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Mass Spectrum and decay properties of the B_c meson

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The properties of B_c mesons are of special interest in quarkonium spectroscopy as they are the only quarkonia consisting of heavy quarks with different flavours and can not annihilate into gluons and so they are more stable. The possibility to study the B_c meson ignited considerable interest in its spectroscopy and other properties in light of recent experimental measurements and theoretical estimates. Using the Gaussian wave function mass spectra and decay properties of $B_c(c\bar{b})$ meson are investigated in the framework of phenomenological quark anti-quark potential (coulomb plus power) model consisting of relativistic corrections to the kinetic energy. The spin-spin, spin-orbit and tensor interactions are employed to obtain the pseudoscalar and vector meson masses incorporating the effect of mixing. The decay constants $(f_{P/V})$ are computed using the wave function at the origin. The lifetime and electromagnetic transition rates are also calculated in this scheme. Our predictions for the B_c meson are in good agreement with experimental results as well as other theoretical models.

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1. Introduction

The $B_c^{\pm}$ meson was first observed by the CDF collaboration via the decay mode $B_c^{\pm} \to J/\psi l^{\pm}\nu$ in 1.8 GeV collisions at the Fermilab Tevatron[1]. They measured the $B_c^{\pm}$ mass to be $6.40 \pm 0.39 \pm 0.13$ GeV. Subsequently Abulencia *et al*[2] and Aaltonen *et al*[3] observed $B_c^{\pm}$ in the decay channel $B_c^{\pm} \to J/\psi\pi^{\pm}$ with masses $6285.7 \pm 5.3(stat) \pm 1.2(syst)$ MeV and $6275.6 \pm 2.9(stat) \pm 2.5(syst)$MeV respectively. The present average value of the B_c meson mass is 6.277 ± 0.006 GeV [4]. The possibility to study the B_c meson ignited considerable interest in its spectroscopy[5, 6, 7, 8, 9]. The properties of B_c mesons are of special interest in quarkonium spectroscopy as they are the only quarkonia consisting of heavy quarks with different flavours and cannot annihilate into gluons and so are more stable with widths less than a hundred keV. Excited B_c meson states which lie below the BD threshold can only undergo radiative or hadronic transitions to the ground state which then decay weakly and are significantly more stable then corresponding charmonium or bottomonium states.

Non-relativistic potential models inspired by the basic QCD characteristics have been quite successful for the prediction of various meson properties with the help of a Schrodinger equation. Masses of the ground as well as excited states can be calculated by making use of the one gluon exchange potential(OGEP) while the decay properties can be evaluated by using the wave functions and the overlap integral. In ref [10] we calculated the properties of the B_c meson within potential model scheme by considering both the quarks to be non-relativistic, however Ref [6] indicates that the charm quark is not heavy enough to be treated non-relativistic. In the present work we extend our previous study by including kinematic relativistic corrections within the kinetic energy of quarks and employ Gaussian wave function in position space as well as momentum space. Decay constant is calculated non-relativistically. We present a study of mass spectra and decay properties of B_c meson in the potential scheme of Coulomb plus power potential with the power index ν=1. We present the details of semi-relativistic treatment in section-2. The decay constants ($f_{P/V}$) of this meson is presented in section-3. Finally we draw our conclusion in sec-4.

2. Theoretical Framework

For the study of the B_c meson we consider the relativistic Hamiltonian in which motion of the quarks inside the B_c meson is relativistic[11, 12]

$$H = \sqrt{\mathbf{p}^2 + m_b^2} + \sqrt{\mathbf{p}^2 + m_{\bar{c}}^2} + V(\mathbf{r}) \tag{2.1}$$

where $\mathbf{p}$ is the relative momentum of the quark-antiquark and m_b is the b quark mass and $m_{\bar{c}}$ is the c quark mass. The Hamiltonian in Eq(2.1) represents the energy of the meson in the meson rest frame. We expand the kinetic energy(K.E.) part of the Hamiltonian up to $\mathcal{O}(p^6)$, as

