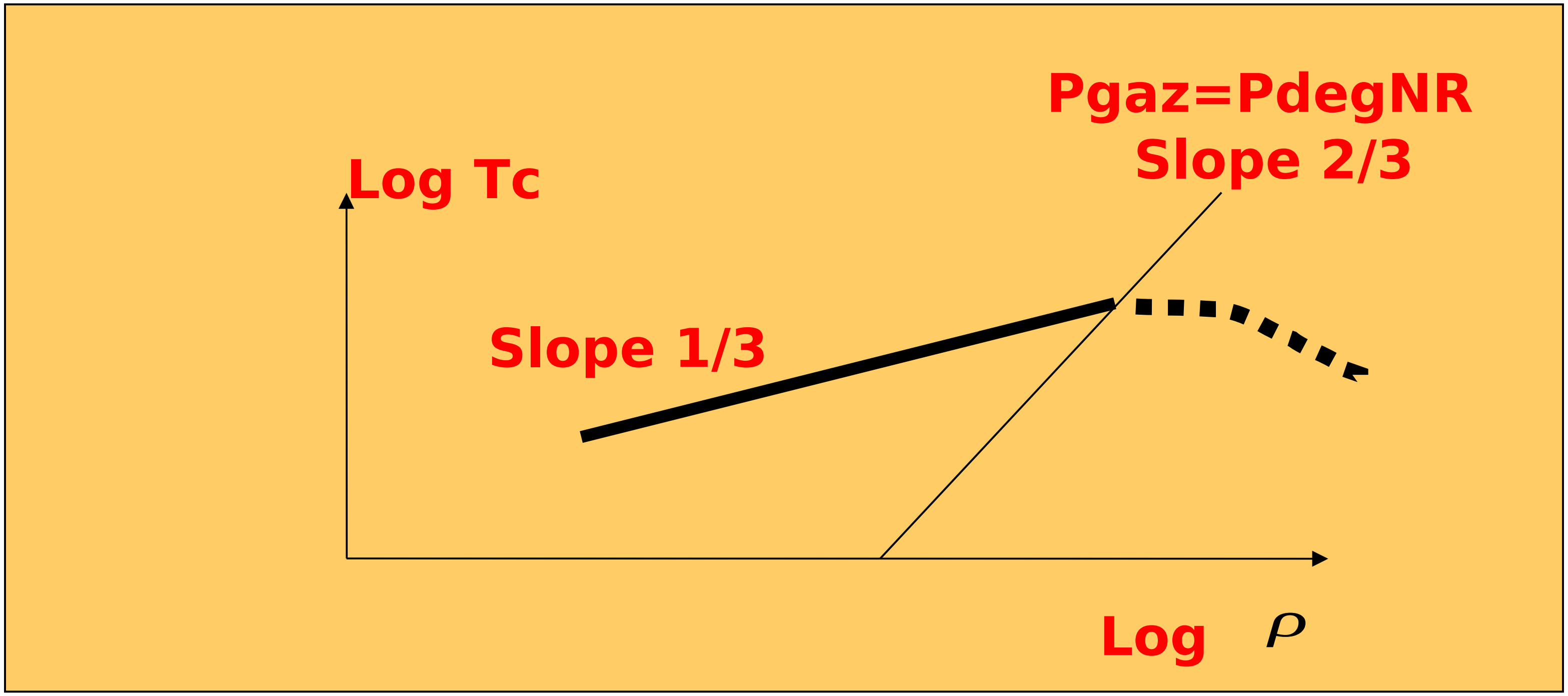
$$P \propto \rho^{5/3}$$

$$\alpha = 3/5 \text{ and } \delta = 0$$

Non → Degenerate Conditions



Evolution of the temperature and density at the centre



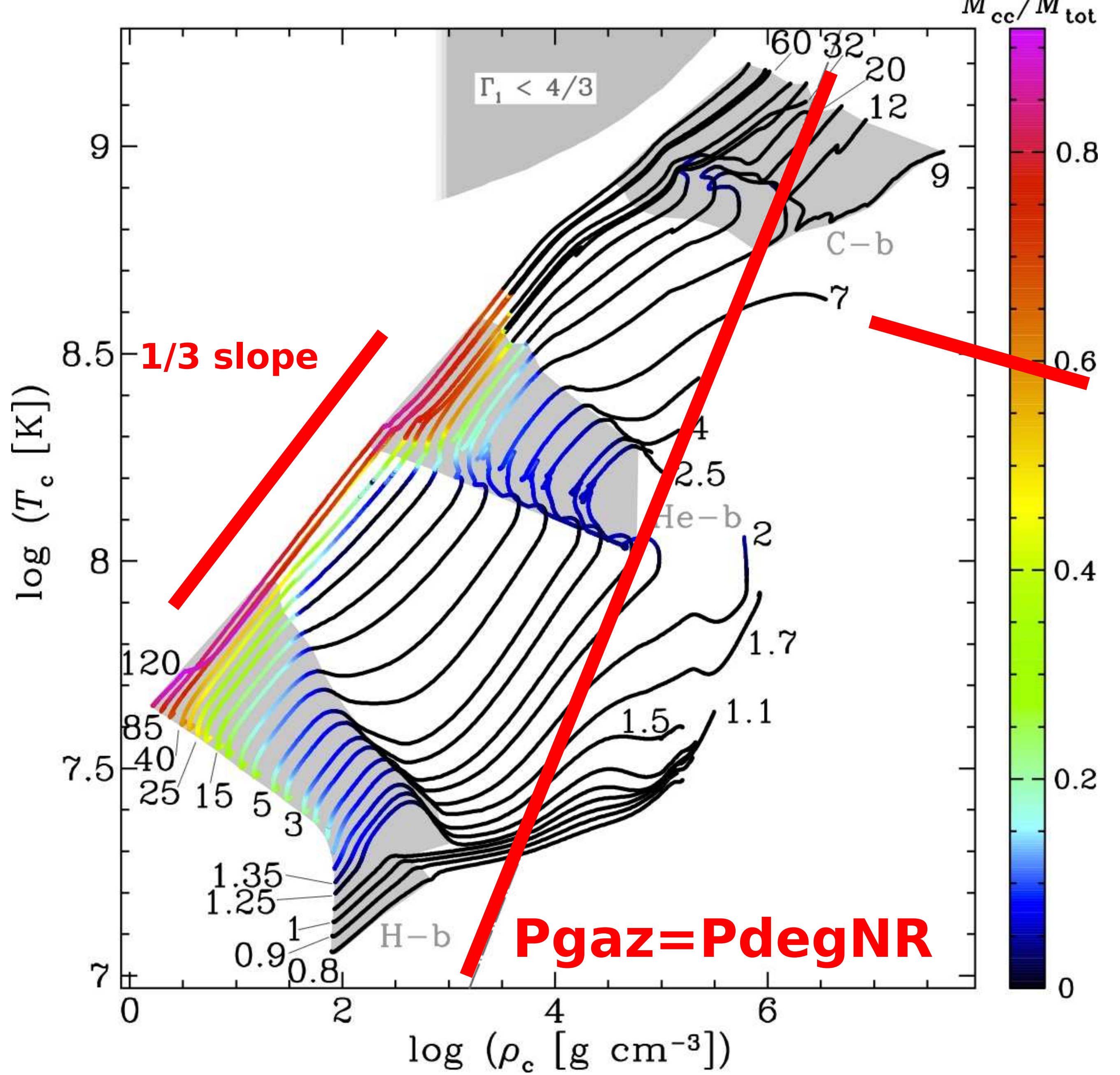
Pgaz=PdegNR

$$\frac{k}{\mu m_H}\rho T = K_1 \left(\frac{\rho}{\mu e}\right)^{5/3} \quad \rightarrow T = K_1 \frac{\mu m_H}{k}\frac{1}{\mu_e^{5/3}}\rho^{2/3}$$

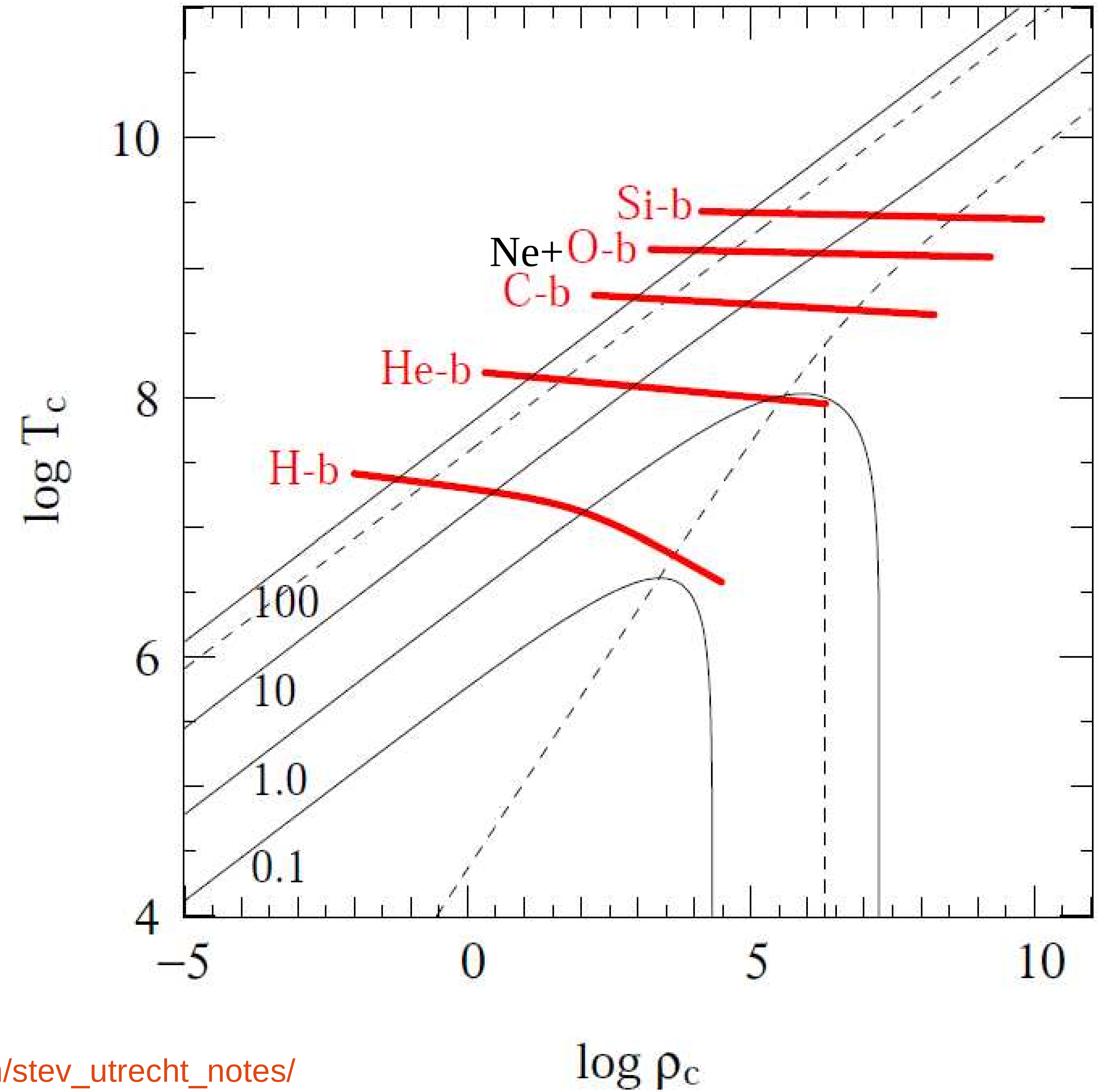
Non → Degenerate Conditions

Models by

Ekström et al. (2012)
A&A, 537, A146)



Mass Domains



Lecture notes from O. Pols taken from:
http://www.astro.ru.nl/~onnop/education/stev_utrecht_notes/

Mass Domains

- 0.08 M_{sun} inferior mass limit for core H-burning : Brown Dwarfs
- 0.08 M_{sun} – 0.5M_{sun}: H burning OK, degenerate before core He-burning (lifetime > Hubble time→ no He white dwarf from single stars)
- 0.5-7M_{sun}: core H OK, core He OK (He-flash below 1.8 M_{sun}), degenerate CO white dwarf
- 7-9 M_{sun}: Core C burning OK→ WD(?) or Complete destruction (?) or collapse through electron captures (?)

- 9 - 150 M_{sun}: core H, He, C, Ne, O, Si-→ Fe cores
- 150-250 M_{sun}: Pair Creation Supernovae

Mass Domains

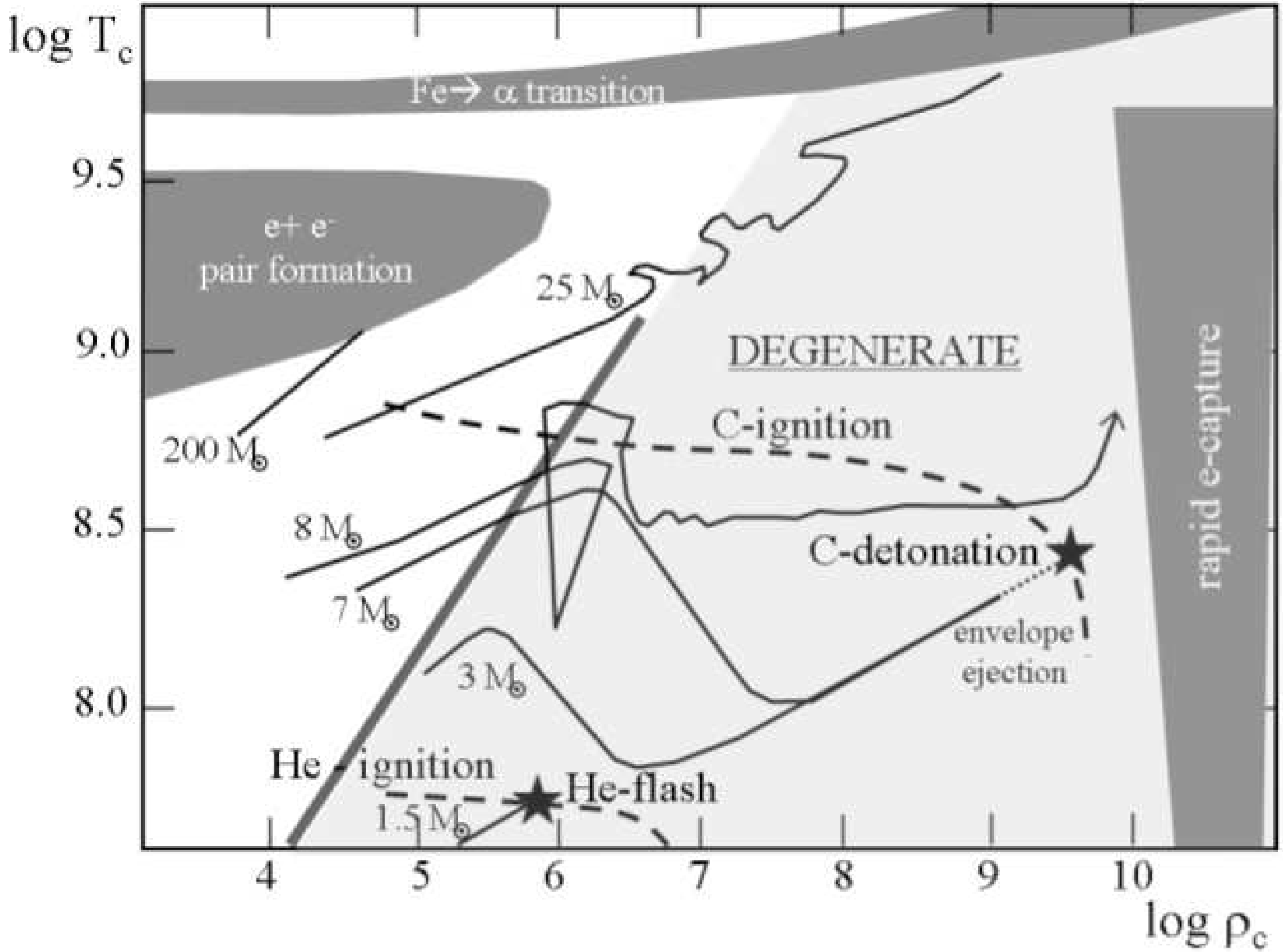


Fig. 26.10. Evolution of central conditions for different masses with indications of instability domains (Sect. 7.8), the Fe–α transition indicates the photodesintegration of Fe nuclei into α particles. The degenerate region is light gray. Dashed lines show the place where nuclear energy generation rates balance neutrino losses. Adapted from T.J. Mazurek and J.C. Wheeler [401]

Massive stars: M > 9 solar masses

Main sequence:

hydrogen burning

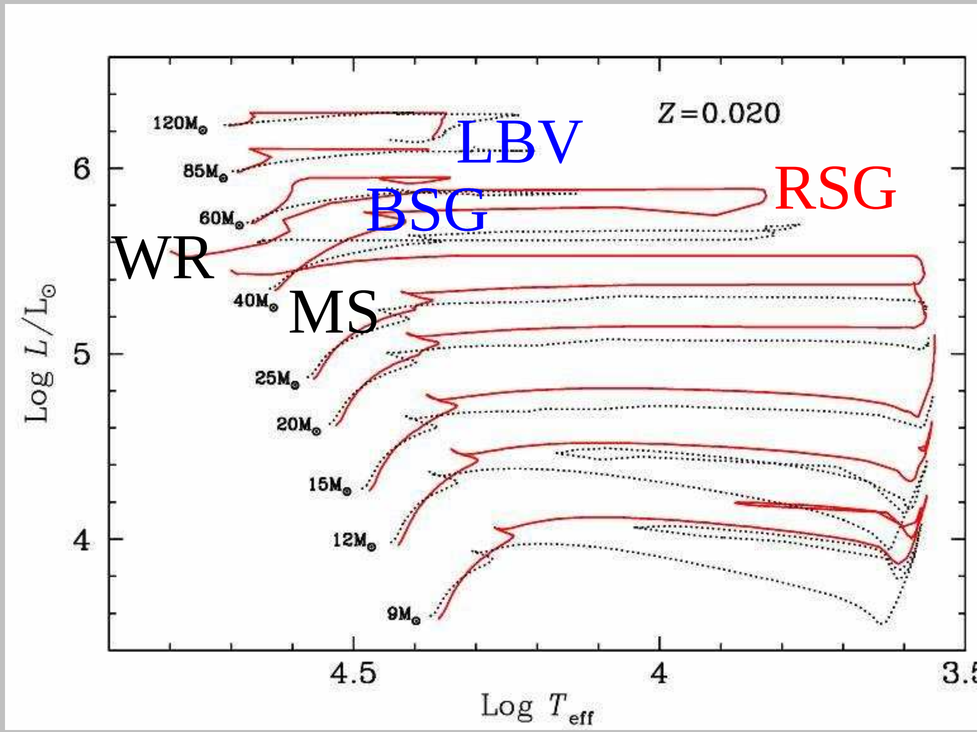
After Main Sequence:

Helium burning

Supergiant stage (red or blue)

