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Mixing angle of K_1 axial vector mesons

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Analyses of various experimental measurements all indicate that the mixing angle θ_{K_1} of $K_1(1270)$ and $K_1(1400)$ is in the vicinity of 33° or 57°. However, whether θ_{K_1} is greater or less than 45° is still quite controversial. For example, there were two very recent studies of the strong decays of K_1 mesons. One group claimed that $\theta_{K_1} \approx 60^\circ$, while the other group obtained $\theta_{K_1} = (33.6 \pm 4.3)^\circ$. Since the determination of the mixing angles $\alpha_{^3P_1}$ and $\alpha_{^1P_1}$ with the former (latter) being the mixing angle of $f_1(1285)$ $(h_1(1170))$ and $f_1(1420)$ $(h_1(1380))$ in the flavor basis through mass relations depends on θ_{K_1}, we show that $\theta_{K_1} \approx 57^\circ$ is ruled out as it leads to a too large deviation from ideal mixing in the $^1P_1$ sector, inconsistent with the lattice calculation of $\alpha_{^1P_1}$ and the observation of strong decays of $h_1(1170)$ and $h_1(1380)$. We find that for $\theta_{K_1} \approx (28-30)^\circ$, the corresponding $\alpha_{^3P_1}$ and $\alpha_{^1P_1}$ agree well with all lattice and phenomenological analyses. This again reinforces the statement that $\theta_{K_1} \sim 33^\circ$ is much more favored than 57°.

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1. Introduction

The mixing of the flavor-SU(3) singlet and octet states of vector and tensor mesons to form mass eigenstates is of fundamental importance in hadronic physics. According to the Appelquist-Carazzone decoupling theorem, in a vectorial theory, as the mass of a particle gets large compared with a relevant scale, say, $\Lambda_{QCD} \simeq 300$ MeV, one can integrate this particle out and define a low-energy effective field theory applicable below this scale [1]. Evidently, even though m_s is not $\gg \Lambda_{QCD}$, there is still a nearly complete decoupling for the case of vector mesons, namely, $\rho(770)$ and $\omega(892)$ states. A similar situation of near-ideal mixing occurs for the $J^{PC} = 2^{++}$ tensor mesons $f_2(1275)$, $f_2'(1525)$ and the $J^{PC} = 3^{--}$ mesons $\omega_3(1670)$, $\phi_3(1850)$ and this can also be understood in terms of approximate decoupling of the light $u\bar{u} + d\bar{d}$ state from the heavier $s\bar{s}$ state.

In the quark model, two nonets of $J^P = 1^+$ axial-vector mesons are expected as the orbital excitation of the $q\bar{q}$ system. In terms of the spectroscopic notation ${}^{2S+1}L_J$, there are two types of P-wave axial-vector mesons, namely, ${}^3P_1$ and ${}^1P_1$. These two nonets have distinctive C quantum numbers for the corresponding neutral mesons, $C = +$ and $C = -$, respectively. Experimentally, the $J^{PC} = 1^{++}$ nonet consists of $a_1(1260)$, $f_1(1285)$, $f_1(1420)$ and K_{1A}, while the 1^{+-} nonet contains $b_1(1235)$, $h_1(1170)$, $h_1(1380)$ and K_{1B}. The non-strange axial vector mesons, for example, the neutral $a_1(1260)$ and $b_1(1235)$ cannot have a mixing because of the opposite C-parities. On the contrary, K_{1A} and K_{1B} are not the physical mass eigenstates $K_1(1270)$ and $K_1(1400)$ and they are mixed together due to the mass difference of strange and light quarks. Following the common convention we write

$$\begin{pmatrix} |K_1(1270)\rangle \\ |K_1(1400)\rangle \end{pmatrix} = \begin{pmatrix} \sin\theta_{K_1} & \cos\theta_{K_1} \\ \cos\theta_{K_1} & -\sin\theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_{1A}\rangle \\ |K_{1B}\rangle \end{pmatrix} . \tag{1.1}$$

Various phenomenological studies indicate that the K_{1A}-K_{1B} mixing angle θ_{K_1} is around either 33° or 57°, [1] but there is no consensus as to whether this angle is greater or less than 45°.

