Spectroscopy and Decay properties of $D$ and $D_s$ mesons with Martin-like confinement potential in Dirac formalism

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We construct quark - antiquark bound states of open charm mesons by assuming that the constituent quark and antiquark are confined by an average Martin-like potential within the Dirac formalism. On fixing the model parameters using the ground state masses, other excited states of the $D$ and $D_s$ meson spectra are computed. The corresponding wave functions are then employed to compute the decay constants of $D$ and $D_s$ meson within the relativistic frame work. These decay constants play an important role in the leptonic and nonleptonic weak decays of the open flavour mesons. The present calculation yields the decay constant of $D$ and $D_s$ meson as $216.02$ MeV and $235.18$ MeV respectively as against the experimental values of $206.7 \pm 8.9$ MeV and $260.0 \pm 5.4$ MeV respectively. The present results are also in accordance with the QCD sum rule predictions ($204 \pm 6$ MeV and $246 \pm 6$ MeV) and Lattice QCD ($218.9 \pm 11.3$ MeV and $260.1 \pm 10.8$ MeV) for $D$ and $D_s$ mesons respectively. The branching ratio of the leptonic decays of $D$ and $D_s$ mesons in all the three leptonic channels are also computed and the results are in good agreement with the experimental values particularly in the case of $D_s$ meson.
1. Introduction

The recent experimental observations of open flavour mesonic states at charm sector have provided a boost to the theoretical attempts towards the understanding of the dynamics of light quarks in the company of heavy flavour quarks. The colour confinement of quarks is understood in terms of multi-gluon exchange processes at the non-perturbative regime of the hadronic size and it is not feasible to compute theoretically from the QCD first principles. The lattice QCD methods, QCD sum rules and potential models etc. are thus being employed to study the hadronic properties. Though lattice calculations are based on ab-initio method at some extent, it requires huge computing power and thus limited to the study of hadrons properties mainly to the ground states and few cases where excited states are being studied. In this context, other theoretical models particularly to potential models become a tool to study the properties of hadrons. The masses of heavy-light system for low-lying $1S$ and $1P_J$ states of $D$ and $D_s$ mesons are known from experiment [1]. Recently, many new resonances of $D$ and $D_s$ systems such as $D^*_0(2400)$, $D(2540)$, $D^*(2610)$, $D_s(2638)$, $D_s(2710)$, $D_j(2750)$, $D^*_j(2760)$, $D_s(2860)$, $D_s(3040)$ are also reported by different experimental groups [2, 3, 4, 5]. But very few of them are comparable with the existing theoretical predictions [6, 7, 8, 9, 10, 11]. In this context, these newly discovered states are very important for theorists to fine tune their models for better understanding of the $Q\bar{q}$ dynamics.

2. Methodology

In the present study, we assume that the constituent quarks in a meson core is independently confined by an average potential of the form

$$V(r) = \frac{1}{2}(1 + \gamma_0)(\lambda \rho^{01} + V_0)$$

To a first approximation, the confining part of the interaction is believed to provide the zeroth-order quark dynamics inside the meson through the quark Lagrangian density

$$\mathcal{L}_q^0(x) = \bar{\psi}_q(x) \left[ \frac{i}{2} \gamma^\mu \partial_\mu - V(r) - m_q \right] \psi_q(x).$$

The solution of Dirac equation resulting from the Lagrangian can be written as two component (positive and negative energies in the zeroth order) form as

$$\psi^{(+)}_{nlj}(\vec{r}) = N_{nlj} \left( \frac{ig(r)/r}{(\sigma \cdot \hat{r})f(r)/r} \right) \mathcal{Y}_{ljm}(\hat{r})$$

$$\psi^{(-)}_{nlj}(\vec{r}) = N_{nlj} \left( \frac{i(\sigma \cdot \hat{r})f(r)/r}{g(r)/r} \right) (-1)^{j+m_l-l} \mathcal{Y}_{ljm}(\hat{r})$$

Here, $N_{nlj}$ is the overall normalization constant, and $\mathcal{Y}_{ljm}(\hat{r})$ are the solid spherical harmonics and is expressed in terms of the spinor wave functions $\chi_{2m_l}$. The reduced radial part $g(r)$ of the upper component and $f(r)$ of the lower component of Dirac spinor $\psi_{nlj}(r)$ are numerically solved and normalized. The optimized model parameters employed for the present study are given below. The quark masses, $m_c = 1.27$ GeV, $m_s = 0.1$ GeV and $m_{u,d} = 0.003$ GeV are taken from PDG.
3. The Decay constant of the charm flavored mesons

Following the procedure adopted by [13] the decay constant can be expressed through the meson wave function $\psi(p)$ in the momentum space as

$$f_p^2 = \frac{3|I_M|^2}{2\pi^2 M_p J_M} \ (3.1)$$

where

$$I_M = \int_0^\infty dp \ p^3 A(p) \sqrt{g_q(p) g_q^*(-p)}, \ J_M = \int_0^\infty dp \ p^3 g_q(p) g_q^*(-p)$$
Table 2: The decay constant $f_\rho$ of $D$ systems (in MeV).

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<tr>
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<th>1S</th>
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<tr>
<td>Present</td>
<td>216.02</td>
<td>185.99</td>
<td>168.14</td>
<td>155.39</td>
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<tr>
<td>Expt./PDG [1]</td>
<td>206.7 ± 8.9</td>
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<tr>
<td>QCDSR [14]</td>
<td>204 ± 6</td>
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<tr>
<td>LQCD [15]</td>
<td>218.9 ± 11.3</td>
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<tr>
<td>LFQM [16]</td>
<td>205.8 ± 8.9</td>
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<tr>
<td>BSM [17]</td>
<td>230 ± 25</td>
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Table 3: The decay constant $f_\rho$ of $D_s$ systems (in MeV).