Wolf-Rayet (WR): M > 20-25 M_o

WR without RSG: M > 40 M_o



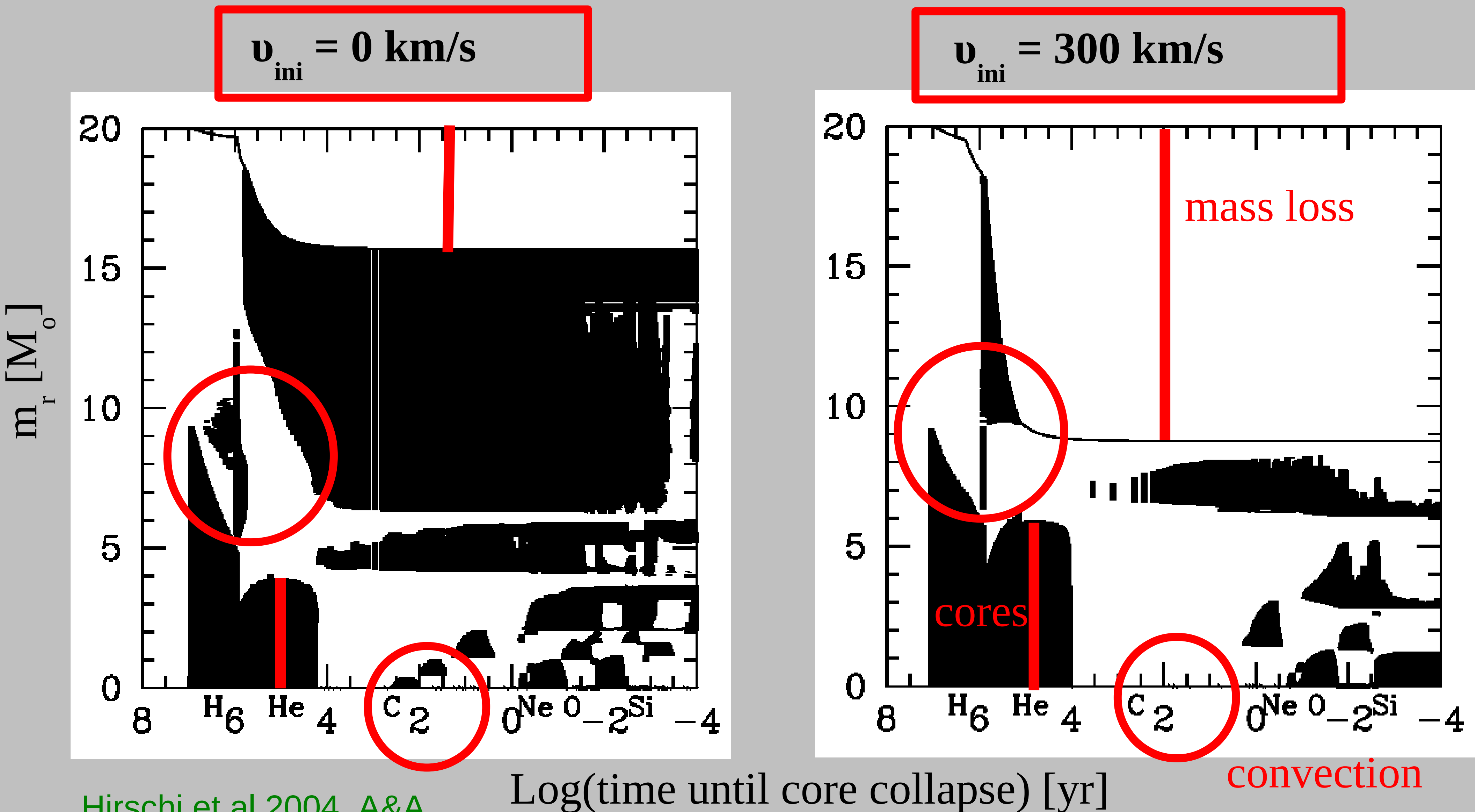
http://www.astro.keele.ac.uk/~hirschi/animation/anim.html

Advanced stages:

carbon, neon, oxygen, silicon burning → iron core

Core collapse → bounce → supernova explosion

Impact of Rotation, $\mathcal{M}_{ini} = 20\ \mathcal{M}_\odot$

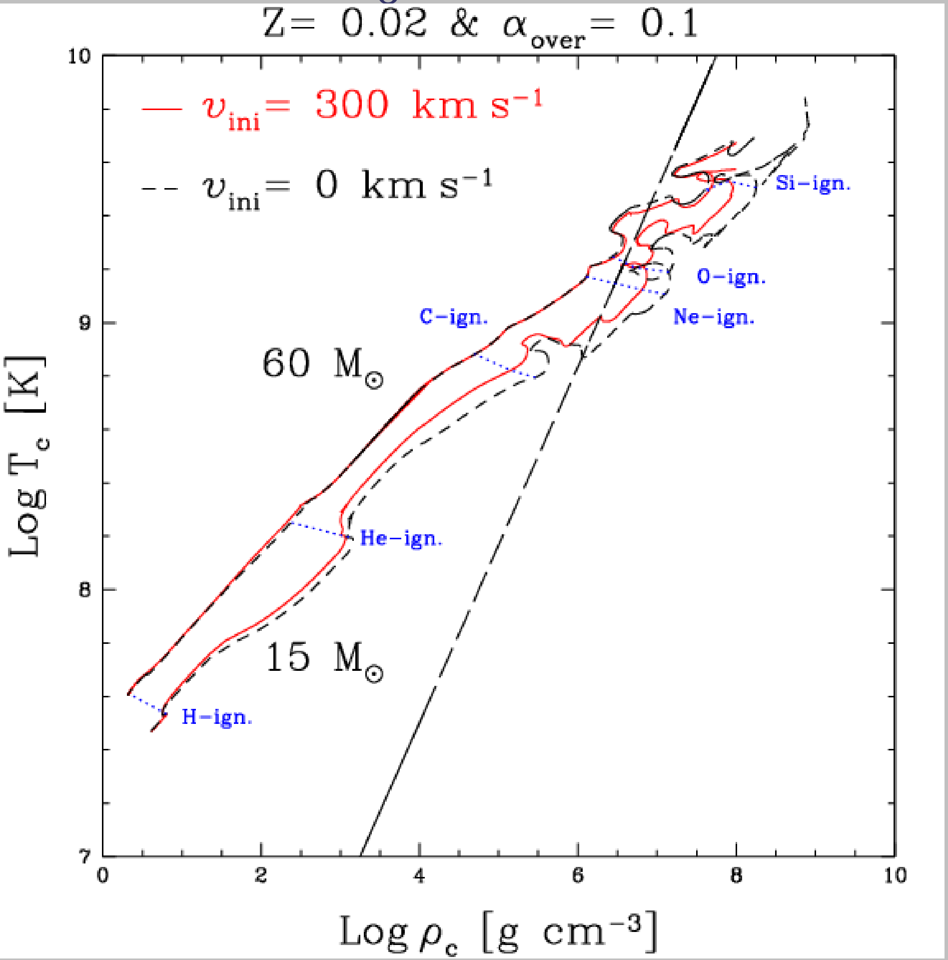


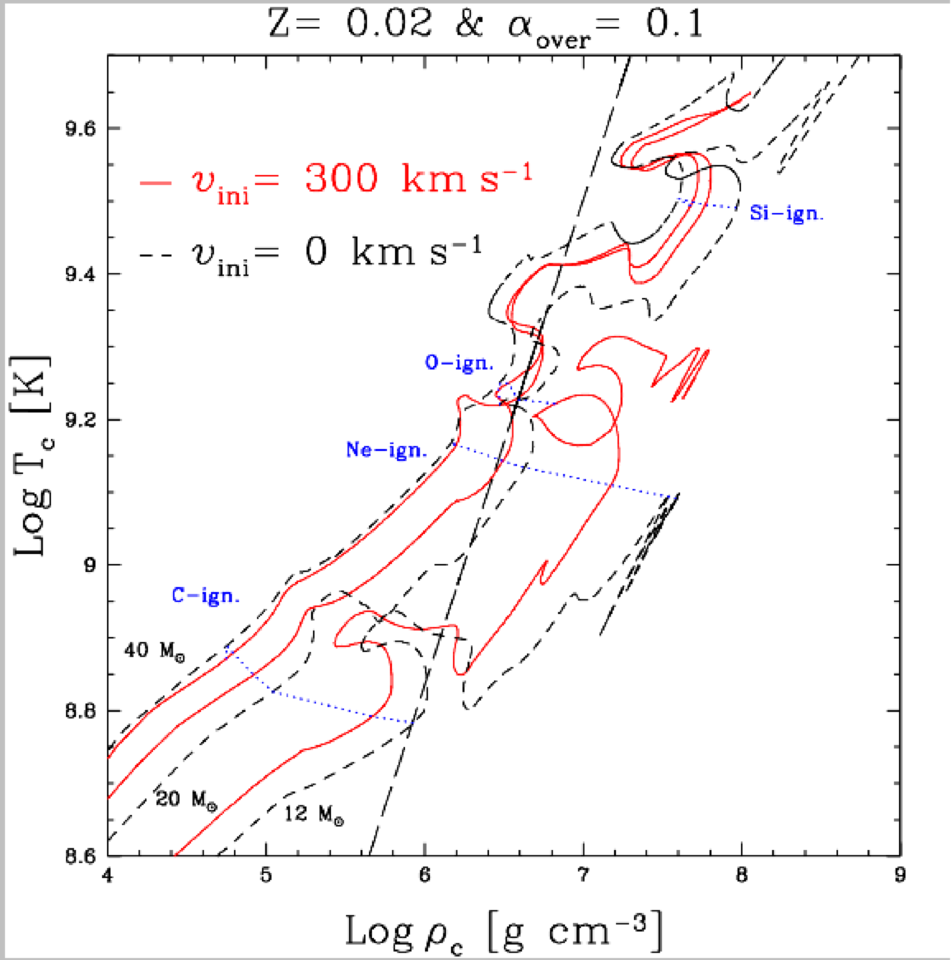
Hirschi et al 2004, A&A

Massive Stars

M<~30 M_o : Rotational mixing dominates → bigger cores

M>~30 M_o : mass loss dominates → ~ or smaller cores





How massive can stars be?

Do very massive stars (VMS: $M>100M_o$) exist?

Very Massive Stars in the Local Universe, 2014, Springer, Ed. Jorick S. Vink

- Star formation: already difficulties with 30 M_o stars but 2/3D simulations are promising (Kuiper et al 11, Krumholz 2014)

- Stellar evolution: possible up to ~ 1,000 M_o (BUT mass loss/rad.) (Baraffe et al 01)

Can we see them?

- Rare and short-lived

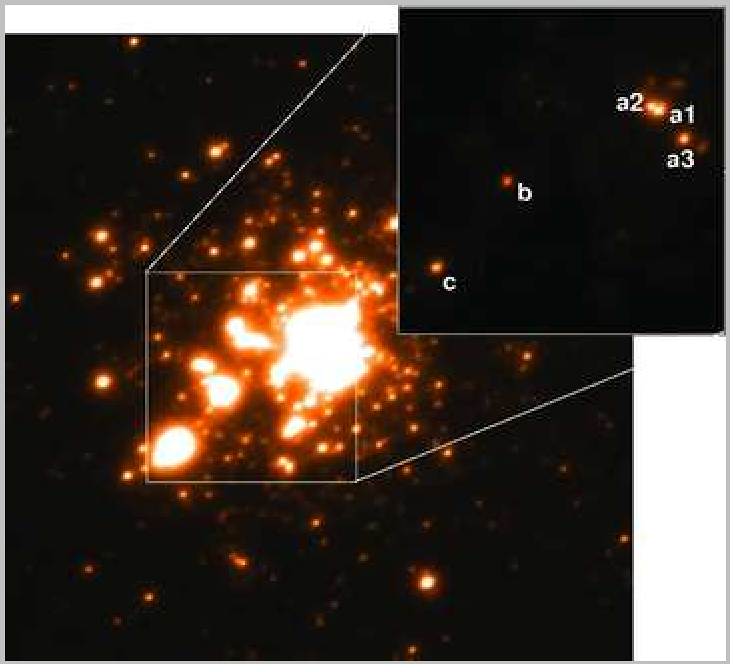
- Need to look at youngest and most massive clusters:

    - Arches: M<~150 Mo

      (Figer 05, Martins et al 08)

    - NGC 3603 & R136: new **M_{max}=320M_o!**

      (Crowther et al 10, MNRAS)



R136 cluster

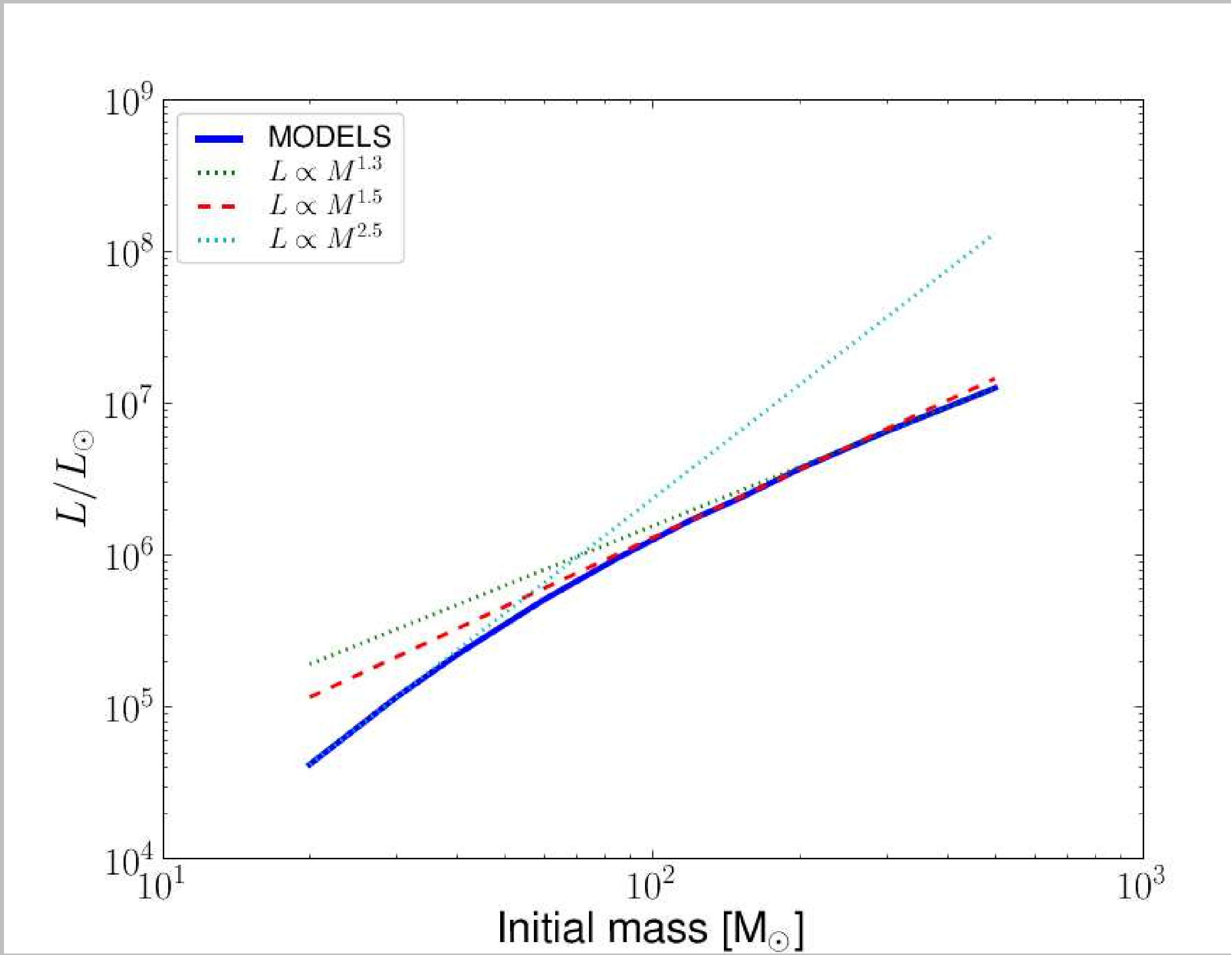
Very luminous stars (~10^7 L_o)

R136a1 ($10^7 L_o$) alone supplies 7% of the ionizing flux of the entire 30 Doradus region!

What is the shape of the luminosity vs mass relation in this mass range?

Textbooks: L ~ M^3 for stars in the solar mass range

Above 100 M_o: L~$M^{1-1.5}$

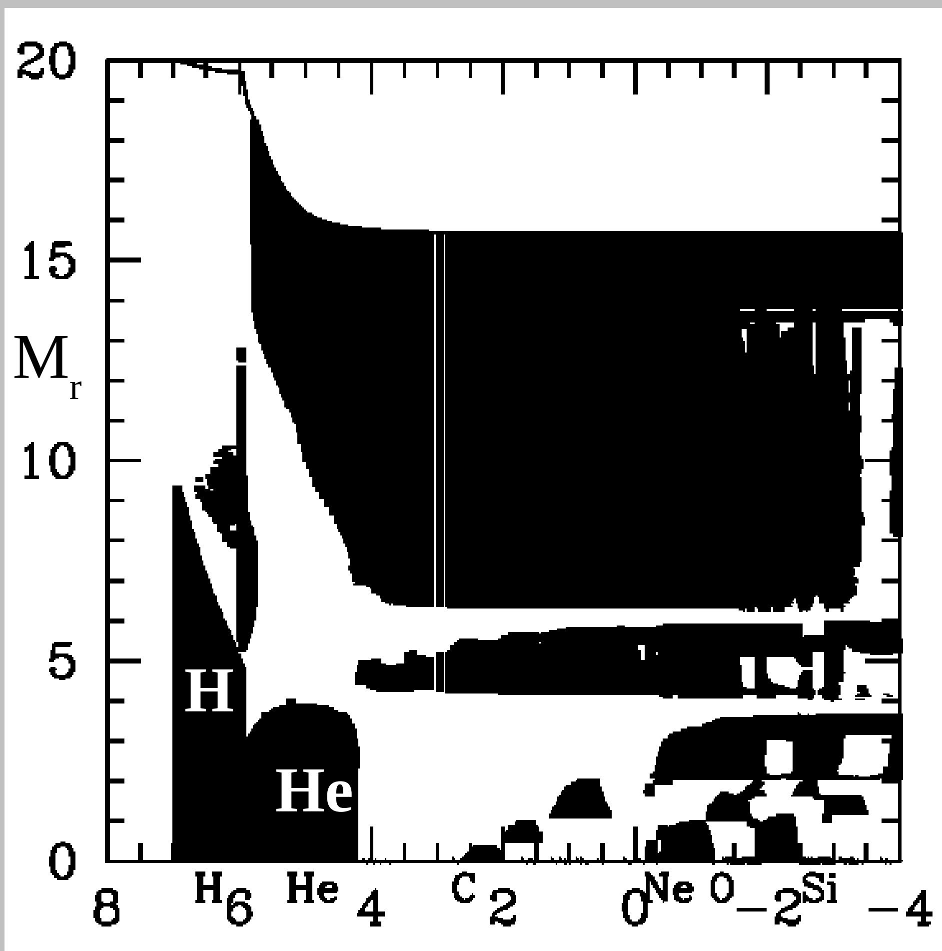


(Yusof et al 2013)

The Evolution of VMS

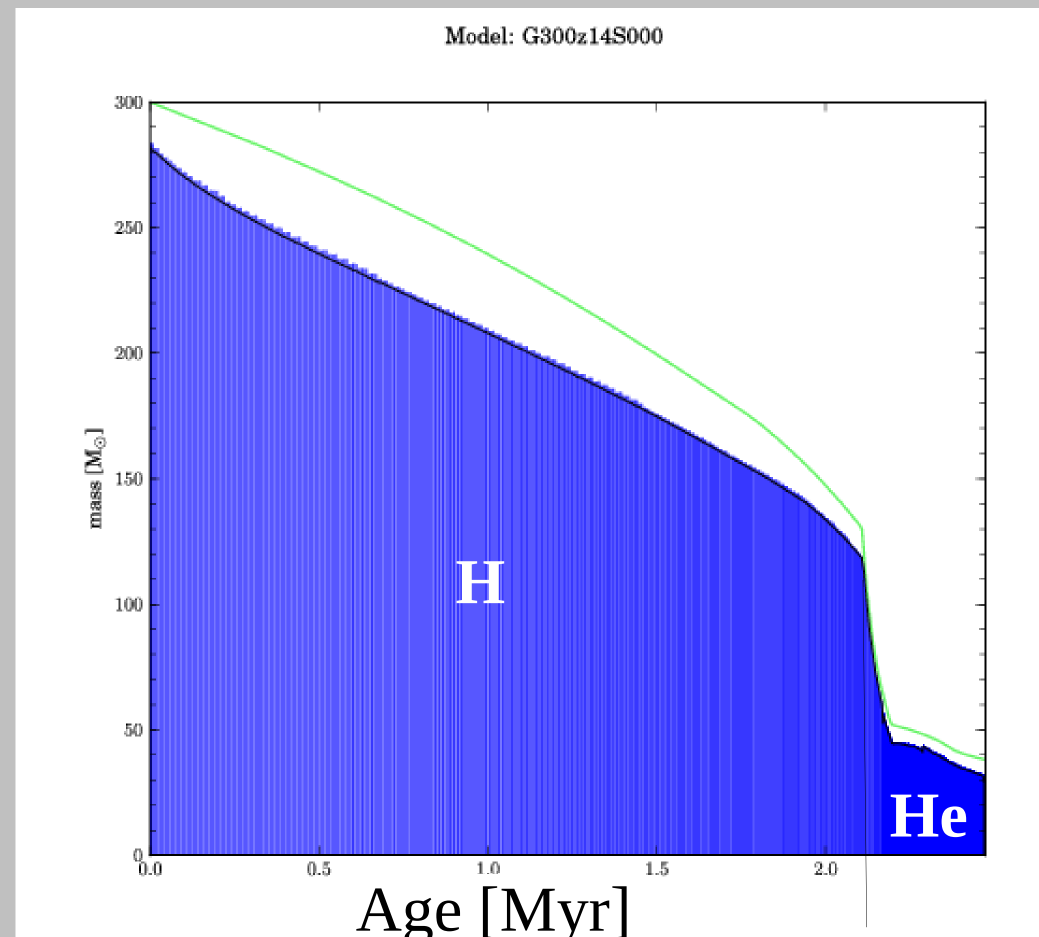
VMS = Very Massive Stars for M > 100 M_o

20 M_o



Log10(Time left until collapse)

300 M_o



(Yusof et al 13 MNRAS, aph1305.2099)

VMS: much larger convective core & mass loss!

