# The role of radially excited charmed mesons in semileptonic $B$ decays 

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We study the $B$ meson semileptonic decays into the first radial excitation of the $D^{(*)}$ mesons. Contrary to recent suggestions, we find the branching ratios are too small to explain the present discrepancy between the inclusive branching fraction and the sum of the different exclusive channels for $b \rightarrow c$ semileptonic decay of the $B$ meson.

[^0]
## 1. Introduction

In Ref. [1], the authors argue that the present discrepancy between the inclusive rate $\mathscr{B}\left(B^{+} \rightarrow\right.$ $X_{c} l^{+} v_{l}$ ) and the sum of the different exclusive channels could be explained by a large decay rate to the first radially excited $D^{\prime}$ and $D^{* *}$ states. Present experimental data give

$$
\begin{equation*}
\mathscr{B}\left(B^{+} \rightarrow X_{c} l^{+} v_{l}\right)-\mathscr{B}\left(B^{+} \rightarrow D^{(*)} l^{+} v_{l}\right)-\mathscr{B}\left(B^{+} \rightarrow D^{(*)} \pi l^{+} v_{l}\right)=(1.45 \pm 0.13) \% . \tag{1.1}
\end{equation*}
$$

and the expectation in Ref. [1] is that $\mathscr{B}\left(B \rightarrow D^{\prime(*)} l^{+} v_{l}\right) \sim \mathscr{O}(1 \%)$.
Evidence for states that could be assigned to the first radial excitation of the $D^{(*)}$ mesons have been found by the BABAR Collaboration [2]. Their results are given in Table 1. The spin-parity of the $D(2550)$ meson has been established to be $J^{\pi}=0^{-}$while for the $D^{*}(2600)$ it is not yet known. The $D^{*}(2600)$ meson decays into $D \pi$ and $D^{*} \pi$ final states, and the helicity-angle distribution of the decay products is consistent with the meson being a $J^{\pi}=1^{-}$state. Besides, its mass is close to the $D^{\prime}$ mass, which makes it a perfect candidate to be its spin partner. Those mesons are naturally

|  | $J^{\pi}$ | Mass (MeV) | Width (MeV) |
| :--- | :---: | :---: | :---: |
| $D(2550)^{0}$ | $0^{-}$ | $2539.4 \pm 4.5 \pm 6.8$ | $130 \pm 12 \pm 13$ |
| $D^{*}(2600)^{0}$ | $?^{?}$ | $2608.7 \pm 2.4 \pm 2.5$ | $93 \pm 6 \pm 13$ |

Table 1: $D^{\prime(*)}$ resonances as measured by the BABAR Collaboration [2].
explained within the quark model as being $2^{1} S_{0}$ and $2^{3} S_{1}$ states.
In Ref. [3] we have studied, in a constituent quark model framework, the semileptonic $B$ decays into the $D(2550)^{0}$ and $D^{*}(2600)^{0}$ mesons assuming the latter are $2 S$ states. To obtain their masses and wave functions we have used the quark model potential of Ref. [4], that provides a general good description of the light and heavy-light meson spectrum. The results for the masses are given in Table 2. The $B$ meson mass is well reproduced but the masses of the $D^{\prime(*)}$ mesons are larger

|  | Mass (MeV) | Mass $_{\text {Exp. }}(\mathrm{MeV})$ |
| :--- | :---: | :---: |
| $B$ | 5275 | $5279.42 \pm 0.17[5]$ |
| $D(2550)^{0}(2 S)$ | 2700 | $2539.4 \pm 4.5 \pm 6.8 \quad[2]$ |
| $D^{*}(2600)^{0}(2 S)$ | 2750 | $2608.7 \pm 2.4 \pm 2.5 \quad[2]$ |

Table 2: Meson masses obtained with the quark model of Ref. [4] compared to experiment.
by some 150 MeV . On the other hand their mass difference agrees with experiment within errors. While the masses of the $D^{\prime(*)}$ mesons are a little bit too large we expect their wave functions to be reasonably good. A test of their goodness was conducted in Ref. [6] by studying the $D^{(*)}$ mesons strong decays. In the calculation physical masses were used in order to get the phase space right. The results, shown in Table 3, are in very good agreement with data by the BABAR Collaboration. In our evaluation of $B \rightarrow D^{\prime(*)}$ semileptonic decays done in Ref. [3] we also used physical masses. This calculation we discuss in the following.

|  | $\Gamma(\mathrm{MeV})$ | $\Gamma_{\text {Exp. }}(\mathrm{MeV})[2]$ |
| :--- | :---: | :---: |
| $D(2550)^{0}$ | 132.07 | $130 \pm 13 \pm 13$ |
| $D^{*}(2600)^{0}$ | 96.91 | $93 \pm 6 \pm 13$ |

Table 3: Strong decay widths obtained with the $3 P 0$ model in Ref.[6] as compared to data.