$$K.E. = \frac{\mathbf{p}^2}{2}\left(\frac{1}{m_b} + \frac{1}{m_c}\right) - \frac{\mathbf{p}^4}{8}\left(\frac{1}{m_b^3} + \frac{1}{m_c^3}\right) + \frac{\mathbf{p}^6}{16}\left(\frac{1}{m_b^5} + \frac{1}{m_c^5}\right) \tag{2.2}$$

and $V(\mathbf{r})$ is the quark-antiquark potential [10, 13, 14],

$$V(r) = -\frac{\alpha_c}{r} + Ar^{\nu} + V_0 \tag{2.3}$$

where A is the potential parameter and ν is a general power index, such that the choice of $\nu = 1$ corresponds to the Coulomb plus linear potential with a constant term V_0. $\alpha_c = (4/3)\alpha_S\left(M^2\right)$, $\alpha_S\left(M^2\right)$ is the strong running coupling constant. We have used the gaussian wave function in the present study. The gaussian wave function in position space has the form

$$R_{nl}(\mu,r) = \mu^{3/2}\left(\frac{2\,(n-1)!}{\Gamma\,(n+l+1/2)}\right)^{1/2}(\mu r)^l \times e^{-\mu^2r^2/2}L_{n-1}^{l+1/2}(\mu^2r^2) \tag{2.4}$$

Here, μ is the variational parameter and L is Laguerre polynomial. For the present study, we employ the Ritz variational scheme. We obtain the expectation values of the Hamiltonian as

$$H\psi = E\psi \tag{2.5}$$

For a chosen value of ν, the variational parameter, μ is determined for each state using the Virial theorem. Gaussian wavefunction in position space has been employed to obtain the expectation value of the potential energy part in the Virial theorem while momentum space wave function has been used to obtain the kinetic energy part. As the interaction potential assumed here does not contain the spin dependent part, it gives the spin averaged masses of the system in terms of the power index ν. The spin averaged mass for the ground state is computed for the values of ν from 0.5 to 2.0. Since the calculations by Ebert *et al.*[5] are relatively recent we have matched matched the spin averaged mass with the value from Ref [5] for the ground state using the equation

$$M_{SA} = M_P + \frac{3}{4}(M_V - M_P) \tag{2.6}$$

where M_V and M_P are the vector and pseudoscalar meson ground state masses taken from ref [5]. The parameters used to calculate the low lying masses of the B_c meson are $A = 0.175GeV$, $m_c = 1.55GeV$, $m_b = 4.88GeV$ and the value of the constant V_0 was found for $\nu = 1$ is -0.282 GeV.

We add separately (in Eq.2.5) the spin-dependent part of the usual one gluon exchange potential (OGEP) between the quark anti quark for computing the hyperfine and spin-orbit shifting of the low-lying S, P and D-states [8, 9]

$$\begin{aligned} V_{SD}(\mathbf{r}) = &\left(\frac{\mathbf{L}\cdot\mathbf{S_b}}{2m_b^2}+\frac{\mathbf{L}\cdot\mathbf{S_{\bar{c}}}}{2m_{\bar{c}}^2}\right)\left(-\frac{dV(r)}{rdr}+\frac{8}{3}\alpha_S\frac{1}{r^3}\right)+\frac{4}{3}\alpha_S\frac{1}{m_bm_{\bar{c}}}\frac{\mathbf{L}\cdot\mathbf{S}}{r^3}+\frac{4}{3}\alpha_S\frac{2}{3m_bm_{\bar{c}}}\mathbf{S_b}\cdot\mathbf{S_{\bar{c}}}4\pi\delta(\mathbf{r}) \\ &+\frac{4}{3}\alpha_S\frac{1}{m_bm_{\bar{c}}}\left\{3(\mathbf{S_b}\cdot\mathbf{n})(\mathbf{S_{\bar{c}}}\cdot\mathbf{n})-(\mathbf{S_b}\cdot\mathbf{S_{\bar{c}}})\right\}\frac{1}{r^3}, \qquad \mathbf{n}=\frac{\mathbf{r}}{r}\end{aligned}$$

where $V(r)$ is the phenomenological potential, the first terms takes into account the relativistic corrections to the potential $V(r)$, the second term accounts spin orbital interaction, third term is usual spin-spin interaction part which is responsible for pseudoscalar and vector meson splitting and fourth term stands for tensor interaction.