The fate of VMS: PCSN/BH/CCSN?

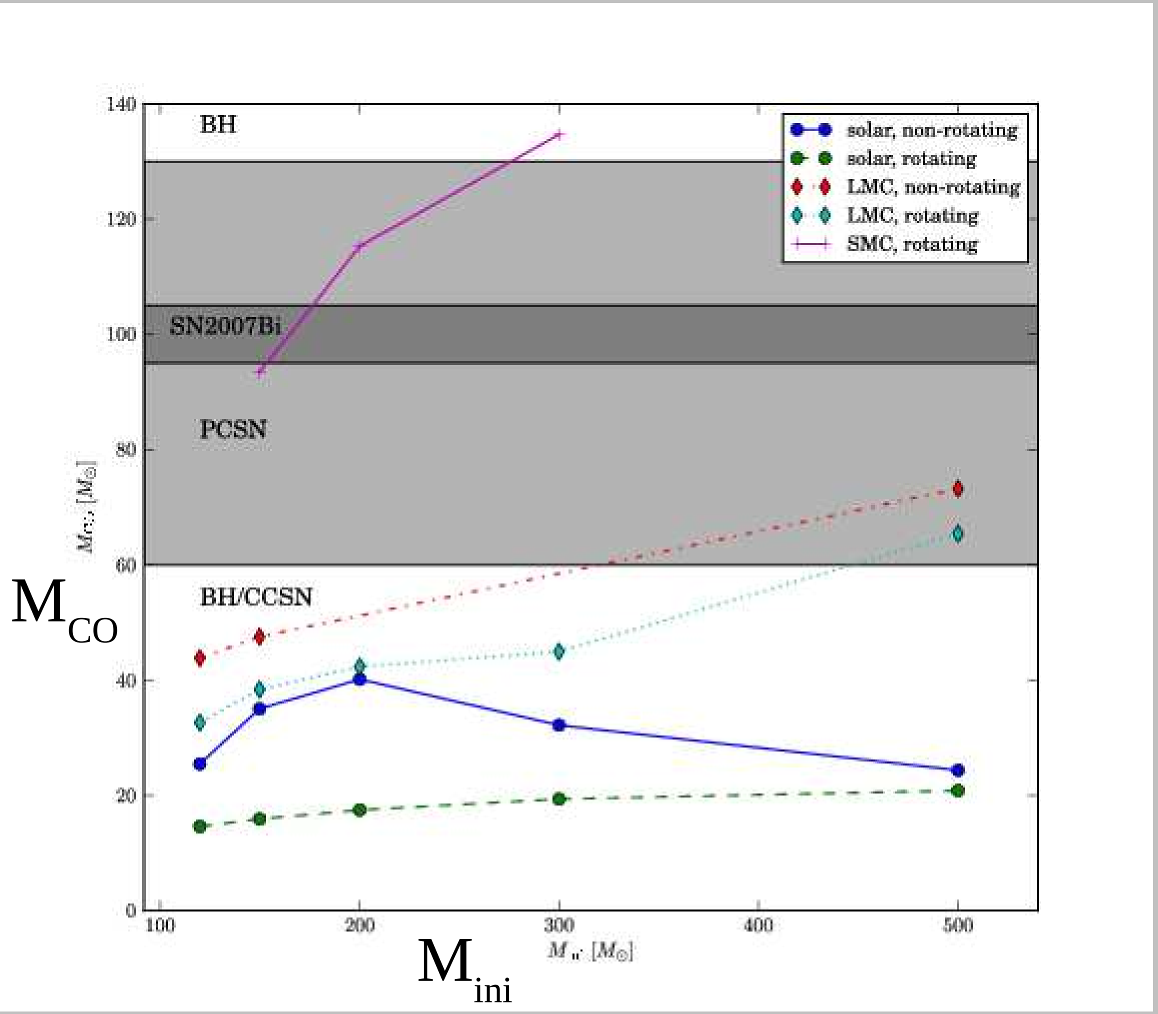
(Yusof et al 13 MNRAS, aph1305.2099)

Z_{solar}: no PCSN

(Rotating) models with Z<Z(LMC) lose less mass,

and enter the PCSN instability region!

BUT mass loss uncertain!



PCSN range from Heger & Woosley (2002)

Consistent with Langer et al (2007): PCSN for $Z<Z_{\odot}/3$

S Process in Massive Stars

Weak s process: (slow neutron capture process) during core He- and shell C-burning

Kaeppeler, et al, 2011, RvMP, 83, 157

He: T > 0.25 GK

(~ 21.6keV)

C: T ~ 1GK

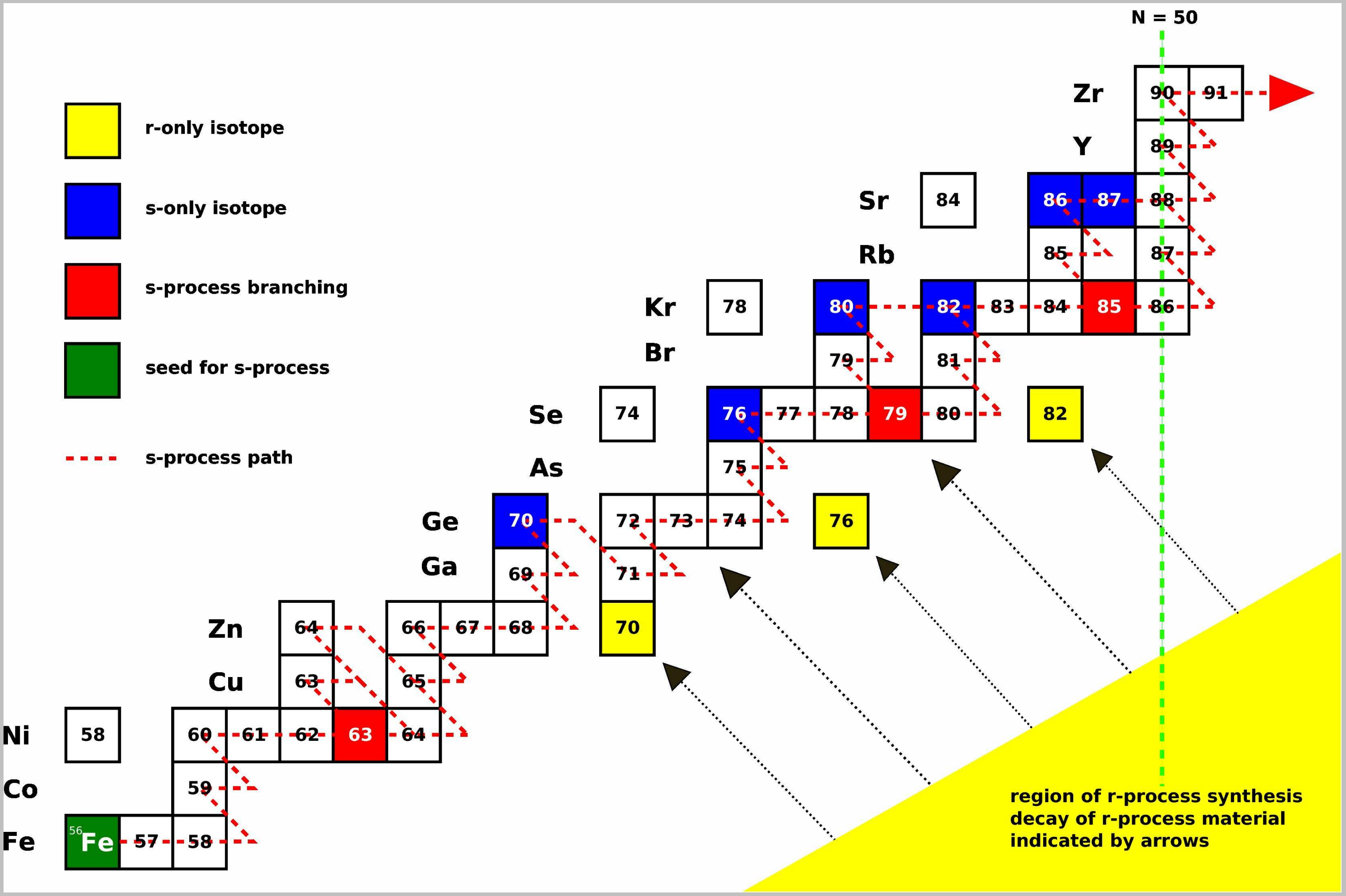
N-source: ^{22}Ne(a,n)

Seed: iron

Poisons:

- He-b.: ^{22}Ne, ^{25}Mg, ^{16}O, ^{12}C

- C-b.: ^{24}Mg, ^{25}Mg, ^{16}O, ^{20}Ne



At solar Z: rotating models may produce up to 3x more s process

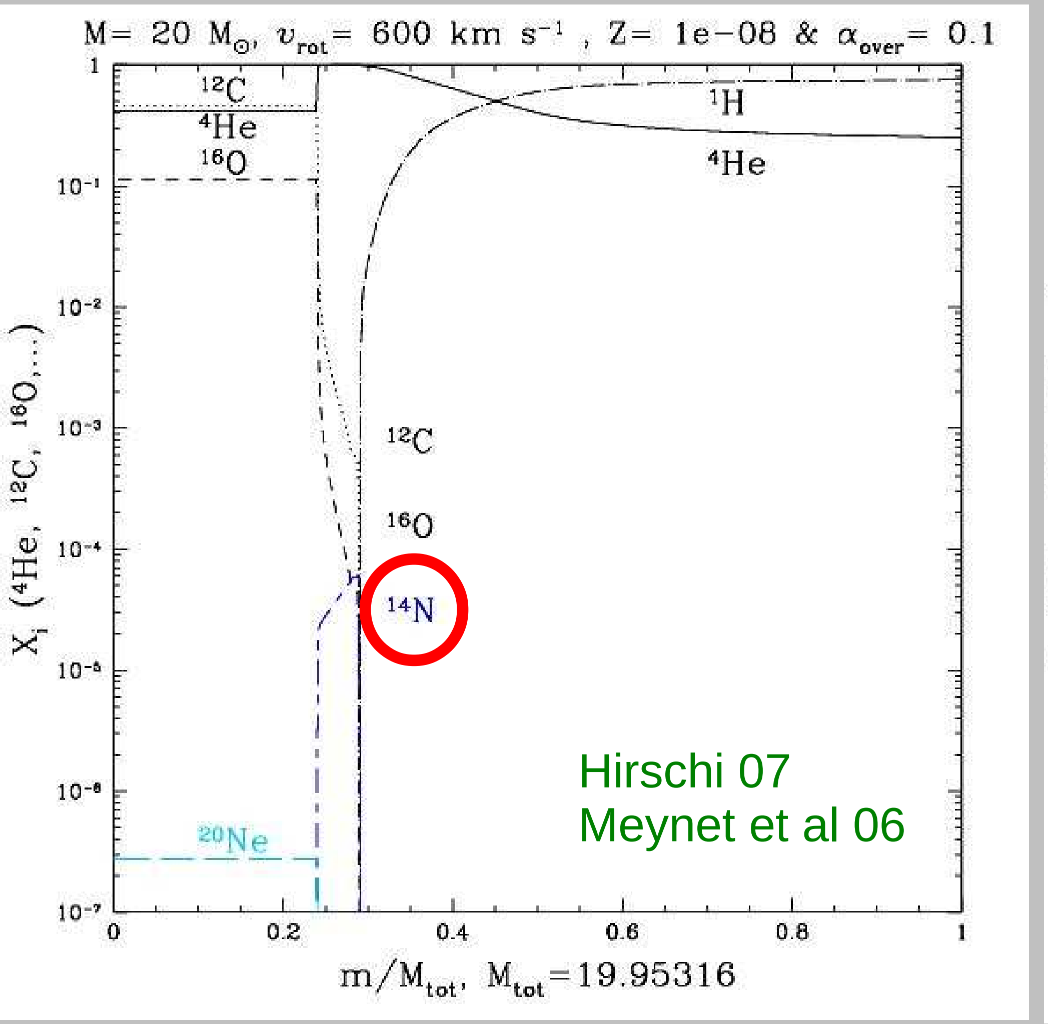
(See also Chieffi, Limongi, 2012ApJS..199...38L)

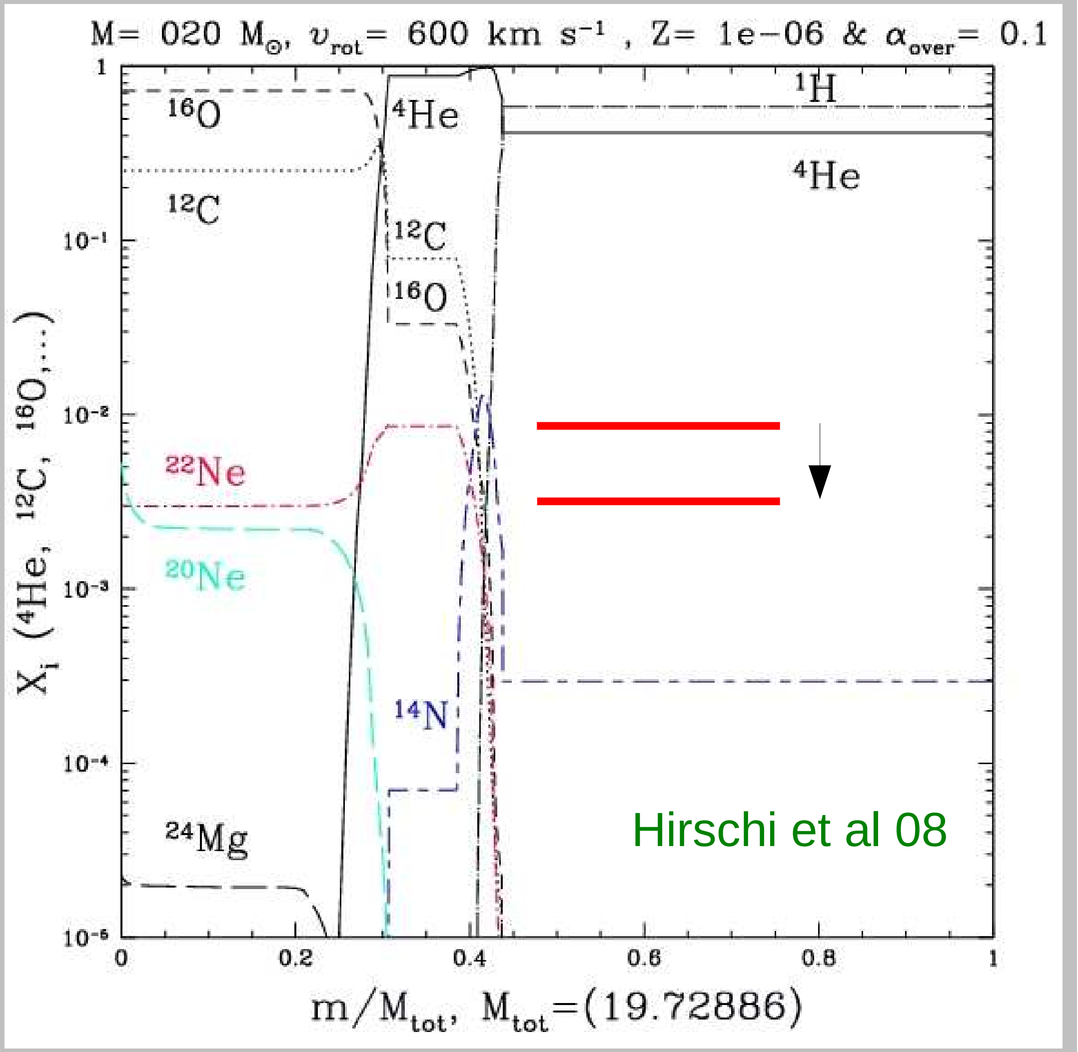
How much s process do massive rotating stars produce at low Z?

