Exercises in Mainline Stellar Nuclear Burning

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1. Introduction

Stellar nucleosynthesis follows a fairly well-understood sequence of burning stages. These are reviewed in a several text books, including [3, 1, 4]. The purpose of this brief paper is to present a set of computational exercises using NucNet Tools to demonstrate these stages in a massive star. The exercises are presented as a series of tasks. In separate manuscript, we presents "solutions" to the exercises.

2. Installing NucNet Tools

To carry out the exercises, it is first necessary to install NucNet Tools. To do so, follow the instructions at http://sourceforge.net/p/nucnet-projects/wiki/libraries/

3. Hydrogen Burning

The first stellar burning phase is hydrogen burning, which converts abundant hydrogen into helium. In a massive star this occurs at higher temperature but lower density than in the Sun.

Task 1: Follow the blog post http://sourceforge.net/u/mbradle/blog/2012/07/modifying-calculation-input-with-nucnet-tools-example-codes/ describing CNO burning. Use the XPath expression "[z <= 84]" to allow for He burning. The calculation will take roughly 15 minutes–more or less time depending on your computer. Reproduce the graph in the blog post. Discuss your calculation and emphasize and explain the significant abundance of ^{14}N at the end.

4. Helium Burning

Once a star burns hydrogen to helium, it contracts and heats until the He ignites. The He burns to C and O (but not much Ne!)

Task 2: Use the output of your hydrogen-burning calculation to run a helium-burning calculation. In particular, to extract the last zone in *my_output.xml* by typing (all on one line):

```
xsltproc http://nucnet-tools.sf.net/data_pub/2015-01-31/extract.xsl
my_output.xml > ../../data_pub/zone_he.xml
```

Since this reads an XSLT stylesheet over the web, you will need to be connected to the internet. You may also download extract.xsl and use it directly by typing:

```
xsltproc extract.xsl my_output.xml > ../../data_pub/zone_he.xml
```

Once that is done, change the property *dt* to 1.e-5, *time* to 0, *tend* to 3.15e11, *t9_0* to 0.300, *rho_0* to 1500 in *zone_he.xml*. Then run the code by typing (all on one line):

```
./run_single_zone ../../data_pub/my_net.xml ../../data_pub/zone_he.xml
my_output_he.xml "[z <= 84]"
```

Plot ^{4}He, ^{12}C, and ^{16}O versus time. Discuss your calculation and, in particular, how the ^{16}O abundance rises late in the calculation. Why does the ^{20}Ne not also grow?

Task 3: Study the s process that occurs during helium burning. Plot ^{14}N, ^{18}O, and ^{22}Ne versus time. Next, plot $n_n = N_A \rho X_n$, the number density of neutrons, as a function of time. Note N_A is Avogadro's number, ρ is, of course, the mass density, and X_n is the neutron mass fraction. Finally, plot elemental abundances at the beginning ("[position() = 1]") and end ("[last()]") of the calculation, as described in http://sourceforge.net/u/mbradle/blog/2013/04/plotting-abundances-versus-nucleon-number/. Note that you will use your output, not *my_new_output.xml* in the blog post. This figure shows the "movement" of abundance upwards in Z during the calculation. Discuss your calculations and figures. There are several blog posts on the s process. You might consider flow diagrams, where appropriate (for example, see http://sourceforge.net/u/mbradle/blog/2013/06/creating-flow-diagrams/). Be sure to explain where the neutrons are coming from (ultimately the ^{14}N).

5. Carbon Burning

The principal products of helium burning are ^{12}C and ^{16}O. As the star contracts and heats, carbon burning begins.

Task 4: Use the *extract.xsl* file and xsltproc to extract the last zone from *my_output_he.xml* as *../../data_pub/zone_c.xml*. Edit that zone file so that you run a network calculation at $T_9 = 0.9$ and density $\rho = 15,000$ g/cc for 10^4 years (be sure to change *dt* to 1.e-5 and *time* to 0 again). Graph the mass fractions of ^{12}C, ^{16}O, ^{20}Ne, and ^{24}Mg as a function of time (we suggest log x vs. linear y and the x range from 0.01 to 1.e4 years). Comment on what is going on. In particular, note the key reactions ^{12}C + ^{12}C and ^{12}C + ^{16}O. Also note that if the star forms a white dwarf after this burning, it will be a O/Ne/Mg white dwarf, not a C/O white dwarf, as it would have been after helium burning.