3. Decay Constants ($f_{P/V}$)

We have calculated the decay constants according to the well known relation between $f_{P,V}$ and the ground state wave function at the origin $\psi_{P/V}(0)$, the Van-Royen-Weisskopf formula[17].

$$f_{P/V}^2 = \frac{12\left|\psi_{P/V}(0)\right|^2}{M_{P/V}} \tag{3.1}$$

Table 1: Mass spectrum of the B_c meson.

States	Our	Expt.[4]	Ref.[5]	Ref.[6]	Ref[7]	Ref[8]	Ref[9]	Ref.[15]	[16]
1^1S_0	6.269	6.277	6.272	6.270	6.271	6.253	6.264	6.275	6.256
1^3S_1	6.334		6.333	6.332	6.338	6.317	6.337	6.331	6.343
1^3P_0	6.716		6.784	6.699	6.699	6.683	6.700	6.770	6.848
$1P_1$	6.751		6.720	6.734	6.741	6.717	6.730	6.781	6.882
$1P_1'$	6.757		6.810	6,749	6.750	6.729	6.736	6.793	6.882
1^3P_2	6.772		6.888	6,762	6.768	6.743	6.747	6.804	6.889
2^1S_0	6.865		6.850	6.835	6.855	6.867	6.856	6.852	6.865
2^3S_1	6.879		6.904	6.881	6.887	6.902	6.899	6.890	6.879
1^3D_1	7.020		7.082	7.072	7.028	7.008	7.012	7.142	7.308
$1D_2$	7.025		7.076	7.077	7.036	7.001	7.009	7.148	7.297
$1D_2'$	7.034		7.090	7.079	7.041	7.016	7.012	7.155	7.304
1^3D_3	7.027		7.092	7.081	7.045	7.007	7.005	7.146	7.286
2^3P_0	7.121		7.155	7.091	7.122	7.088	7.108	7.166	7.309
$2P_1$	7.148		7.067	7.126	7.145	7.113	7.135	7.176	7.337
$2P_1'$	7.153		7.206	7.145	7.150	7.124	7.142	7.185	7.339
2^3P_2	7.164		7.244	7.156	7.164	7.134	7.153	7.194	7.345
3^1S_0	7.250		7.225	7.193	7.250			7.236	6.939
3^3S_1	7.257		7.255	7.235	7.272			7.268	6.997
2^3D_1	7.360		7.426						
$2D_2$	7.363		7.405						
$2D_2'$	7.371		7.429						
2^3D_3	7.366		7.444						
3^3P_0	7.449		7.445						
$3P_1$	7.473		7.333						
$3P_1'$	7.479		7.517						
3^3P_2	7.489		7.567						
4^1S_0	7.571		7.524					7.542	
4^3S_1	7.576		7.544					7.570	
5^1S_0	7.857		7.784					7.804	
5^3S_1	7.861		7.800					7.829	

The computed f_P and f_V for B_c meson are tabulated in Tables 2 and 3.

4. Conclusion

Mass spectra and decay constants of the B_c meson were calculated in non-relativistic potential model scheme by incorporating kinematic relativistic corrections within the hamiltonian of the system. Masses for the radially and orbitally excited states were evaluated. In the absence of precise experimental measurements for excited states we compared our results with other theoretical model predictions. For the spin averaged masses we find that our model is able to reproduce the

Table 2: Pseudoscalar decay constants(in GeV)

State	Our	[6]	[15]	[18]
1S	0.432	0.562	0.439	0.402
2S	0.190			
3S	0.135			
4S	0.108			
5S	0.092			

Table 3: Vector decay constants(in GeV)

State	Our	[6]	[15]	[18]
1S	0.434	0.562	0.463	0.431
2S	0.190			
3S	0.135			
4S	0.108			
5S	0.092			

masses fairly well at potential index $\nu = 1$, which corresponds to linear confinement. The subsequent calculations are therefore done at only $\nu = 1$. For the complete mass spectrum our results are generally in good agreement with those of the others with a difference in most cases of 80 MeV or less. The decay constants are in good agreement with other models.

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