Rotation induced mixing @ low Z

Before H-shell boost

Xi @ end of He burning

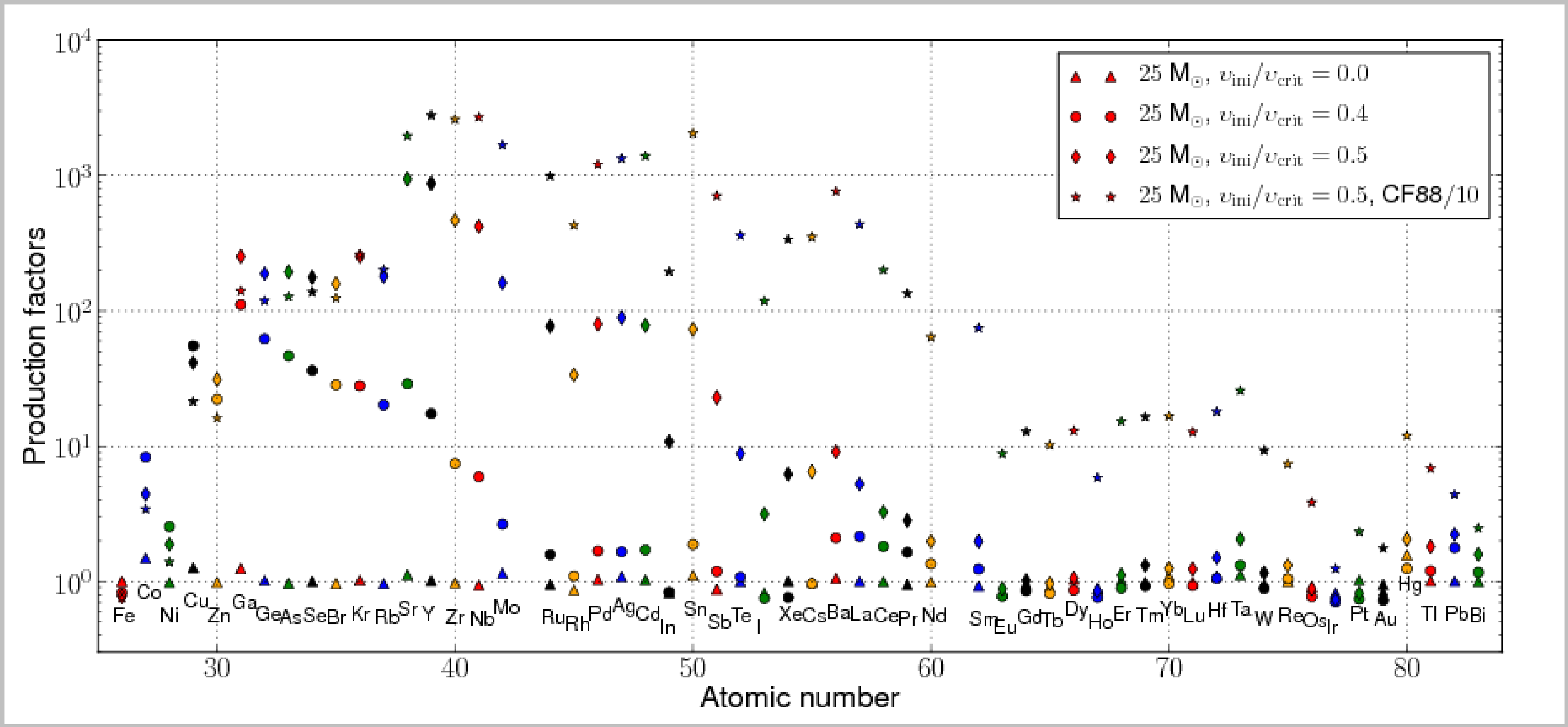




--> s process ???

New S-Process Models of Massive Rotating Stars

$Z=10^{-5}$, rotating models with different $^{17}O(a,g)$ rates; V_{ini}



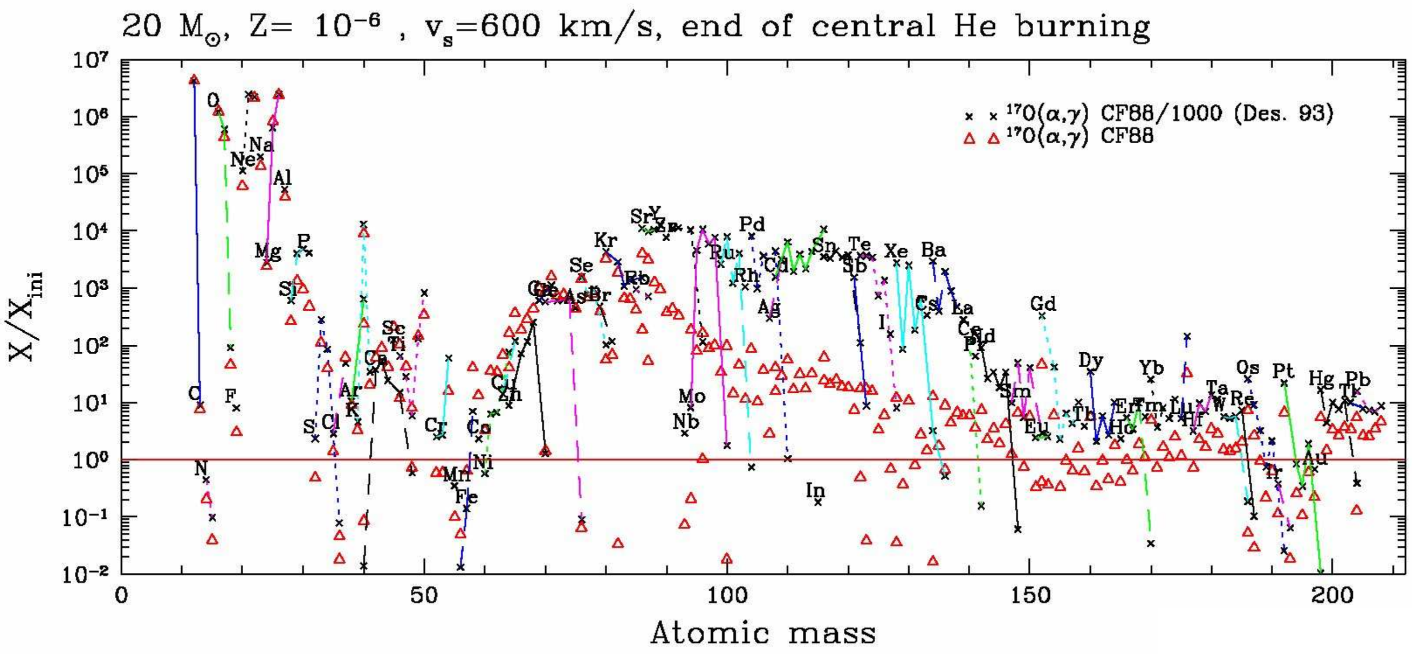
Frischknecht et al, A&A letter 2011, 2014 in prep

- STELLAR EVOLUTION CALCULATIONS WITH 600/700-ISOTOPE NETWORK!

- **^{22}Ne production almost primary but still varies with Z & especially V_{ini}.** M_{ini}
- Secondary seeds (Fe) limit production (^{22}Ne cannot act as seed)
- Strong variations in [Sr,Y/Ba] up to 2 dex dep. on Z,V_{ini}, and $^{17}O(a,g)$

- Possibility of explosive n-capture process in He-shell

S Process in Massive Stars: Nuclear Physics Uncertainty



Hirschi et al 2008, NICX

Pignatari et al 08, ApJ letter, 687,95

$^{16}O(n,\gamma)^{17}O$:

- ^{16}O poison if $^{17}O(\alpha,\gamma)^{21}Ne$ dom.
- ^{16}O absorber if $^{17}O(\alpha,n)^{20}Ne$ dom.

Measurement of $^{17}O(a,g)^{21}Ne$ at TRIUMF

Taggart et al NICXI:

$^{17}O(a,g)$ lower than CF88!

Best et al 2011 (@ Notre Dame):

But much higher than Descouvemont 1993!

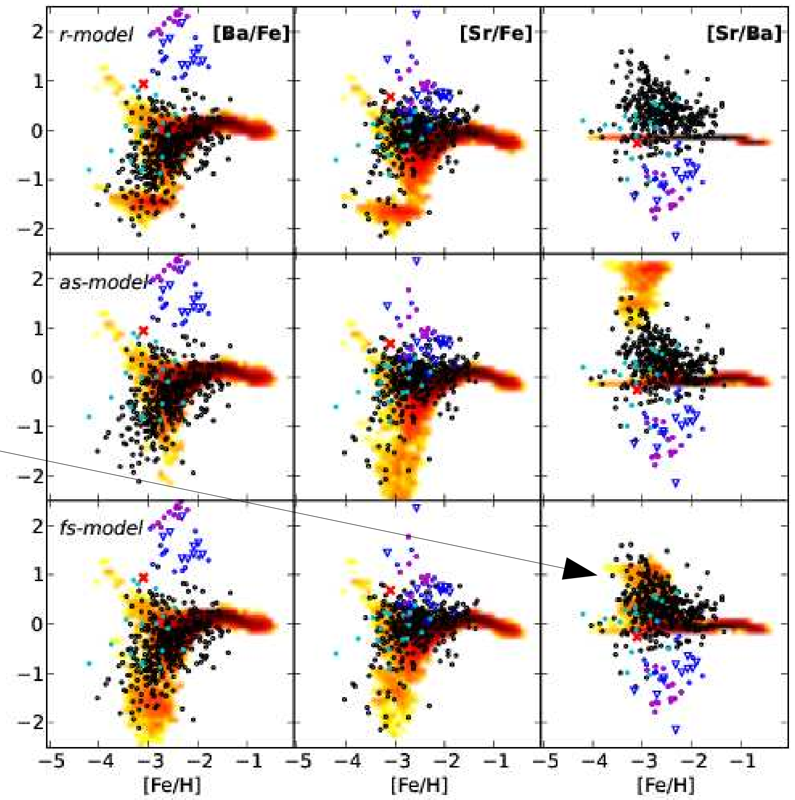
New S-Process Models Compared to EMP * & Bulge GC

* New models also explain abundances in one of the oldest clusters in galactic bulge Chiappini et al, Nature Letter, 2011

Inhomogeneous GCE models by
Cescutti et al 2013 A&A,553,51

- Strong variations in [Sr/Ba] > 1 dex matches well observed range for EMP stars (black circles)!

(no main s process included so cannot explain CEMP-s stars in blue)

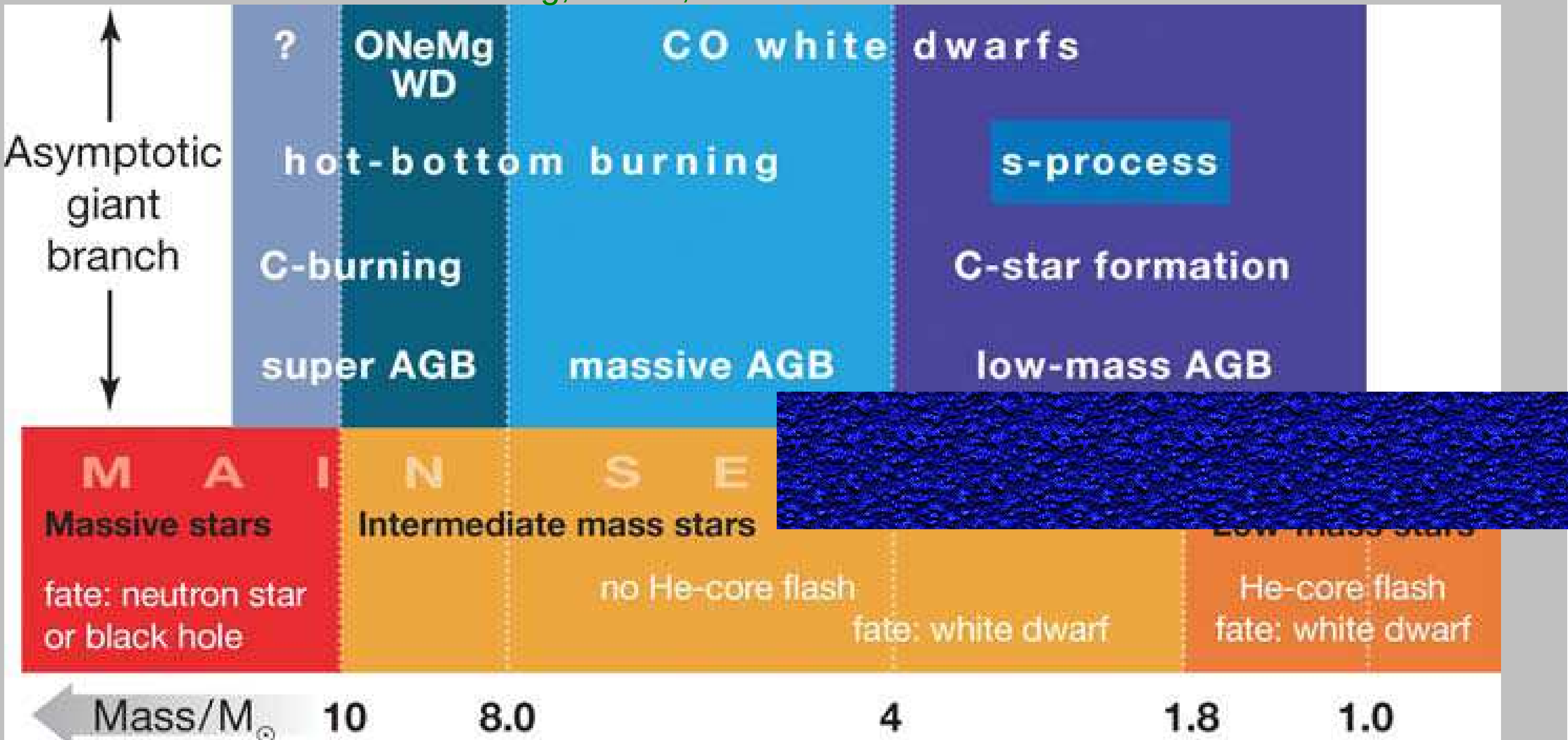


Model name	panels in Fig. 5	s-process	r-process
r-	Upper	No s-process from massive stars	standard + extended r-process site (8 - 30 $M_\odot$)
as-	middle	average rotators ($v_{ini}/v_{critic} = 0.4$)	standard r-process site (8 - 10 $M_\odot$)
fs-	lower	fast rotators ($v_{ini}/v_{critic} = 0.5$) and 1/10 for $^{17}O(\alpha,\gamma)$ reaction rate	standard r-process site (8 - 10 $M_\odot$)

(EMP *: Frebel et al 2010)

Intermediate & Low-Mass Stars

Herwig, ARAA, 2005



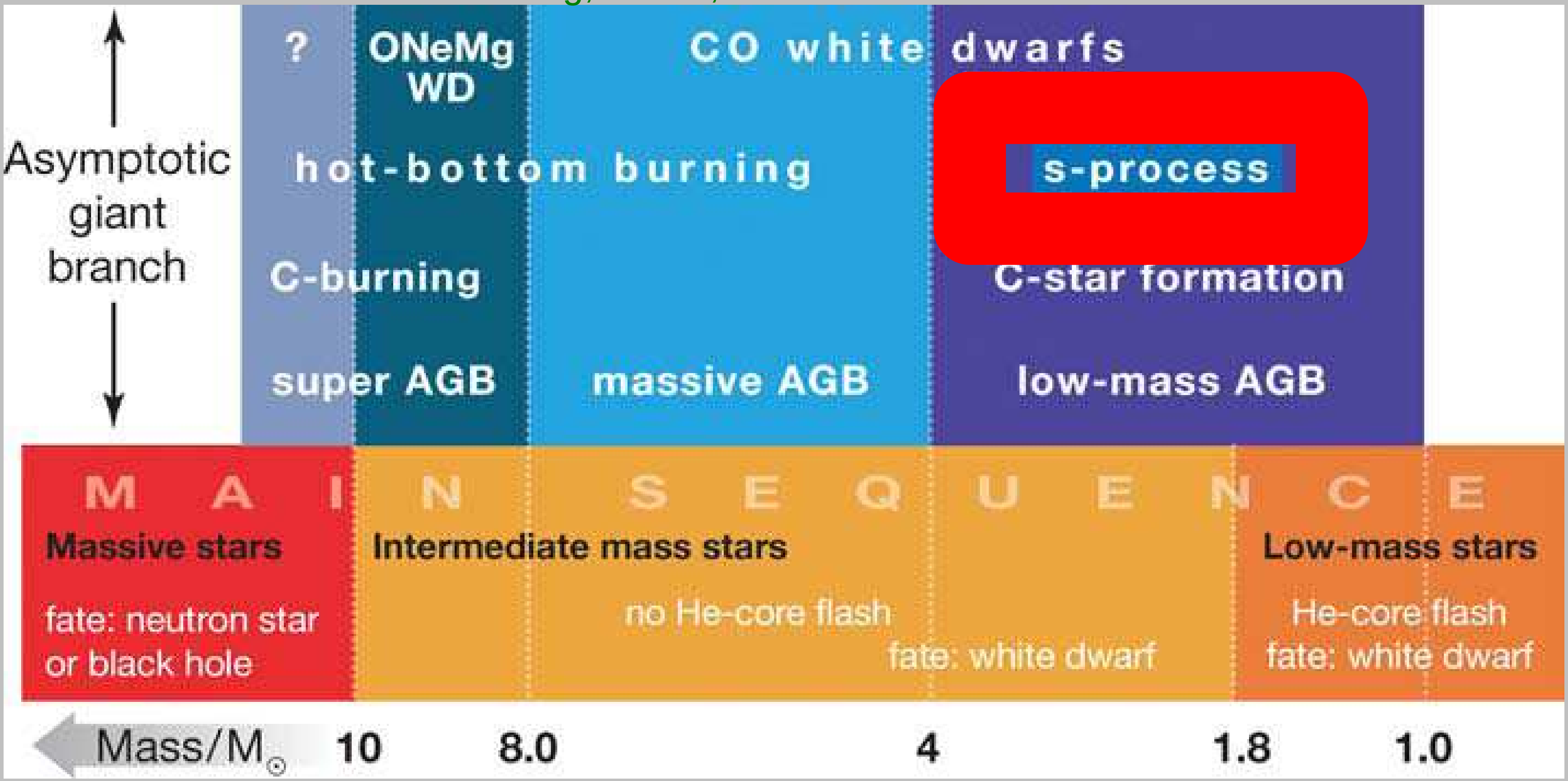
Intermediate-mass stars: 1.8 - 9 M_o do not ignite C-burning in centre

(C-flash for SAGB stars, see later)

Low-mass stars: 0.5-1.8M_o do not ignite He-burning in centre (He-flash)

Intermediate & Low-Mass Stars

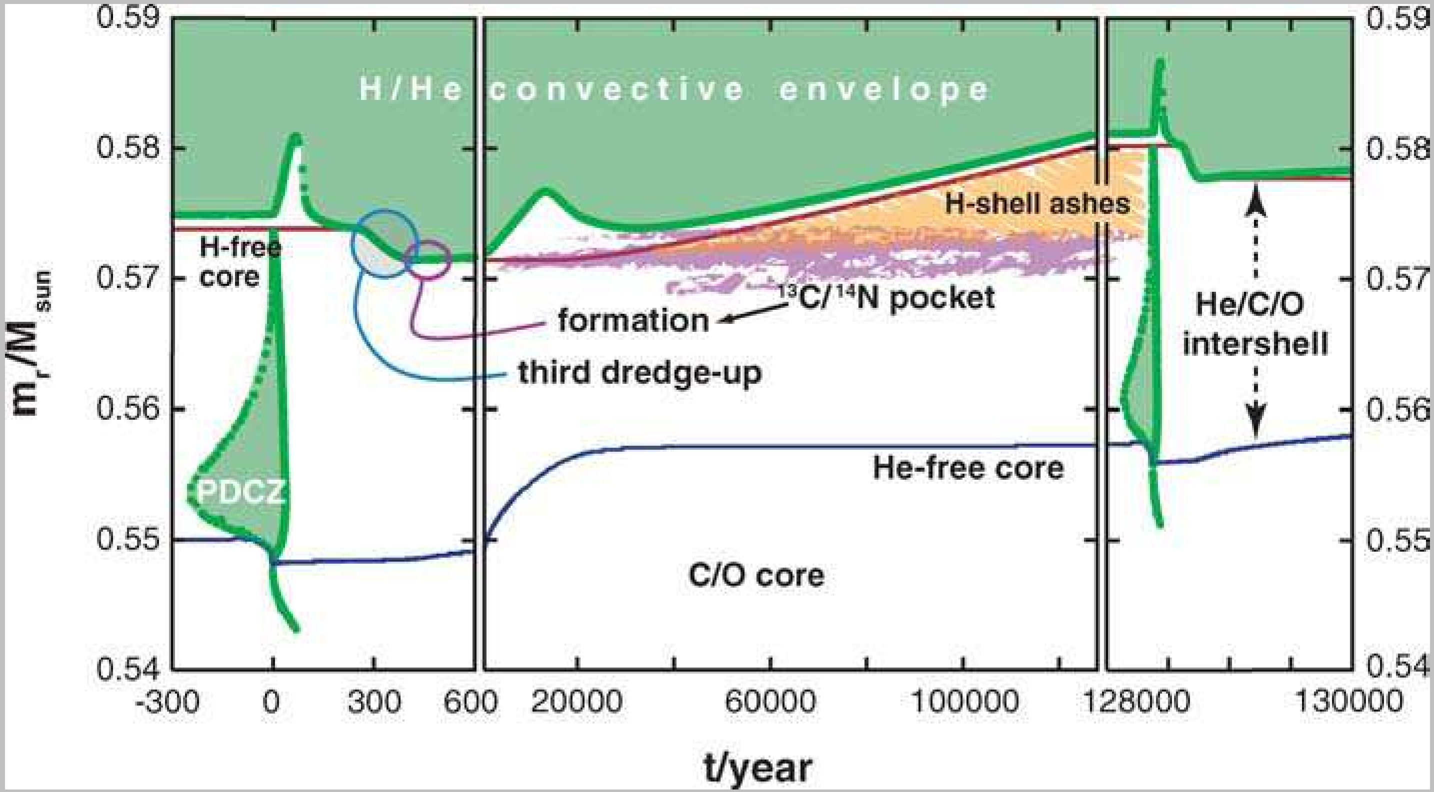
Herwig, ARAA, 2005



AGB phase & s process in both intermediate-mass stars and low-mass stars!