We have shown in [2] that the mixing angle θ_{K_1} can be pinned down based on the observation that when the $f_1(1285)$-$f_1(1420)$ mixing angle $\theta_{{}^3P_1}$ and the $h_1(1170)$-$h_1(1380)$ mixing angle $\theta_{{}^1P_1}$ are determined from the mass relations, they depend on the masses of K_{1A} and K_{1B}, which in turn depend on θ_{K_1}. Since nearly ideal mixing occurs for vector, tensor and 3^{--} mesons except for pseudoscalar mesons where the axial anomaly plays a unique role, this feature is naively expected to hold also for axial-vector mesons. Lattice calculations of $\theta_{{}^1P_1}$ and the phenomenological analysis of the strong decays of $h_1(1170)$ and $h_1(1380)$ will enable us to discriminate the two different solutions for θ_{K_1}. In this talk we will elaborate on this in more detail.

2. Mixing of axial-vector mesons

There exist several estimations on the mixing angle θ_{K_1} in the literature. From the early experimental information on masses and the partial rates of $K_1(1270)$ and $K_1(1400)$, Suzuki found

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[1] As discussed in [2] and many early publications, the sign ambiguity of θ_{K_1} can be removed by fixing the relative sign of the decay constants of K_{1A} and K_{1B}. We shall choose the convention of decay constants in such a way that θ_{K_1} is always positive.

two possible solutions $\theta_{K_1} \approx 33^\circ$ and 57° [3]. A similar constraint $35^\circ \lesssim \theta_{K_1} \lesssim 55^\circ$ was obtained in Ref. [4] based solely on two parameters: the mass difference between the $a_1(1260)$ and $b_1(1235)$ mesons and the ratio of the constituent quark masses. An analysis of $\tau \to K_1(1270)\nu_\tau$ and $K_1(1400)\nu_\tau$ decays also yielded the mixing angle to be $\approx 37^\circ$ or 58° [5].[2] Another determination of θ_{K_1} comes from the $f_1(1285)$-$f_1(1420)$ mixing angle $\theta_{^3P_1}$ to be introduced shortly below which can be reliably estimated from the analysis of the radiative decays $f_1(1285) \to \phi\gamma, \rho^0\gamma$ [6]. A recent updated analysis yields $\theta_{^3P_1} = (19.4^{+4.5}_{-4.6})^\circ$ or $(51.1^{+4.5}_{-4.6})^\circ$ [7].[3] As we shall see below, the mixing angle $\theta_{^3P_1}$ is correlated to θ_{K_1}. The corresponding θ_{K_1} is found to be $(31.7^{+2.8}_{-2.5})^\circ$ or $(56.3^{+3.9}_{-4.1})^\circ$. Therefore, all the analyses yield a mixing angle θ_{K_1} in the vicinity of either 33° or 57°.

However, there is no consensus as to whether θ_{K_1} is greater or less than 45°. It was found in the non-relativistic quark model that $m^2_{K_{1A}} < m^2_{K_{1B}}$ [10, 11, 12] and hence θ_{K_1} is larger than 45°. Interestingly, θ_{K_1} turned out to be of order 34° in the relativized quark model of [13]. Based on the covariant light-front model [14], the value of 51° was found by the analysis of [15]. From the study of $B \to K_1(1270)\gamma$ and $\tau \to K_1(1270)\nu_\tau$ within the framework of light-cone QCD sum rules, Hatanaka and Yang advocated that $\theta_{K_1} = (34 \pm 13)^\circ$ [16]. There existed two recent studies of strong decays of $K_1(1270)$ and $K_1(1400)$ mesons with different approaches. One group obtained $\theta_{K_1} \approx 60^\circ$ based on the $^3P_0$ quark-pair-creation model for K_1 strong decays [17], while the other group found $\theta_{K_1} = (33.6 \pm 4.3)^\circ$ using a phenomenological flavor symmetric relativistic Lagrangian [18]. In short, there is a variety of different values of the mixing angle cited in the literature. It is the purpose of this work to pin down θ_{K_1}.

We next consider the mixing of the isosinglet 1^3P_1 states, $f_1(1285)$ and $f_1(1420)$, and the 1^1P_1 states, $h_1(1170)$ and $h_1(1380)$ in the quark flavor and octet-singlet bases:

$$\begin{pmatrix} |f_1(1285)\rangle \\ |f_1(1420)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{^3P_1} & \sin\theta_{^3P_1} \\ -\sin\theta_{^3P_1} & \cos\theta_{^3P_1} \end{pmatrix} \begin{pmatrix} |f_1\rangle \\ |f_8\rangle \end{pmatrix} = \begin{pmatrix} \cos\alpha_{^3P_1} & \sin\alpha_{^3P_1} \\ -\sin\alpha_{^3P_1} & \cos\alpha_{^3P_1} \end{pmatrix} \begin{pmatrix} |f_q\rangle \\ |f_s\rangle \end{pmatrix}, \tag{2.1}$$