## 2. $B \rightarrow D^{\prime(*)}$ semileptonic decays

The hadronic matrix elements of the weak decays can be written in terms of form factor as

$$
\begin{aligned}
& \frac{\left\langle D^{\prime}\left(p^{\prime}\right)\right| \bar{\Psi}_{c}(0) \gamma^{\mu}\left(1-\gamma_{5}\right) \Psi_{b}(0)|B(p)\rangle}{\sqrt{m_{B} m_{D^{\prime}}}}=h_{+}(w)\left(v+v^{\prime}\right)^{\mu}+h_{-}(w)\left(v-v^{\prime}\right)^{\mu} \\
& \frac{\left\langle D^{\prime *}\left(p^{\prime}\right)\right| \bar{\Psi}_{c}(0) \gamma^{\mu}\left(1-\gamma_{5}\right) \Psi_{b}(0)|B(p)\rangle}{\sqrt{m_{B} m_{D^{\prime *}}}}=h_{V}(w) \varepsilon^{\mu v \alpha \beta} \varepsilon_{v}^{*} v_{\alpha}^{\prime} v_{\beta} \\
&-i\left[-h_{A_{1}}(w)(w+1) \varepsilon^{* \mu}+h_{A_{2}}(w)\left(\varepsilon^{*} \cdot v\right) v^{\mu}+h_{A_{3}}(w)\left(\varepsilon^{*} \cdot v\right) v^{\prime \mu}\right]
\end{aligned}
$$

Now, neglecting lepton masses, one has for the semileptonic decay width

$$
\begin{aligned}
\frac{d \Gamma}{d w}\left(B^{+} \rightarrow D^{\prime} l^{+} v_{l}\right)= & \frac{G_{F}^{2}\left|V_{c b}\right|^{2} m_{B}^{5}}{48 \pi^{3}}\left(w^{2}-1\right)^{3 / 2} r^{3}(1+r)^{2} G^{2}(w) \\
\frac{d \Gamma}{d w}\left(B^{+} \rightarrow D^{\prime *} l^{+} v_{l}\right)= & \frac{G_{F}^{2}\left|V_{c b}\right|^{2} m_{B}^{5}}{48 \pi^{3}}\left(w^{2}-1\right)^{3 / 2}(w+1)^{2} r^{* 3}\left(1-r^{*}\right)^{2} \\
& \times\left[1+\frac{4 w}{w+1} \frac{1-2 w r^{*}+r^{* 2}}{\left(1-r^{*}\right)^{2}}\right] F^{2}(w)
\end{aligned}
$$

where $w=v \cdot v^{\prime}$ is the product of four-velocities of the two mesons, $r^{(*)}=m_{D^{\prime(*)}} / m_{B}$, and the $F$ and $G$ functions are given by

$$
\begin{aligned}
G(w)= & h_{+}(w)-\frac{1-r}{1+r} h_{-}(w) \\
F^{2}(w)= & \left\{\left(1-r^{*}\right)^{2}+\frac{4 w}{w+1}\left(1-2 w r^{*}+r^{* 2}\right)\right\}^{-1}\left\{2\left(1-2 w r^{*}+r^{* 2}\right)\left[h_{A_{1}}^{2}(w)+\frac{w-1}{w+1} h_{V}^{2}(w)\right]\right. \\
& \left.+\left[\left(1-r^{*}\right) h_{A_{1}}(w)+(w-1)\left(h_{A_{1}}(w)-h_{A_{3}}(w)-r^{*} h_{A_{2}}(w)\right)\right]^{2}\right\}
\end{aligned}
$$