6. Neon Burning

Following carbon burning, the star will contract, heat, and undergo neon burning. This burning stage comprises ^{20}Ne $+ \gamma \rightarrow^{16}$O $+^4$He and ^{20}Ne $+^4$He $\rightarrow^{24}$Mg $+ \gamma$, which effectively is ^{20}Ne $+^{20}$Ne $\rightarrow^{16}$O $+^{24}$Mg.

Task 5: Using the output from your carbon burning calculation, run a neon burning calculation at $T_9 = 1.4$ and $\rho = 1.5 \times 10^5$ for 100 years. Graph the mass fractions of ^{16}O, ^{20}Ne, ^{24}Mg, and ^{28}Si as a function of time (same layout as the carbon burning plot). Note the rise of ^{28}Si late from ^{24}Mg $+^4$He.

7. Oxygen Burning

After neon burning, the next stage is oxygen burning. The principal reaction is ^{16}O $+^{16}$O $\rightarrow^{32}$S*. The excited ^{32}S nucleus then de-excites mostly to ^{28}Si (and an alpha) or ^{32}S.

Task 6: Use the output from your neon burning to compute oxygen burning at $T_9 = 2.1$ and $\rho = 1.5 \times 10^6$ g/cc for one year. Graph the mass fractions of ^{16}O, ^{28}Si, and ^{32}S as a function of time and comment.

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Also occurring during oxygen burning is the "gamma" process. The initially present heavy nuclei are disintegrated. First neutrons then protons and alphas are emitted. The nuclei "flow" down the proton-rich side of the stable nuclei. If this matter "freezes out", proton-rich nuclei are formed (the so-called "p-nuclei").

Task 7: Graph the elemental abundances at the beginning and end of your oxygen burning calculation. Note how nuclei "melt" towards the iron-group region.

8. Silicon Burning

The final mainline burning stage is Si burning. Here ^{28}Si converts to ^{56}Ni, which decays to ^{56}Fe. The burning proceeds through a complicated sequence–some Si disintegrates and the light particles that result capture onto the remaining Si to create heavier nuclei. In fact the burning proceeds through a quasi-equilibrium [2, 5].

Task 8: Use your output from your oxygen burning to run silicon burning at $T_9 = 3.5$ and $\rho = 1.5e7$ g/cc for one year. Plot the mass fractions of ^{28}Si, ^{56}Fe, and ^{56}Ni vs. time and discuss.

You might be surprised that the abundance of ^{56}Ni is never particularly large. The reason is that the silicon burning is sufficiently slow that weak decays have time to occur during the burning and thus change the neutron richness of the nuclei present.

Task 9: Plot Y_e as a function of time during the calculation. Y_e is the number of net electrons per nucleon. By charge neutrality, this is the number of protons per nucleon. $Y_e = 0.5$ corresponds to equal numbers of neutrons and protons. The initial composition of our "star" had $Y_e = 0.8554$, which is proton rich. Hydrogen burning changed Y_e to about 0.5, where it remained until silicon burning. To compute Y_e, use the analysis code *compute_abundance_moment_in_zones*:

```
../analysis/compute_abundance_moment_in_zones my_output_si.xml "" z 1
```

Note that ^{56}Fe has $Y_e = 26/56 = 0.464$. A large abundance of ^{56}Ni occurs in *explosive* silicon burning because the burning is rapid and weak decays do not have time to occur.

9. Conclusion

Of course stellar burning really occurs in the context of an evolving star. The temperature and density are not constant, as treated in these exercises, but rather evolve as nuclear fuel is consumed. Furthermore, in many cases, stellar layers are convective, which tends to mix the composition of those layers during the burning. Nevertheless, we hope that by carrying out the above exercises, one will get a good sense of the nature of the mainline stellar burning phases. Once comfortable with the codes, the interested user can try other conditions (e.g., temperature and density) or can modify reaction rates (e.g., see the blog post http://sourceforge.net/u/mbradle/blog/2013/02/modifying-reaction-rates-during-network-calculation/).

References

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