and

$$\begin{pmatrix} |h_1(1170)\rangle \\ |h_1(1380)\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{^1P_1} & \sin\theta_{^1P_1} \\ -\sin\theta_{^1P_1} & \cos\theta_{^1P_1} \end{pmatrix} \begin{pmatrix} |h_1\rangle \\ |h_8\rangle \end{pmatrix} = \begin{pmatrix} \cos\alpha_{^1P_1} & \sin\alpha_{^1P_1} \\ -\sin\alpha_{^1P_1} & \cos\alpha_{^1P_1} \end{pmatrix} \begin{pmatrix} |h_q\rangle \\ |h_s\rangle \end{pmatrix}, \tag{2.2}$$

where $f_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$, $f_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$, $f_q = (u\bar{u} + d\bar{d})/\sqrt{2}$, $f_s = s\bar{s}$ and likewise for h_1, h_8, h_q and h_s. The mixing angle α in the flavor basis is related to the singlet-octet mixing angle θ by the relation $\alpha = 35.3^\circ - \theta$. Therefore, α measures the deviation from ideal mixing. Applying the Gell-Mann Okubo relations for the mass squared of the octet states

$$m_8^2(^3P_1) \equiv m^2_{^3P_1} = \frac{1}{3}(4m^2_{K_{1A}} - m^2_{a_1}), \qquad m_8^2(^1P_1) \equiv m^2_{^1P_1} = \frac{1}{3}(4m^2_{K_{1B}} - m^2_{b_1}), \tag{2.3}$$

we obtain the following mass relations for the mixing angles $\theta_{^1P_1}$ and $\theta_{^3P_1}$ (for details, see [2])

$$\tan\theta_{^3P_1} = \frac{m^2_{^3P_1} - m^2_{f_1'}}{\sqrt{m^2_{^3P_1}(m^2_{f_1} + m^2_{f_1'} - m^2_{^3P_1}) - m^2_{f_1} m^2_{f_1'}}},$$

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[2] Note that the mixing angle results in [5] based on CLEO [8] and OPEL [9] data differ from the the ones obtained in the CLEO paper [8].

[3] From the same radiative decays, it was found $\theta_{^3P_1} = (56^{+4}_{-5})^\circ$ in [6]. This has led some authors (e.g. [10]) to claim that $\theta_{K_1} \sim 59^\circ$. However, another solution, namely, $\theta_{^3P_1} = (14.6^{+4}_{-5})^\circ$ corresponding to a smaller θ_{K_1}, was missed in [6].

Table 1: The values of the $f_1(1285)$-$f_1(1420)$ and $h_1(1170)$-$h_1(1380)$ mixing angles in the quark flavor (upper) and octet-singlet (lower) bases calculated using Eq. (2.4) for some representative K_{1A}-K_{1B} mixing angle θ_{K_1}.

θ_{K_1}	57°	51°	45°	33°	30°	28°
$\alpha_{^3P_1}$	16.5°	9.6°	2.4°	-13.7°	-18.9°	-23.5°
$\alpha_{^1P_1}$	-53.0°	-44.6°	-21.1°	-6.4°	-3.8°	-2.4°
$\theta_{^3P_1}$	52°	45°	38°	22°	16°	12°
$\theta_{^1P_1}$	-18°	-9°	14°	29°	32°	33°

$$\tan\theta_{^1P_1} = \frac{m^2_{^1P_1} - m^2_{h'_1}}{\sqrt{m^2_{^1P_1}(m^2_{h_1} + m^2_{h'_1} - m^2_{^1P_1}) - m^2_{h_1} m^2_{h'_1}}}\,, \tag{2.4}$$

where f_1 and f'_1 (h_1 and h'_1) are the short-handed notations for $f_1(1285)$ and $f_1(1420)$ ($h_1(1170)$ and $h_1(1380)$), respectively, and

$$\begin{aligned} m^2_{K_{1A}} &= m^2_{K_1(1400)}\cos^2\theta_{K_1} + m^2_{K_1(1270)}\sin^2\theta_{K_1}\,, \\ m^2_{K_{1B}} &= m^2_{K_1(1400)}\sin^2\theta_{K_1} + m^2_{K_1(1270)}\cos^2\theta_{K_1}\,. \end{aligned} \tag{2.5}$$

It is clear that the mixing angles $\theta_{^3P_1}$ and $\theta_{^1P_1}$ depend on the masses of K_{1A} and K_{1B} states, which in turn depend on the K_{1A}-K_{1B} mixing angle θ_{K_1}. Table 1 exhibits the values of $\alpha_{^3P_1}$, $\theta_{^3P_1}$ and $\alpha_{^1P_1}$, $\theta_{^1P_1}$ calculated using Eq. (2.4) for some representative values of θ_{K_1}.