To evaluate the form factors we followed Ref. [7].
In the limit of infinitely heavy quark masses, heavy quark symmetry (HQS) predicts [8]

$$
h_{-}(w)=h_{A_{2}}(w)=0, \quad h_{+}(w)=h_{V}(w)=h_{A_{1}}(w)=h_{A_{3}}(w)=G(w)=F(w)=\xi(w)
$$

with $\xi(w)$ the Isgur-Wise function. Besides, at zero recoil one should have in the equal mass limit ( $m_{b}=m_{c}$ ) that $\xi(1)=1$ for decays into $D^{(*)}$ mesons and $\xi(1)=0$ for decays into $D^{\prime(*)}$ mesons. These zero recoil values are determined by the overlap of the initial and final meson wave functions. Deviations from the above are expected for finite and unequal heavy quark masses. The form factors we obtained are shown in Fig. 1. As seen in the figure the deviations from HQS predictions are more important in our case for decays into $D^{\prime(*)}$. In Fig. 2 we show the $F$ and


Figure 1: Form factor for the semileptonic decays $B \rightarrow D^{(*)}$ (left) and $B \rightarrow D^{\prime(*)}$ (right).


Figure 2: $F$ and $G$ functions for the $B \rightarrow D^{\prime(*)}$ decay. We also show results at zero recoil and maximum recoil taken from Refs. [9] and [1] respectively.
$G$ functions for $B \rightarrow D^{\prime(*)}$ decays. In the same figure we show values at maximum recoil taken from Ref. [1], where they were evaluated using light cone sum rules (LCSR), and at zero recoil taken from the relativistic calculation of Ref. [9]. We see our values at zero recoil deviate from the results in Ref. [9] which indicates different overlaps of the wave functions in the two calculations. At maximum recoil our results are also much smaller than the ones obtained in Ref. [1] using LCSR, although the big uncertainties in the latter does not allow to be very conclusive.

In Table 4 we show our final result for the branching ratios $\mathscr{B}\left(B \rightarrow D^{\prime(*)}\right)$. The uncertainties were obtained by varying the parameters of the quark model within $10 \%$ of their central value. The total branching ratio of $0.11 \pm 0.02 \%$ is too small to explain the discrepancy mentioned in the introduction and contradicts the expectations in Ref. [1].

Small results were also obtained in the calculations of Refs. [10, 11]. In Ref. [10] the authors used heavy quark effective theory and they evaluated the Isgur-Wise functions in the quark model using Gaussian wave functions. In Ref. [11] the authors used an updated version of the Isgur-Scora-Grinstein-Wise quark model [12] into which they included, among other things, HQS constraints. These works predict respectively $0.065 \%$ and $0.17 \%$ for the total branching ratio. Another common

|  | $(\%)$ |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | This work | $[10]$ | $[11]$ | $[9]$ |
| $B^{+} \rightarrow D^{\prime 0} l^{+} v_{l}$ | $0.012 \pm 0.006$ | 0.019 | 0 | 0.22 |
| $B^{+} \rightarrow D^{* 0} l^{+} v_{l}$ | $0.097 \pm 0.015$ | 0.046 | 0.17 | 0.18 |
| Sum | $0.11 \pm 0.02$ | 0.065 | 0.17 | 0.40 |

Table 4: Branching ratios, in $\%$, for $B \rightarrow D^{\prime(*)}$ semileptonic decays.
feature of these two calculations as well as ours is that $\mathscr{B}\left(B \rightarrow D^{\prime(*)}\right) \gg \mathscr{B}\left(B \rightarrow D^{\prime}\right)$. A much larger value of $0.4 \%$ for the total branching ratio is obtained in the relativistic calculation of ref. [9]. However, it is still a factor of 2-3 too small to fully explain the discrepancy. It is worth noting that in this case the branching ratios for both decays are rather similar being $\mathscr{B}\left(B \rightarrow D^{\prime}\right)$ the largest of the two. Our calculation as well as the ones in Refs. [10, 11] use impulse approximation, in which the light quark is just a spectator. In Ref. [9] they also included two body exchange current operators. It would be interesting to see how much our results would be affected by the inclusion of those terms.

We can summarize the present contribution by saying that, contrary to expectations in Ref. [1], present calculations of the branching ratios for $B$ decay into the first radial excitation of $D^{(*)}$ indicate that they are not enough to saturate the inclusive decay rate.

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