Intermediate & Low-Mass Stars

The plot you usually see at conferences for AGB stars:

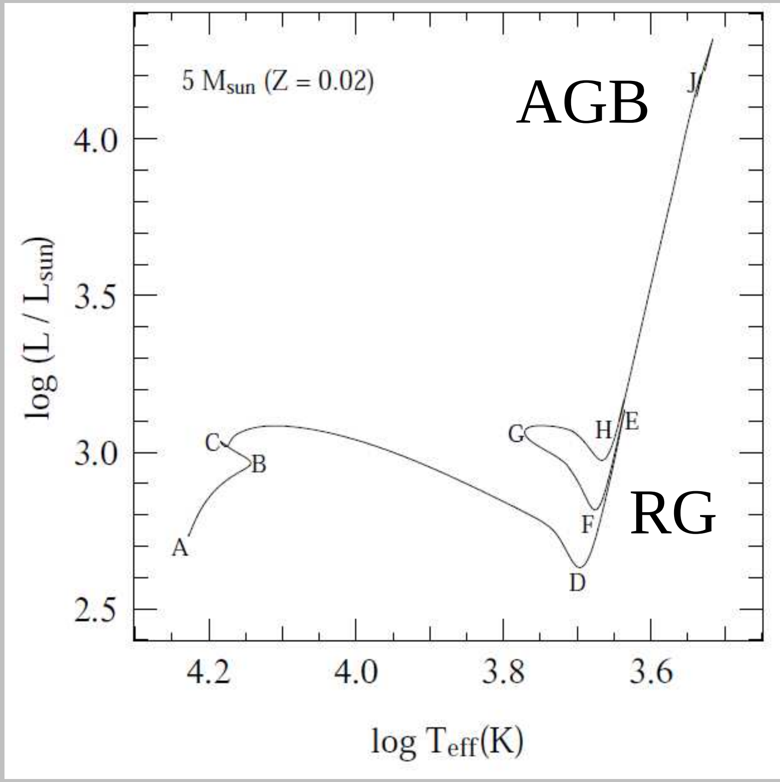


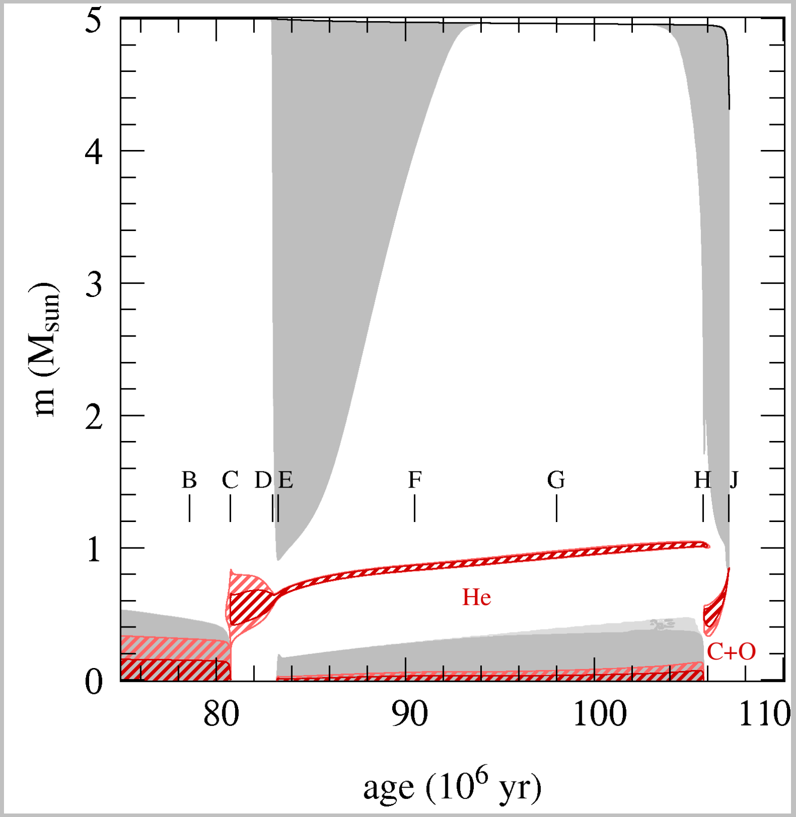
Herwig, ARAA, 2005

Where does it fit in the star's evolution?

Intermediate & Low-Mass Stars

5 M_o star: Evolution through H- and He-burning

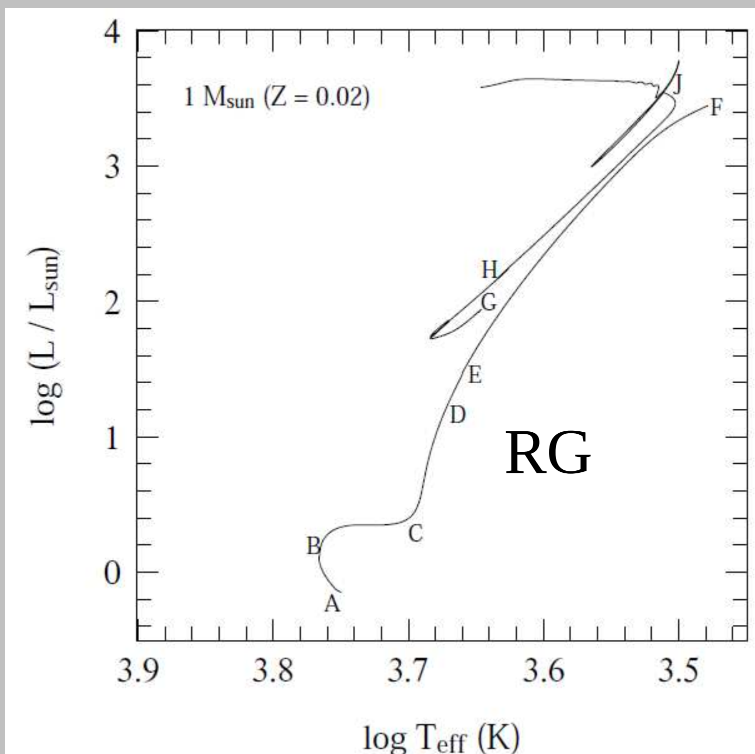


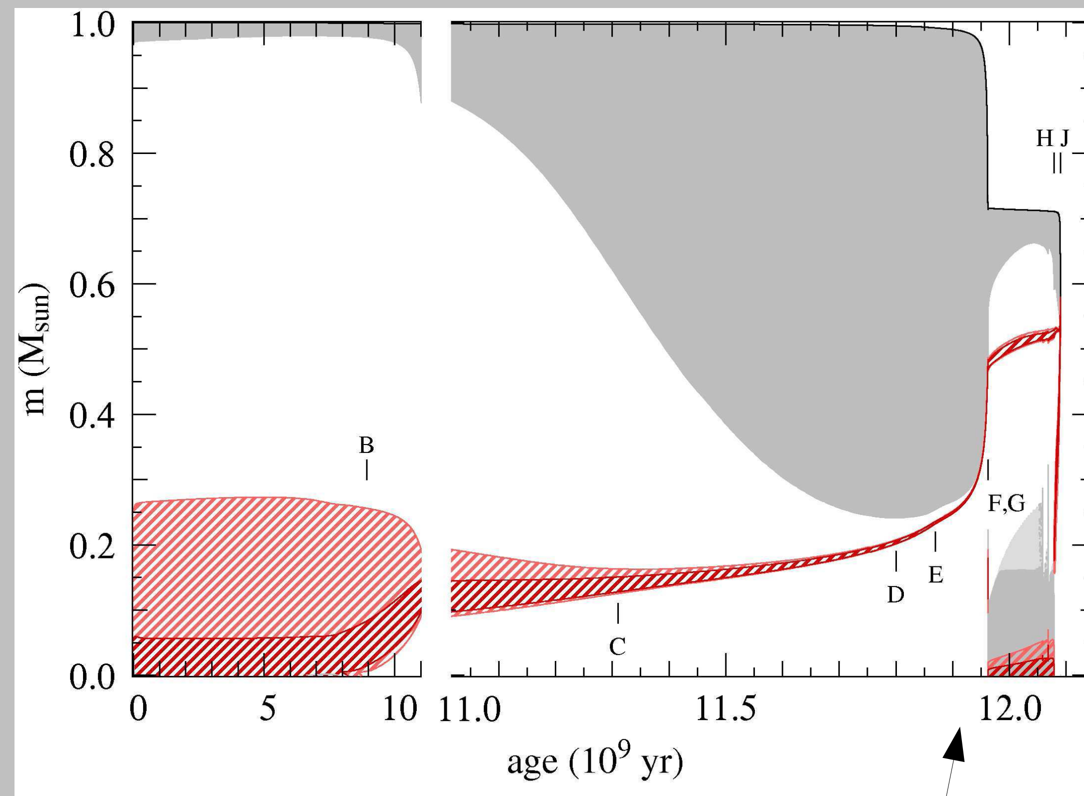


From SE notes, O. Pols (2009)

Intermediate & Low-Mass Stars

1 M_o star: Evolution through H- and He-burning



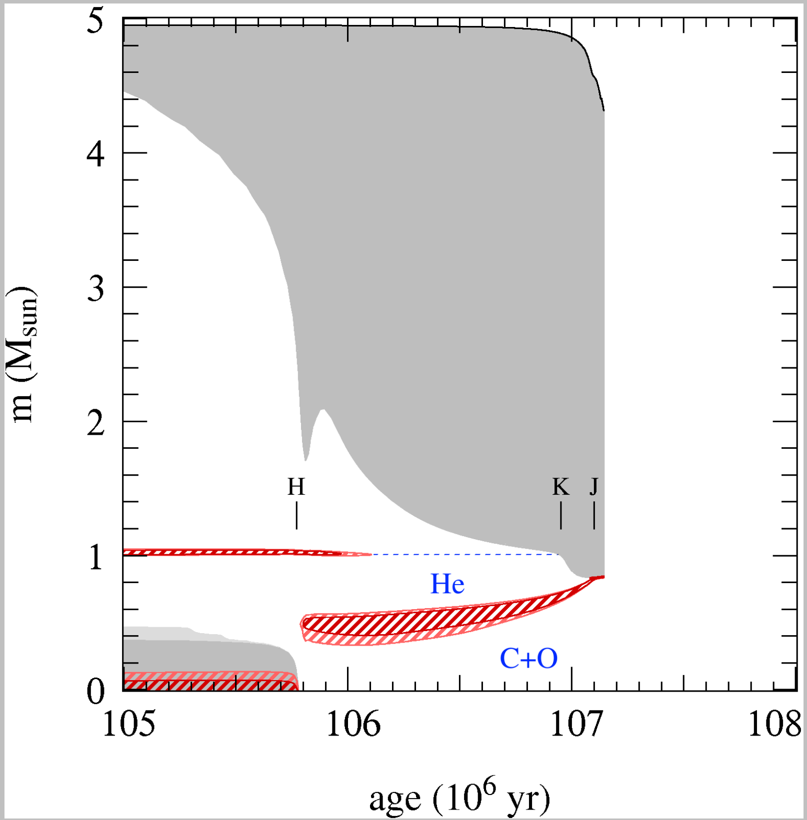


From SE notes, O. Pols (2009)

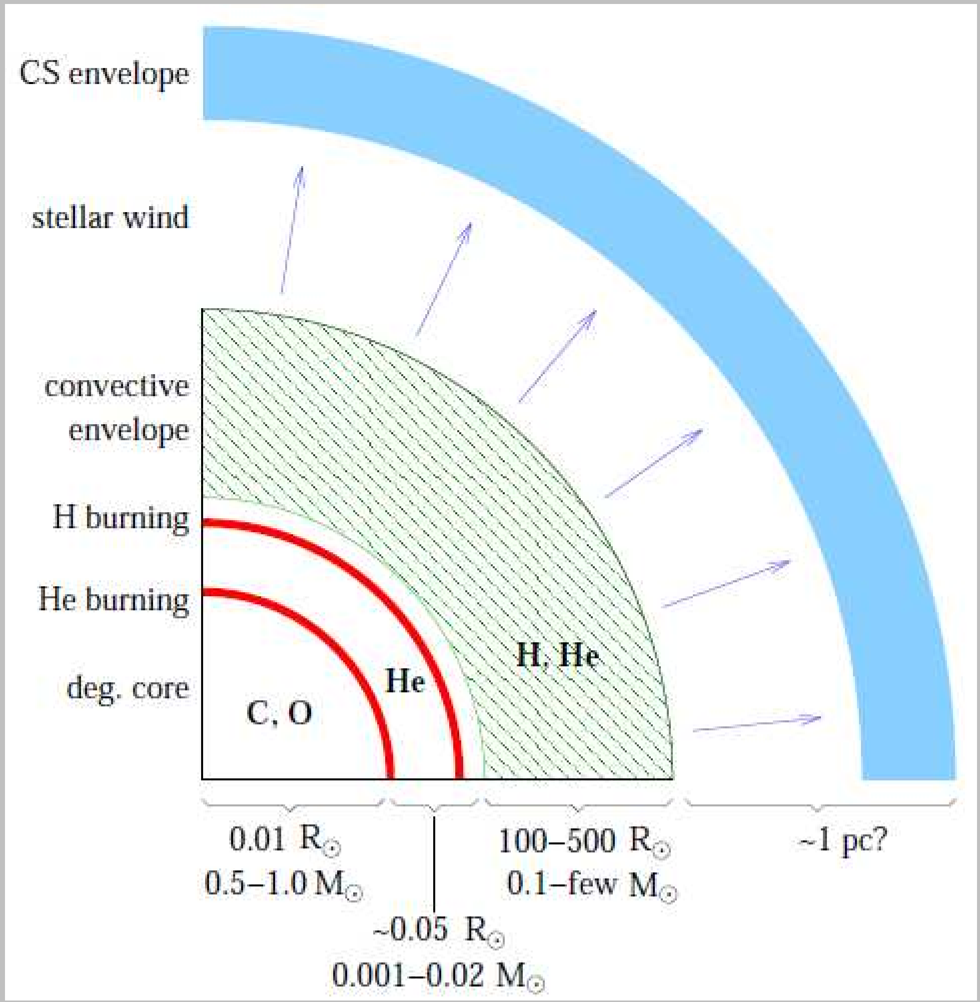
He-flash at point F → G

Intermediate & Low-Mass Stars

5 M_o star: AGB phase



Structure in AGB phase

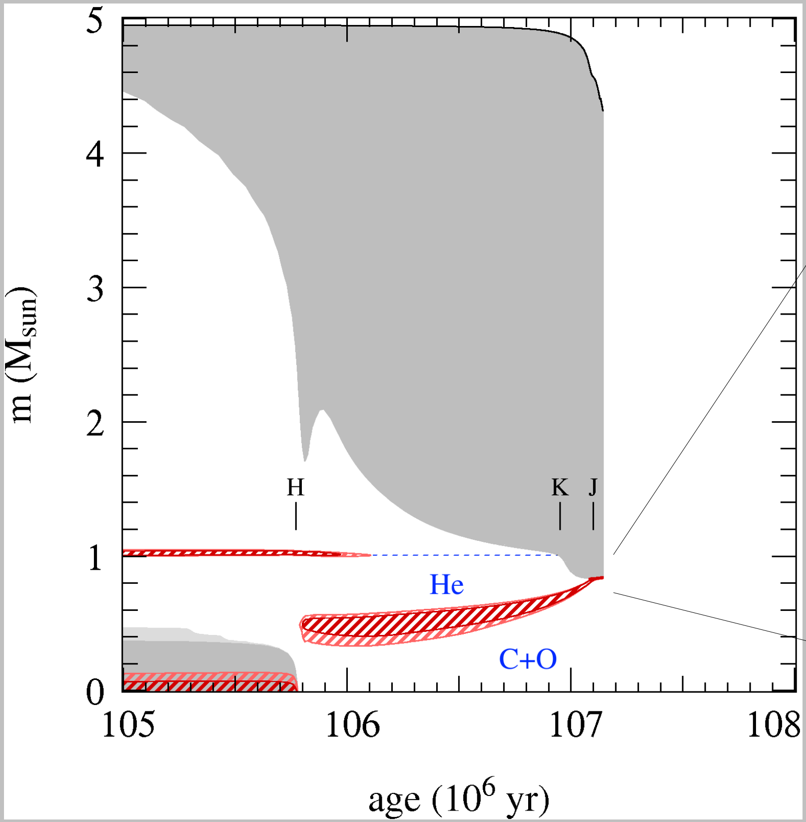


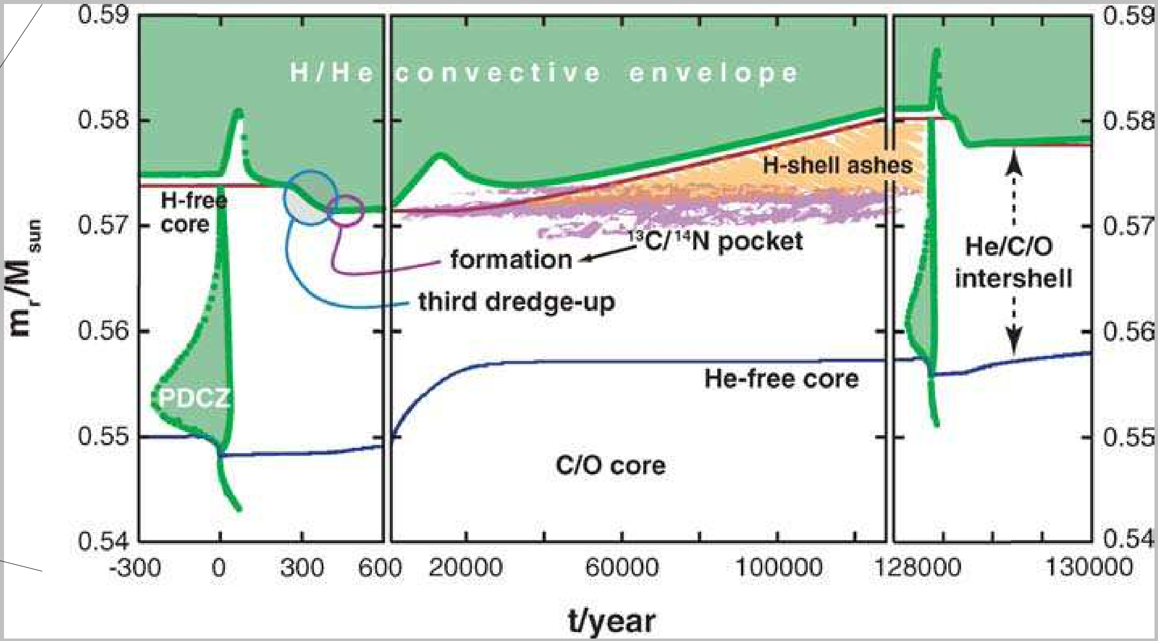
From SE notes, O. Pols (2009)