3. Discussion

We see from Table 1 that the K_{1A}-K_{1B} mixing angle $\theta_{K_1} \approx 57^\circ$ corresponds to $\alpha_{^1P_1} = -53^\circ$ which is too far away from ideal mixing for the $^1P_1$ sector. Indeed, it is in violent disagreement with the lattice result $\alpha_{^1P_1} = \pm(3\pm1)^\circ$ obtained by the Hadron Spectrum Collaboration [19]. Since only the modes $h_1(1170) \to \rho\pi$ and $h_1(1380) \to K\bar{K}^*, \bar{K}K^*$ have been seen so far, this implies that the quark content is primarily $s\bar{s}$ for $h_1(1380)$ and $q\bar{q}$ for $h_1(1170)$. Indeed, if $\theta_{K_1} = 57^\circ$, we will have $h_1(1170) = 0.60n\bar{n} - 0.80s\bar{s}$ and $h_1(1380) = 0.80n\bar{n} + 0.60s\bar{s}$ with $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$. It is obvious that the large $s\bar{s}$ content of $h_1(1170)$ and $n\bar{n}$ content of $h_1(1380)$ cannot explain why only the strong decay modes $h_1(1170) \to \rho\pi$ and $h_1(1380) \to K\bar{K}^*, \bar{K}K^*$ have been seen thus far. Therefore, it is evident that $\theta_{K_1} \approx 57^\circ$ is ruled out.

Can we conclude that θ_{K_1} is less than 45° ? Let's examine the mixing angle $\alpha_{^3P_1}$. There are some information available. First, the radiative decay $f_1(1285) \to \phi\gamma$ and $\rho\gamma$ yields $\alpha_{^3P_1} = \pm(15.8^{+4.5}_{-4.6})^\circ$ [7]. An updated lattice calculation gives $\alpha_{^3P_1} = \pm(27\pm2)^\circ$ [20]. A study of $B_{d,s} \to J/\psi f_1(1285)$ decays by LHCb leads to $\alpha_{^3P_1} = \pm(24.0^{+3.1+0.6}_{-2.6-0.8})^\circ$ [21]. Hence, $\alpha_{^3P_1}$ lies in the range $\pm(15\sim27)^\circ$. Unlike the $^1P_1$ sector, the deviation of $f_1(1285)$-$f_1(1420)$ mixing from the ideal one is sizable. Nevertheless, the quark content is still primarily $s\bar{s}$ for $f_1(1420)$ and $q\bar{q}$ for $f_1(1285)$. Indeed, $K^*\bar{K}$ and $K\bar{K}\pi$ are the dominant modes of $f_1(1420)$ whereas $f_1(1285)$ decays mainly to the $\eta\pi\pi$ and 4π states. It is clear from from Table 1 that when $\theta_{K_1} \approx (28-30)^\circ$, the corresponding

$\alpha_{^3P_1}$ and $\alpha_{^1P_1}$ agree well with all lattice and phenomenological analyses. This in turn reinforces the statement that $\theta_{K_1} \sim 33^\circ$ is much more favored than 57°.

Two remarks are in order: (i) The K_1 mixing angle $\theta_{K_1} \approx 57^\circ$ leads to acceptable $\alpha_{^3P_1}$ but too large $\alpha_{^1P_1}$. (ii) In the octet-singlet basis, the mixing angles are of order $\theta_{^3P_1} \sim 15^\circ$ and $\theta_{^1P_1} \sim 32^\circ$.

4. Conclusions

The K_1 mixing angle $\theta_{K_1} \approx 57^\circ$ is ruled out as it will lead to a too large deviation from ideal mixing in the $^1P_1$ sector, inconsistent with the observation of strong decays of $h_1(1170)$ and $h_1(1380)$ and a recent lattice calculation of $\theta_{^1P_1}$. We found when $\theta_{K_1} \approx (28-30)^\circ$, the corresponding $\alpha_{^3P_1}$ and $\alpha_{^1P_1}$ agree well with all lattice and phenomenological analyses. This again implies that $\theta_{K_1} \sim 33^\circ$ is much more favored than 57°.

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