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<tr>
<td>Present</td>
<td>235.18</td>
<td>203.34</td>
<td>184.40</td>
<td>170.82</td>
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<tr>
<td>Expt./PDG [1]</td>
<td>260.0 ± 5.4</td>
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<tr>
<td>QCDSR [14]</td>
<td>246 ± 6</td>
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<tr>
<td>LQCD [15]</td>
<td>260.1 ± 10.8</td>
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<td>LFQM [16]</td>
<td>264.5 ± 17.5</td>
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<tr>
<td>BSM [17]</td>
<td>248 ± 27</td>
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$$A(p) = \frac{(E_{p_1} + m_{q_1})(E_{p_2} + m_{q_2}) - p^2}{[E_{p_1}E_{p_2}(E_{p_1} + m_{q_1})(E_{p_2} + m_{q_2})]^{3/2}} \text{ and } E_{p_i} = \sqrt{p^2 + m^2}$$

The computed values are listed in Table 2 and 3.

4. The branching ratio of the leptonic decays of $D$ and $D_s$ mesons

Charged mesons produced from a quark and anti-quark can decay to charged lepton pair via a virtual $W^\pm$ boson. The theoretical predications are very clean due to the absence of hadrons in the final state. The total leptonic width of $D$ and $D_s$ mesons are given by [18, 10, 11]

$$\Gamma(D^+_q \rightarrow l^+ \nu_l) = \frac{G_F^2}{8\pi} f_{D_q}^2 |V_{cq}|^2 m_l^2 \left(1 - \frac{m_l^2}{M_{D_q}^2}\right)^2 M_{D_q} \text{ where } q = s, d, u.$$  \hspace{1cm} (4.1)

where Fermi coupling constant ($G_F$) is $1.664 \times 10^{-5}$ and the relevant CKM parameters are taken from PDG [1] as $V_{cs} = 1.006$ and $V_{cd} = 0.230$. The branching ratios of the total leptonic widths are obtained as $BR = \Gamma(D^+_q \rightarrow l^+ \nu_l) \times \tau$, where the lifetime $\tau$ of the $D$ and $D_s$ mesons are taken from PDG [1]. The results are shown in Table 4.

Table 4: The leptonic Branching Ratio (BR) of $D$ and $D_s$ mesons.

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<td>(keV)</td>
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<tr>
<td>$D_s \rightarrow \tau \nu_\tau$</td>
<td>$13.258 \times 10^{-8}$</td>
<td>$6.090 \times 10^{-8}$</td>
<td>$5.844 \times 10^{-2}$</td>
<td>$4.3 \times 10^{-2}$</td>
<td>$5.43 \times 10^{-2}$</td>
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<tr>
<td>$D_s \rightarrow \mu \nu_\mu$</td>
<td>$13.469 \times 10^{-9}$</td>
<td>$6.240 \times 10^{-9}$</td>
<td>$5.937 \times 10^{-3}$</td>
<td>$4.41 \times 10^{-3}$</td>
<td>$5.90 \times 10^{-3}$</td>
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<tr>
<td>$D_s \rightarrow e \bar{\nu}_e$</td>
<td>$3.157 \times 10^{-13}$</td>
<td>-</td>
<td>$1.391 \times 10^{-7}$</td>
<td>-</td>
<td>$&lt; 1.2 \times 10^{-4}$</td>
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<tr>
<td>$D^+ \rightarrow \tau \nu_\tau$</td>
<td>$15.288 \times 10^{-10}$</td>
<td>$4.720 \times 10^{-10}$</td>
<td>$2.433 \times 10^{-3}$</td>
<td>$7.54 \times 10^{-4}$</td>
<td>$&lt; 1.2 \times 10^{-3}$</td>
</tr>
<tr>
<td>$D^+ \rightarrow \mu \nu_\mu$</td>
<td>$5.641 \times 10^{-10}$</td>
<td>$1.795 \times 10^{-10}$</td>
<td>$8.977 \times 10^{-4}$</td>
<td>$2.87 \times 10^{-4}$</td>
<td>$3.82 \times 10^{-4}$</td>
</tr>
<tr>
<td>$D^+ \rightarrow e \bar{\nu}_e$</td>
<td>$1.323 \times 10^{-14}$</td>
<td>-</td>
<td>$2.105 \times 10^{-8}$</td>
<td>-</td>
<td>$&lt; 8.8 \times 10^{-6}$</td>
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5. Results and Discussions

The predicted masses of $S$-wave of $D_s$ meson state $2^3S_1$ (2716 MeV) and $2^1S_0$ (2633 MeV) are in very good agreement with experimental results $2710^{+12}_{-7}$ MeV [1] and 2638 MeV [12] respectively. $2^3S_1$ (2606 MeV) state and $2^1S_0$ (2523 MeV) of $D$ meson are in good agreement with experimental [1] values of 2608 MeV and 2539 MeV reported by BABAR [4]. The expected results of other $S$-wave excited states of $D$ and $D_s$ meson are in accordance with the other results [8, 9]. The decay constant of $D$ meson 216 MeV and $D_s$ meson 235 MeV are also in agreement with the LQCD [15] result and with the QCD sum rule result [14] respectively. The predicted leptonic branching ratios of the $D_s$ meson are in excellent agreement with the experimental results over other theoretical predictions [19].

6. Acknowledgements

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