Intermediate & Low-Mass Stars

5 M_o star: AGB phase

Structure in AGB phase





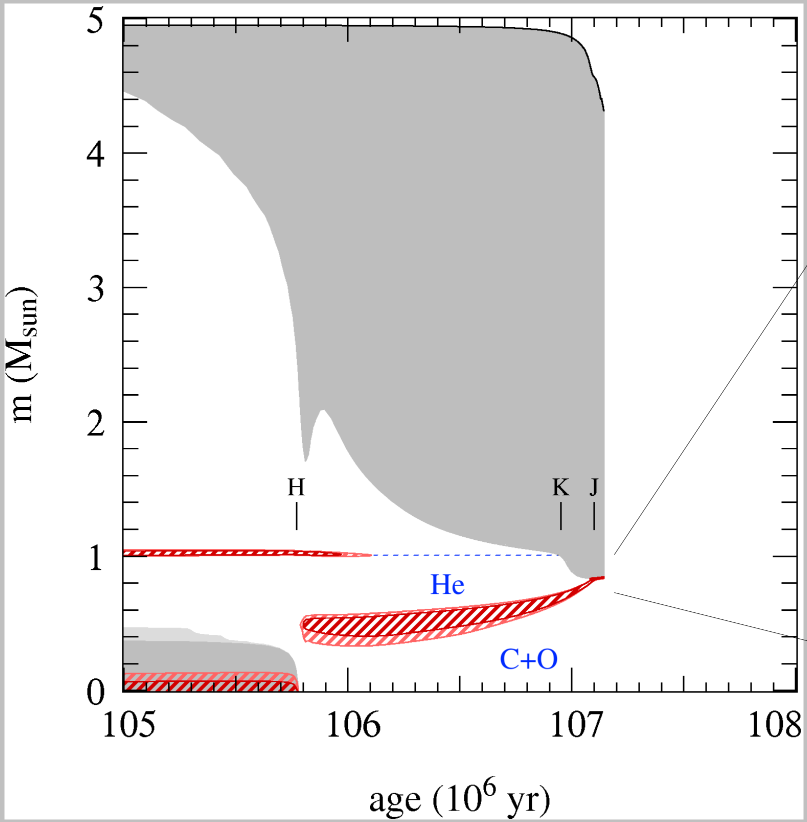
Herwig, ARAA, 2005

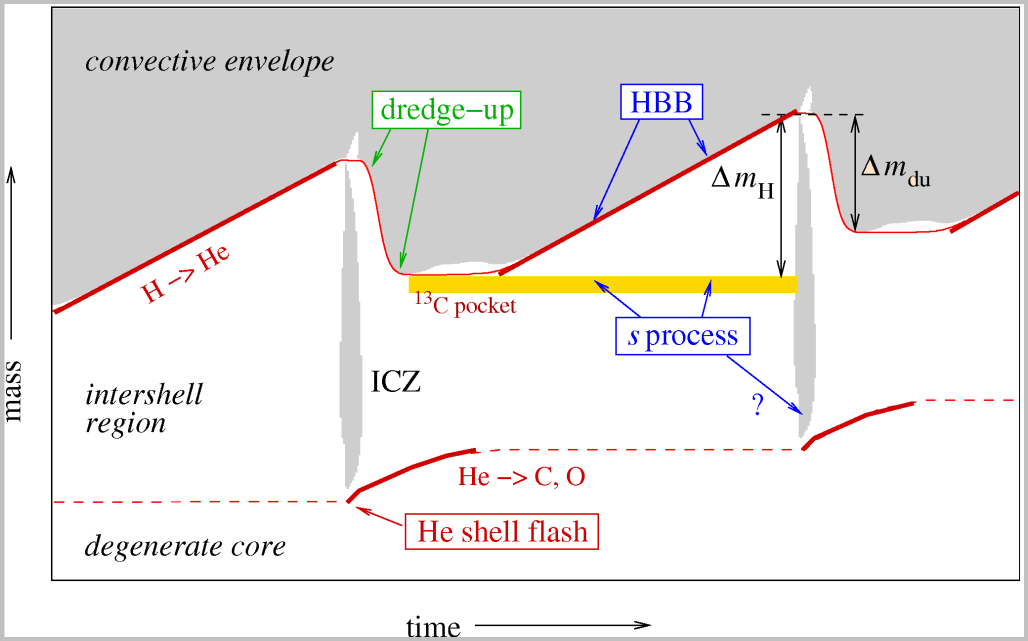
From SE notes, O. Pols (2009)

Intermediate & Low-Mass Stars

5 M_o star: AGB phase

Structure in AGB phase

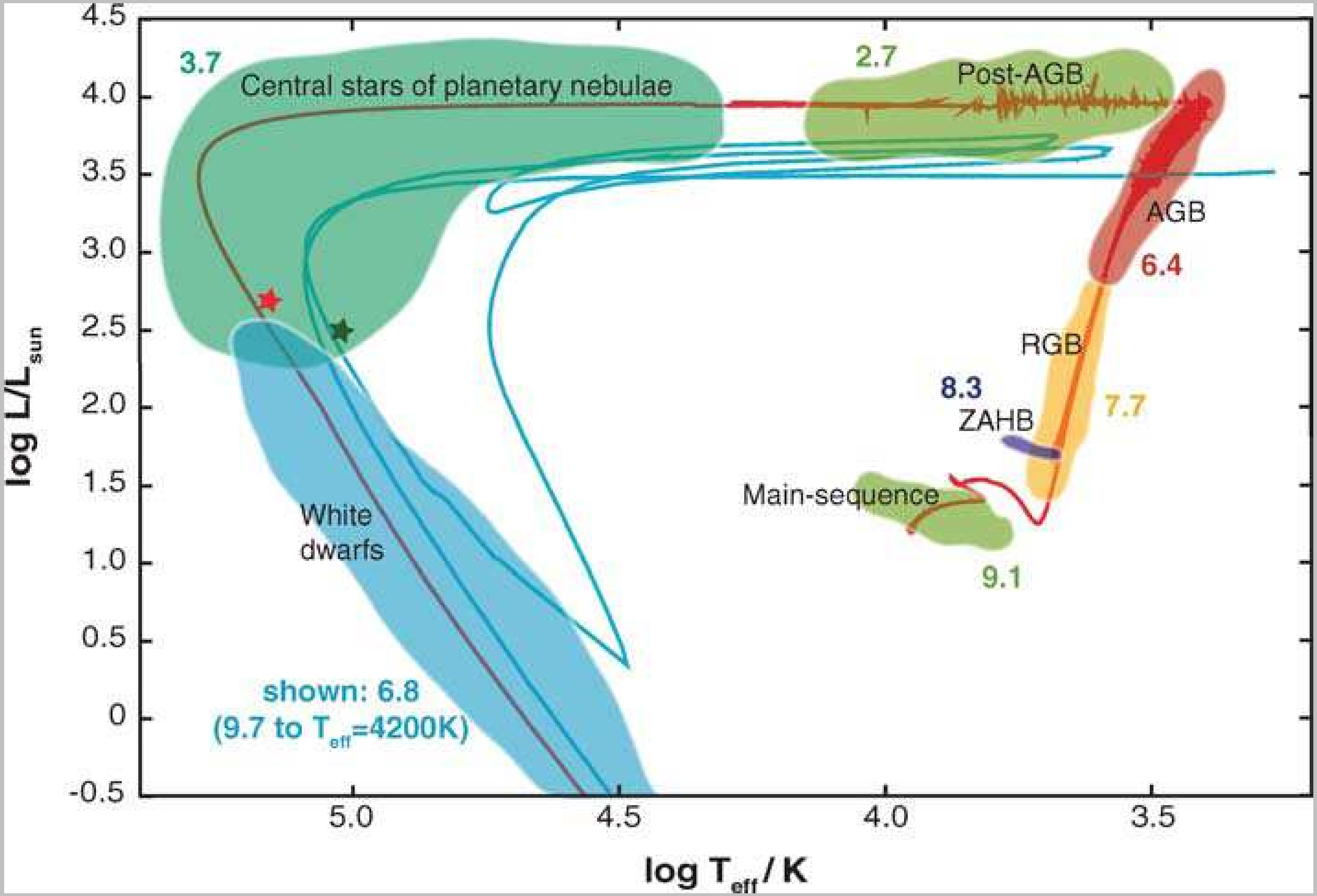




From SE notes, O. Pols (2009)

Intermediate & Low-Mass Stars

2 M_o star: post-AGB phase

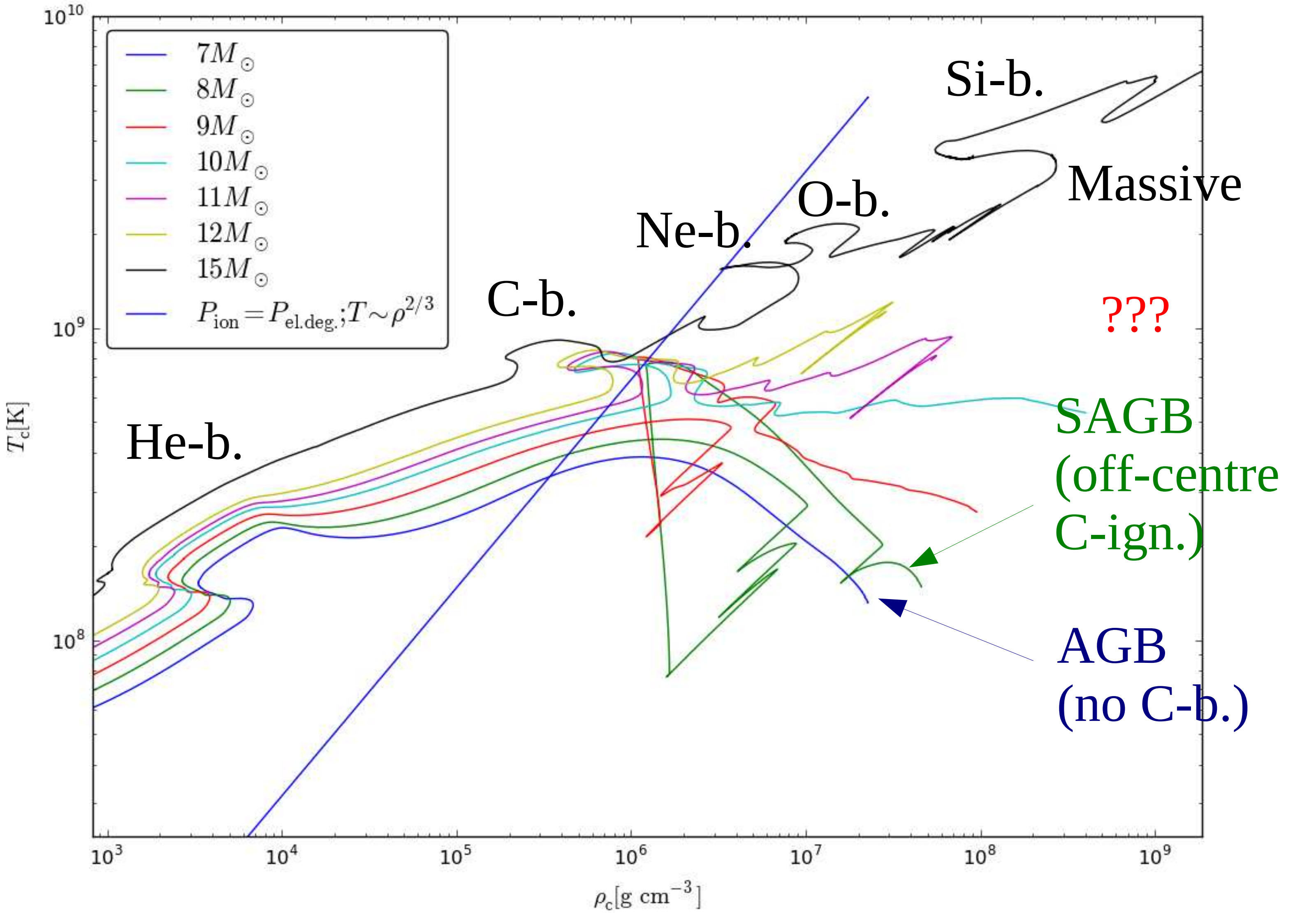


Herwig, ARAA, 2005

Massive/AGB Stars Transition

7-15 M_o models ← MESA stellar evolution code: http://mesa.sourceforge.net/

Paxton et al 10



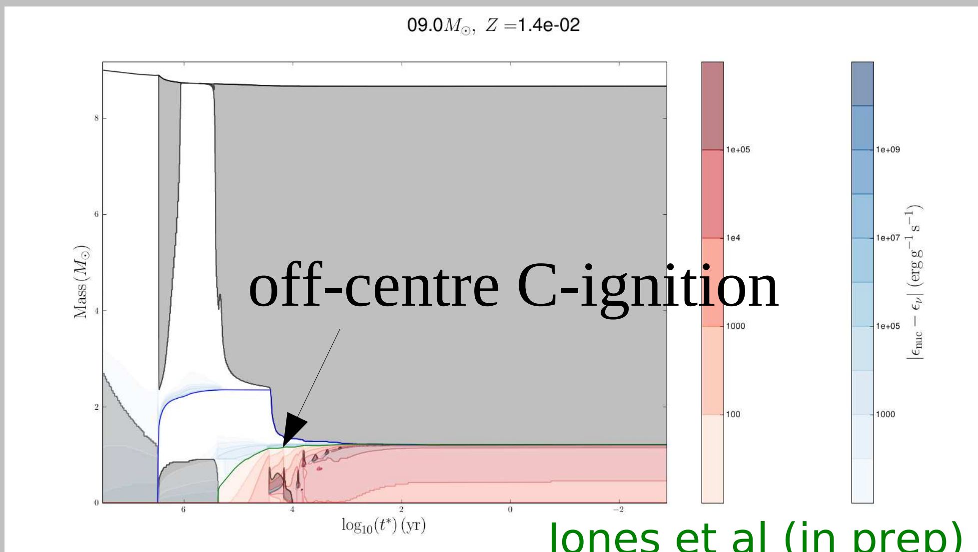
Jones et al 2013; see also Mueller et al 12, Umeda et al 12

SAGB & ECSN progenitors

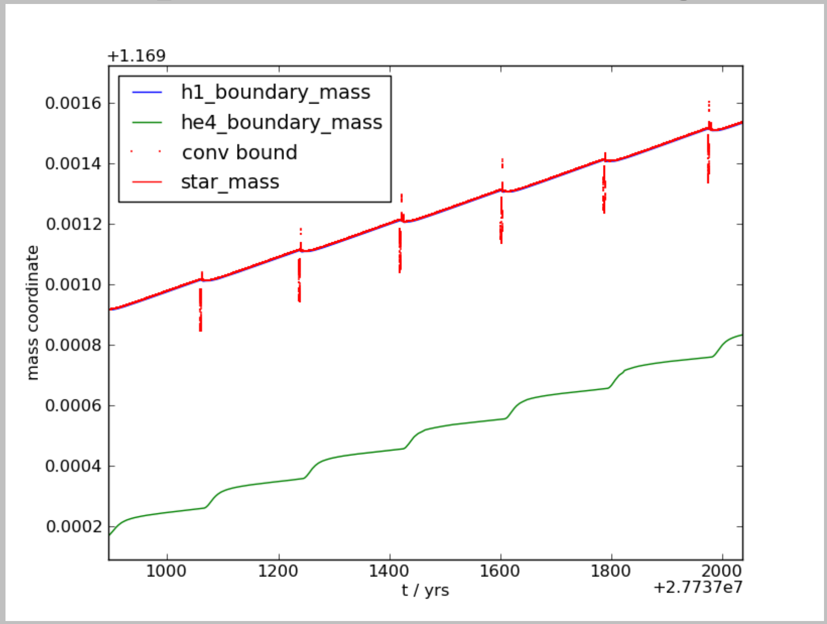
$M_{up} \leq M \leq M_{mas}$; $M_{up} \approx 8 M_{sun}$, $M_{mas} \approx 10 M_{sun}$ (TRANSITION MASSES)

Early evolution like AGBs;

TP-phase → core growth
Dep. on Mdot ↔ mixing



Jones et al (in prep)



Jones et al (subm.)

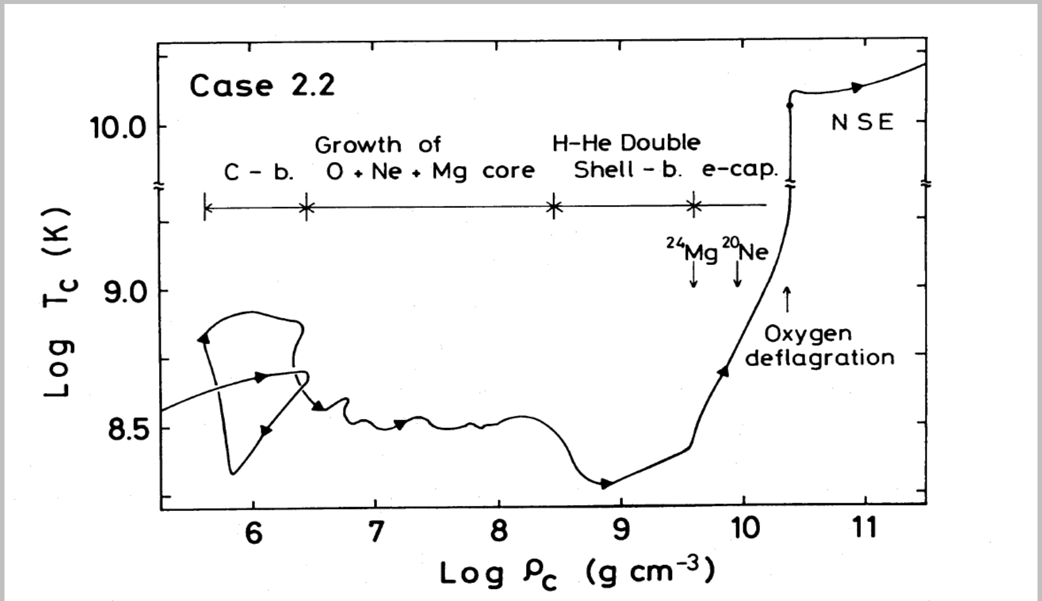
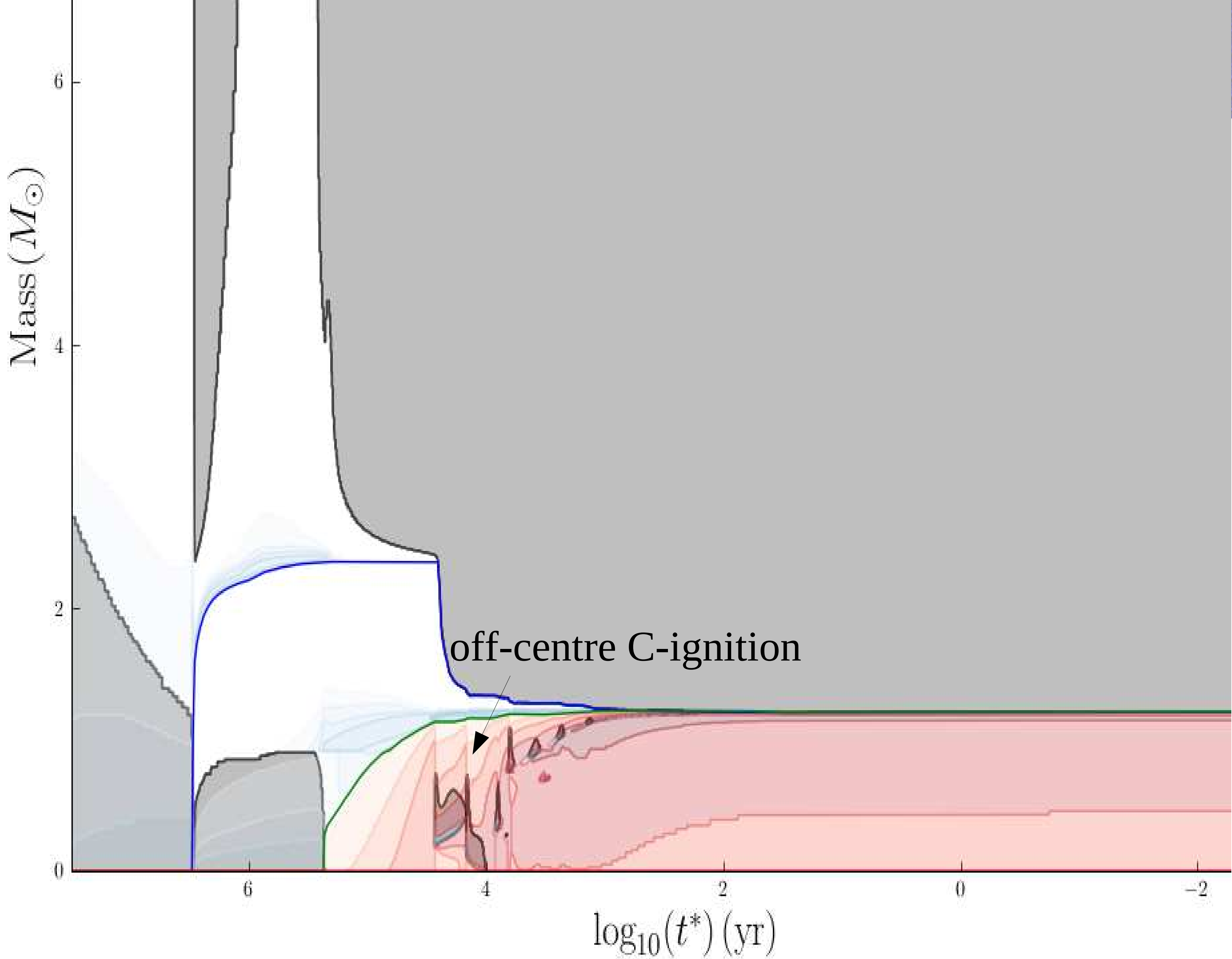


FIG. 4.—Evolutionary track in the central density and temperature diagram

Critical ONeMg core mass = M_{crit} = 1.375
(Miyaji et al. 1980; Nomoto 1984)
See also: Miyaji (1980); Nomoto(1984, 1987); Miyaji & Nomoto (1987); Garcia-Berro, Ritossa and Iben (1990s); Eldridge & Tout (2004); L. Siess (2006, 2007, 2009, 2010), Poelarends (2008); Doherty et al. (2010) ...



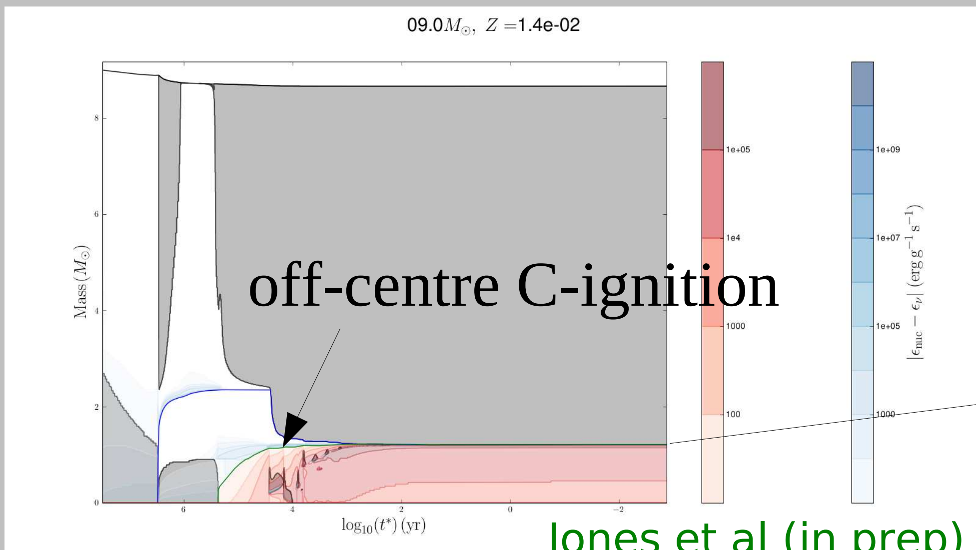
Mass $(M_\odot)$
6
4
2
0
6
4
2
0
−2
$\log_{10}(t^*)$ (yr)
off-centre C-ignition

SAGB & ECSN progenitors

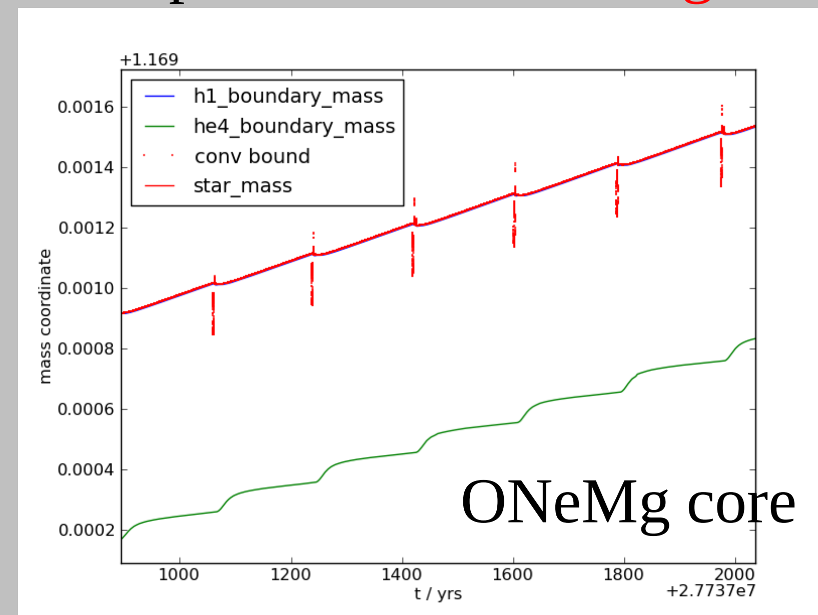
$M_{up} \leq M \leq M_{mas}$; $M_{up} \approx 8M_{sun}$, $M_{mas} \approx 10M_{sun}$ (TRANSITION MASSES)

Early evolution like AGBs;

TP-phase → core growth
Dep. on Mdot ↔ mixing



Jones et al (in prep)



Jones et al (in prep)

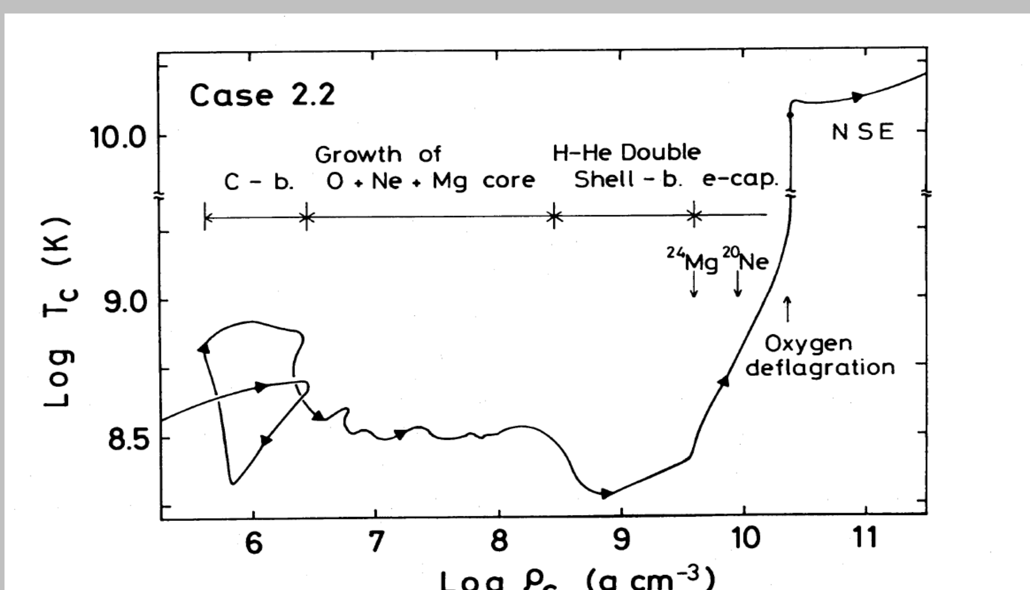


FIG. 4.—Evolutionary track in the central density and temperature diagram

Critical ONeMg core mass = M_{crit} = ~1.375

(Miyaji et al. 1980; Nomoto 1984)

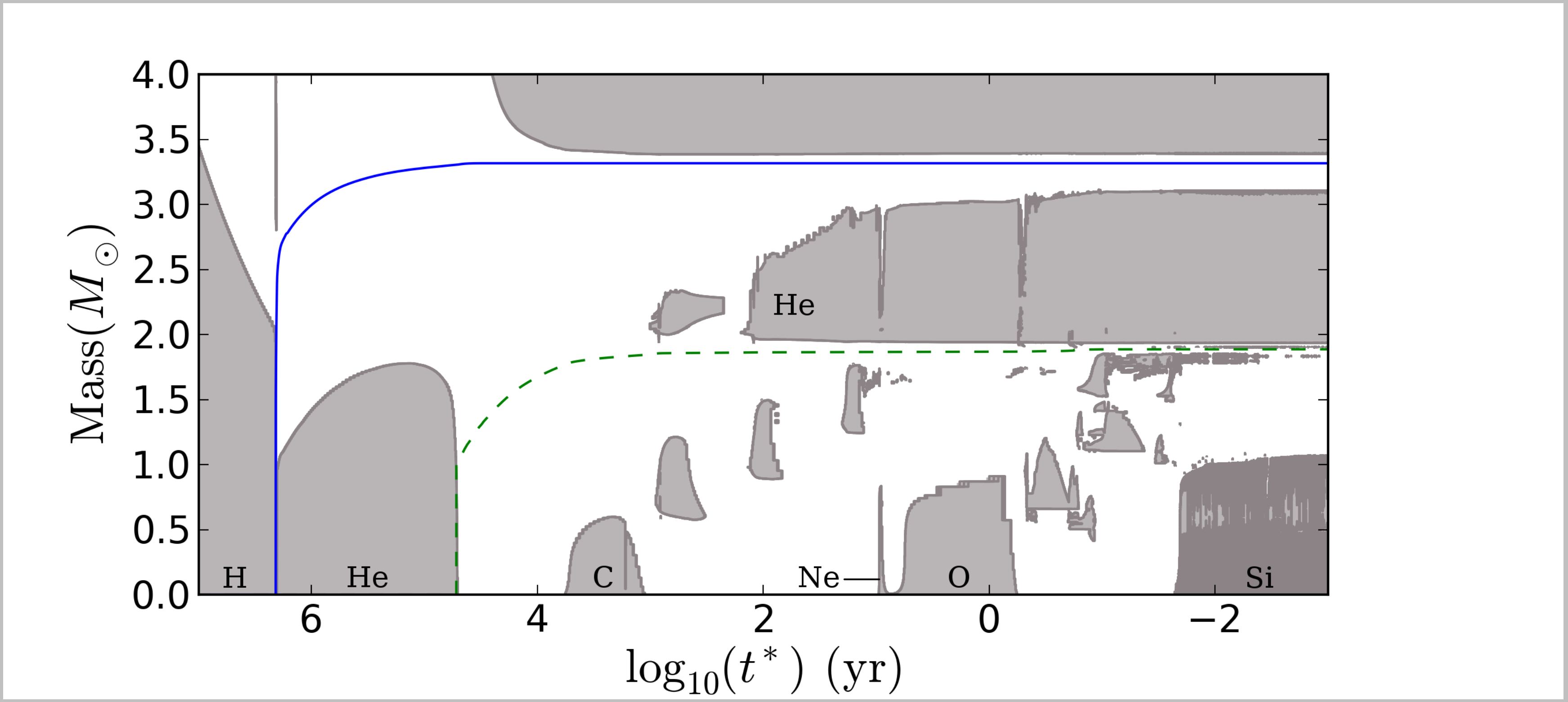
See also: Miyaji (1980); Nomoto(1984, 1987); Miyaji & Nomoto (1987); Garcia-Berro, Ritossa and Iben (1990s); Eldridge & Tout (2004); L. Siess (2006, 2007, 2009, 2010), Poelarends (2008); Doherty et al. (2010) ...

Can Massive Stars produce ECSN ?

7-15 M_o models ← MESA stellar evolution code: http://mesa.sourceforge.net/

Paxton et al 10,12

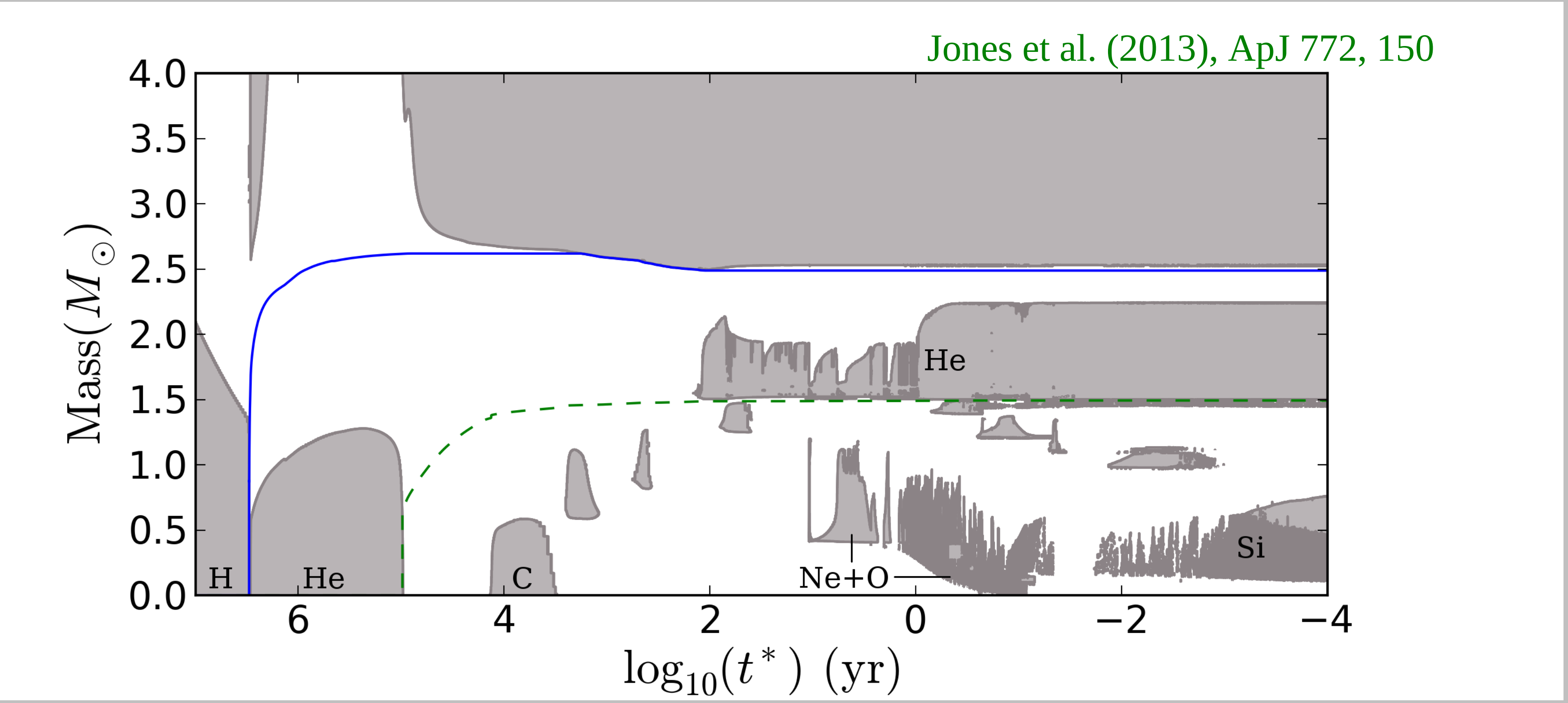
12 M_o is a typical massive star:



All burning stages ignited centrally. Fate: Fe-CCSN

Jones et al. (2013), ApJ 772, 150;
see also Mueller et al 12, Umeda et al 12, Takahashi et al 13

9.5 M_o still a massive star:

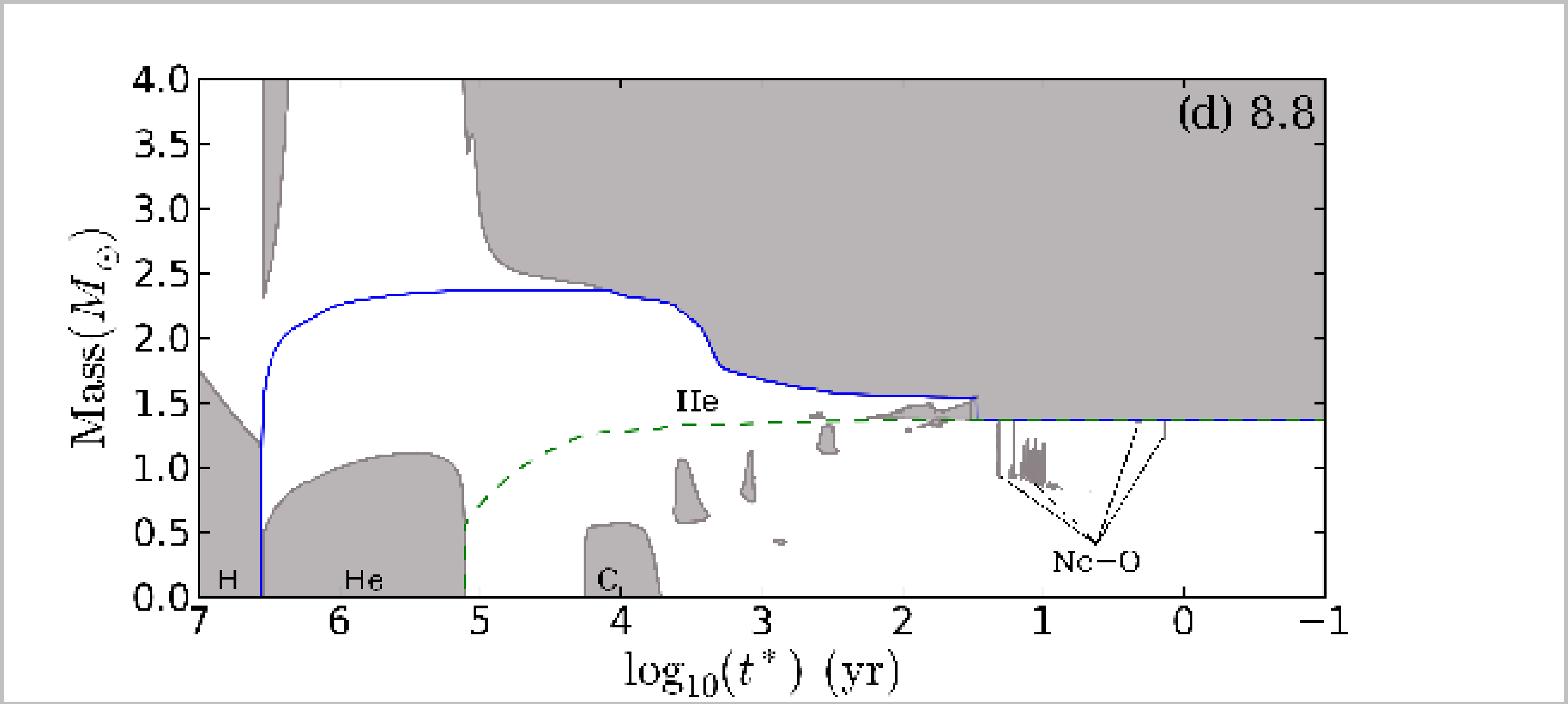


Ne-Si burning stages ignited off-centre. Fate: still Fe-CCSN

Simulations include 114-isotope network!

Can Massive Stars produce ECSN ?

8.8 M_o failed massive star:

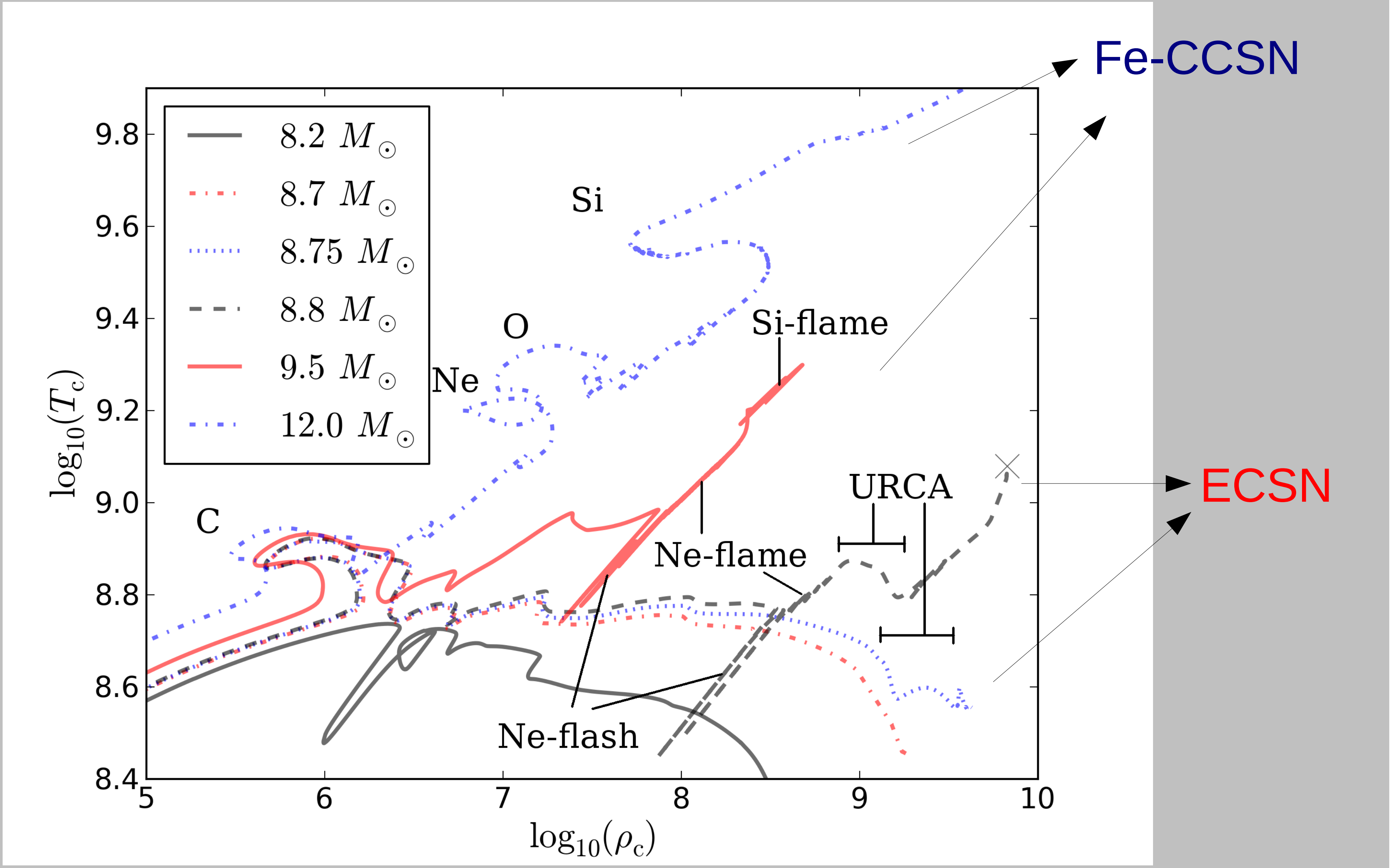


Jones et al. (2013), ApJ 772, 150
See also Nomoto 84: case 2.6
Timmes et al 92,94
Eldridge & Tout 04

Ne-b. starts off-centre but does not reach the centre.
MESA → Oxygen deflagration
Agile-Bolztran for collapse + explosion Fischer et al (in prep)
Fate: ECSN

Key uncertainties: convective boundary mixing, mass loss

Fate of Least-Massive MS: ECSN/Fe-CCSN?



Jones et al. (2013), ApJ 772, 150

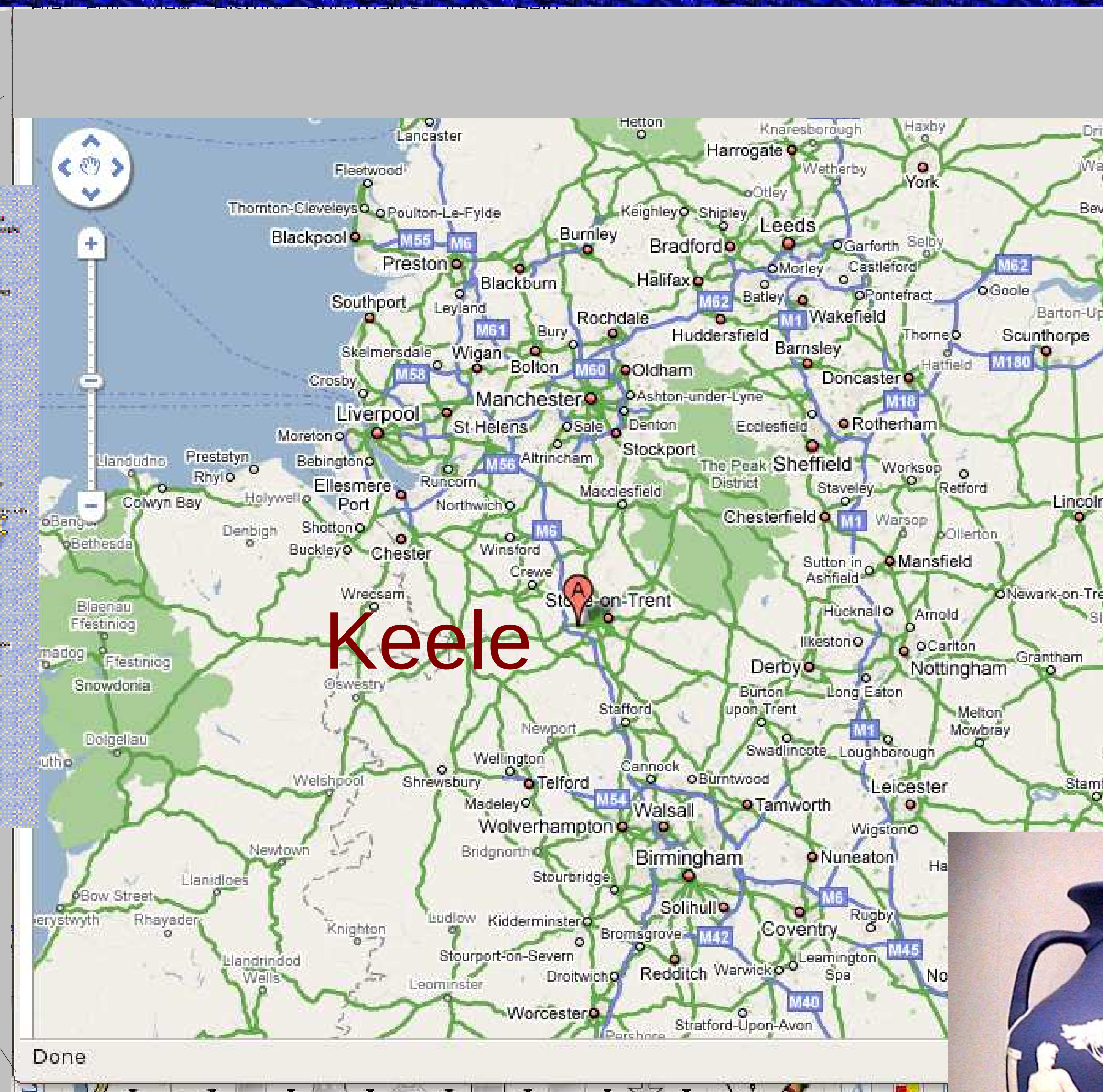
Both SAGB and failed massive stars may produce ECSN

L3-4

- EOS and partial degeneracy
- Standard massive stars
- The most massive stars
- Weak s-process
- Intermediate- and low-mass stars
- Stars at the boundary between massive and intermediate-mass stars

Keele is Not Kiel (Germany) But Where is it?

West Midlands:



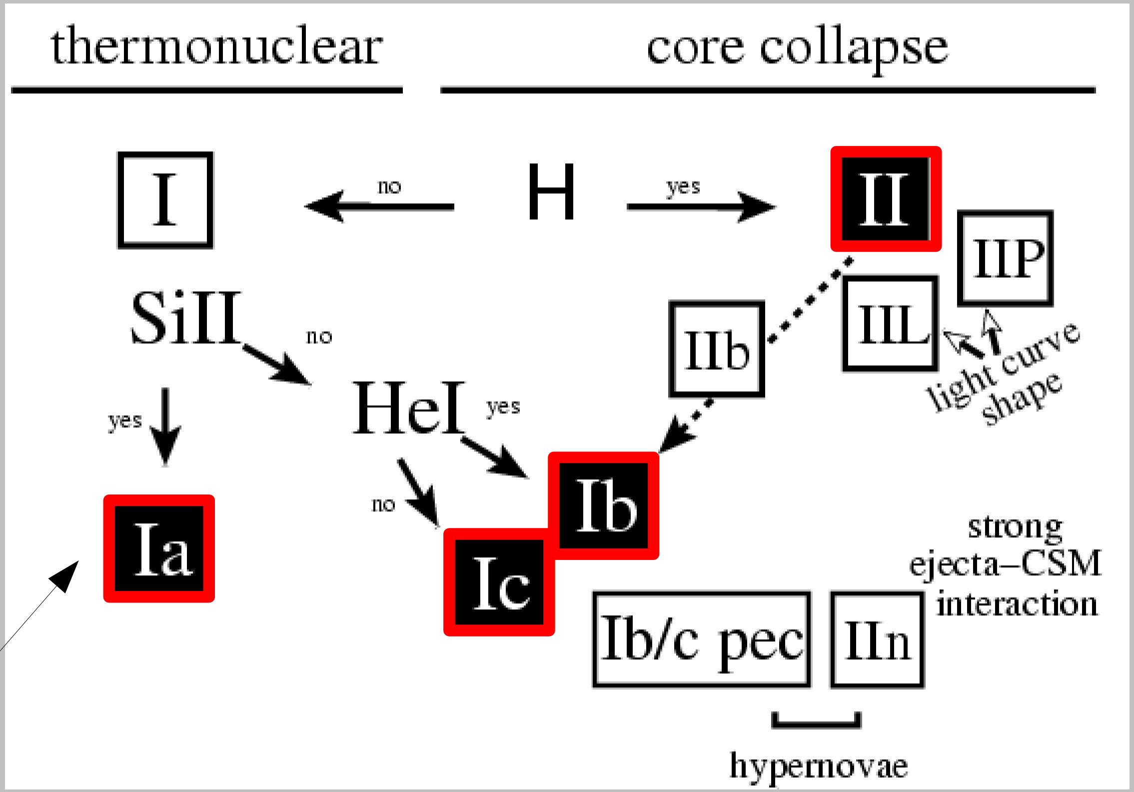
Keele area

is famous for pottery: Wedgwood, ...

and football: Stoke city fc in premier league

Supernova Explosion Types

Massive stars: → SN II (H envelope),
Ib (no H), Ic (no H & He) ← WR

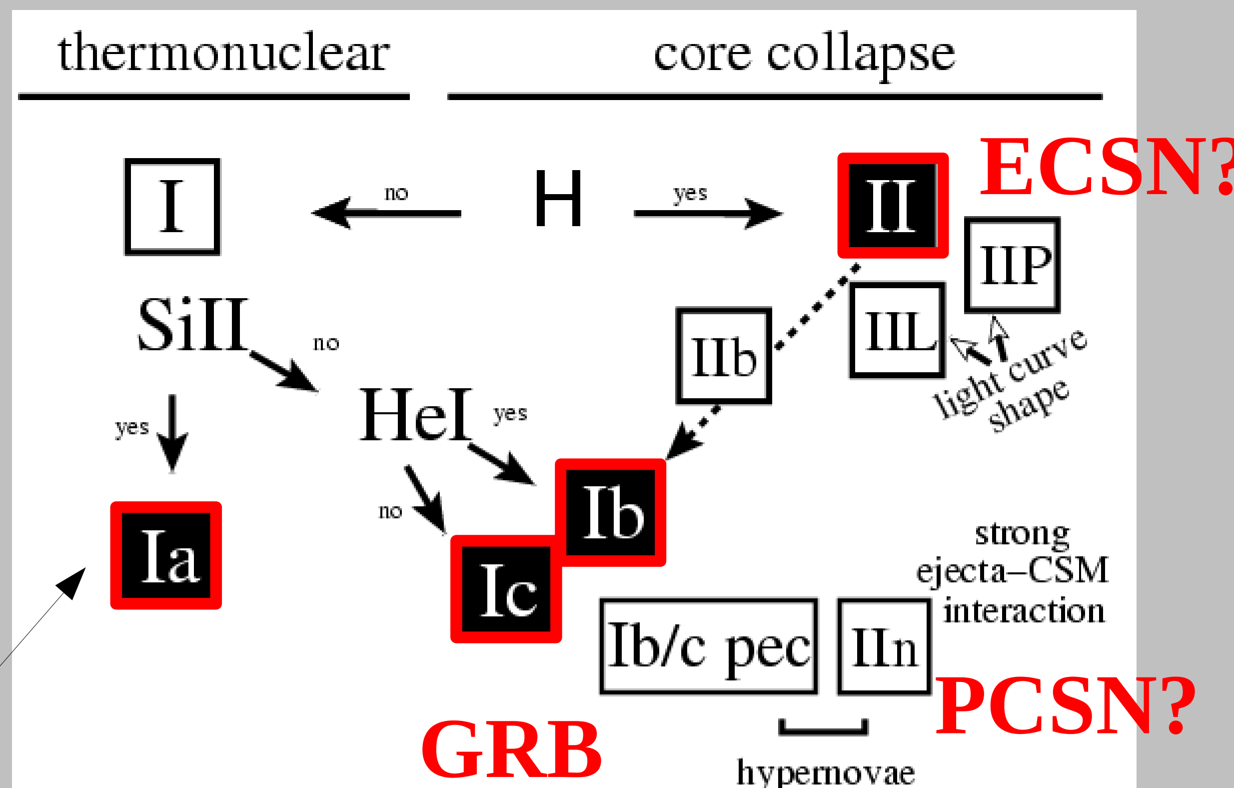


White dwarfs (WD):
in binary systems
Accretion →
Chandrasekhar
mass → SN Ia

(Turatto 03)

Supernova Explosion Types

Massive stars: → SN II (H envelope),
Ib (no H), Ic (no H & He) ← WR



White dwarfs (WD):
in binary systems
Accretion →
Chandrasekhar
mass → SN Ia

(Turatto 03)

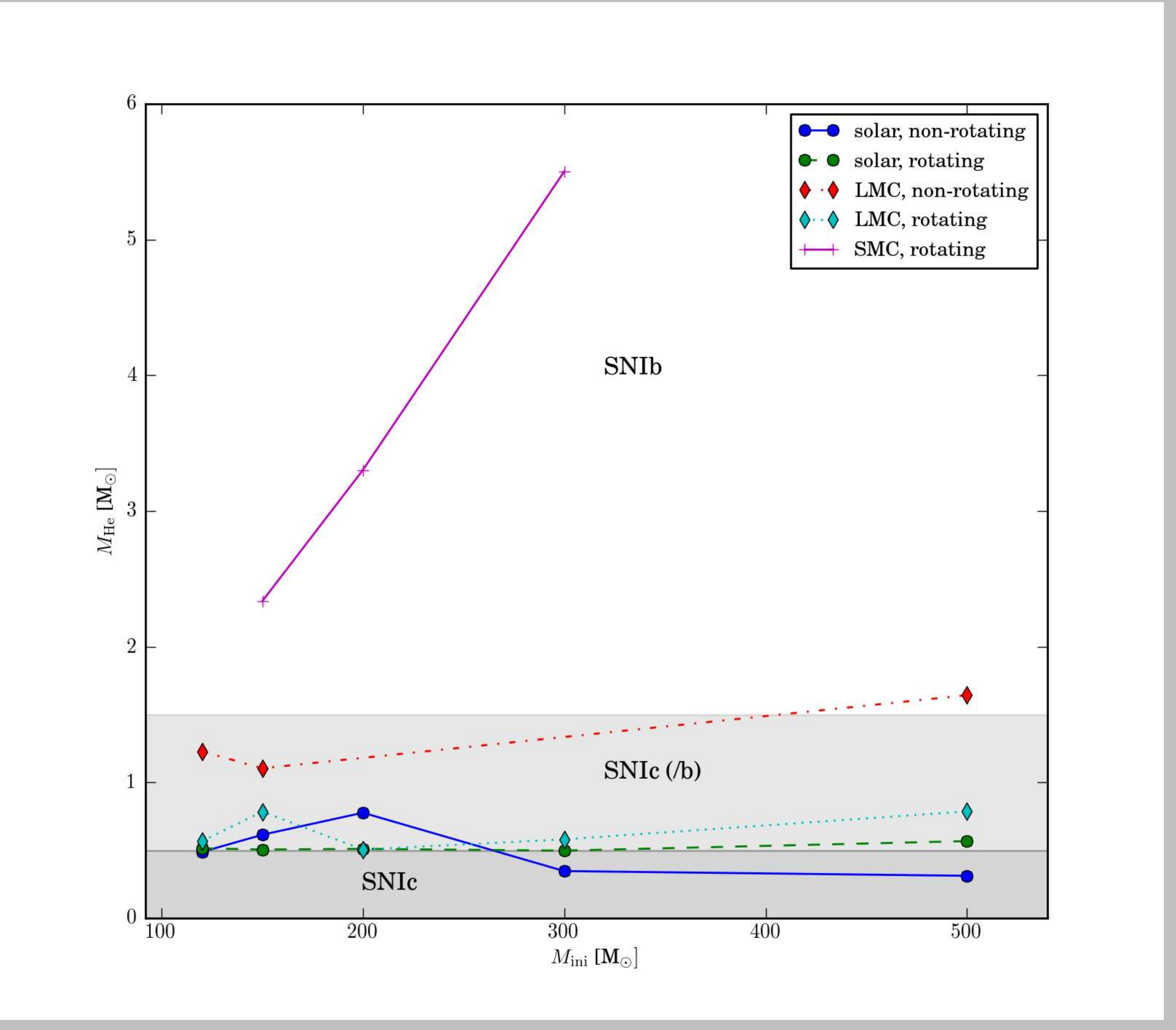
The fate of VMS: SNII/SNIb-c?

SN type:

- NO SNIIn predicted!
~ NOT ok for SN2006gy
(e.g. Woosley et al 2007)

- SNIc at solar Z,

- SNIb/c at Z(SMC)
~ ok for SN2007bi
(Gal-Yam 2009)
BUT see Dessart et al 12,13+ Panstarrs results



(Yusof et al 13 MNRAS, aph1305.2099)