Pion-assisted \( N\Delta \) and \( \Delta\Delta \) dibaryons

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\( N\Delta \) and \( \Delta\Delta \) dibaryon candidates are discussed and related quark-based calculations are reviewed. New hadronic calculations of \( L = 0 \) nonstrange dibaryon candidates are reported. For \( N\Delta \), \( I(J^P) = 1(2^+) \) and \( 2(1^+) \) \( S \)-matrix poles slightly below threshold are found by solving \( \pi NN \) Faddeev equations with relativistic kinematics, and for \( \Delta\Delta \) several \( S \)-matrix poles below threshold are found by solving \( \pi N\Delta \) Faddeev equations with relativistic kinematics in which the \( N\Delta \) interaction is dominated by the \( 1(2^+) \) and \( 2(1^+) \) resonating channels. In particular, the \( I(J^P) = 0(3^+) \) \( \Delta\Delta \) dibaryon candidate \( \Omega_2(2370) \) observed recently by the WASA@COSY Collaboration is naturally explained in terms of long-range physics dominated by pions, nucleons and \( \Delta \)'s. These calculations are so far the only ones to reproduce the relatively small width \( \approx 70 \text{ MeV} \) of \( \Omega_2(2370) \). Predictions are also made for the location and width of \( \Omega_3 \), the \( I(J^P) = 3(0^+) \) exotic partner of \( \Omega_2(2370) \).

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1. Introduction

QCD-motivated studies of six-quark (6q) dibaryons started with Jaffe’s prediction of the deeply bound $uuddss \ I(J^P) = 0(0^+) \ H$ dibaryon [1] using the color-magnetic (CM) one-gluon exchange interaction $V_{CM} = \sum_{<j} - (\lambda_i \cdot \lambda_j) (s_i \cdot s_j) v(r_{ij})$, where $v(r_{ij})$ is a flavor conserving $qq$ short-range potential which for a totally symmetric $L = 0$ wavefunction is approximated by its matrix element $\mathcal{M}_0$. From the $\Delta - N$ mass difference of $\approx 300 \text{ MeV}$, one estimates $\mathcal{M}_0 \sim 75 \text{ MeV}$. Leading dibaryon candidates for strangeness $\mathcal{F}$ ranging from $0$ to $-3$ are listed in Table 1, where $\delta < V_{CM}$ is the contribution of $V_{CM}$ to the dibaryon mass with respect to its contribution to the dibaryon’s constituents $B$ and $B'$. Dibaryon candidates with quarks heavier than $s$ are not covered here.

<table>
<thead>
<tr>
<th>$\mathcal{F}$</th>
<th>SU(3)$_t$</th>
<th>$I$</th>
<th>$J^P$</th>
<th>$BB'$ structure</th>
<th>$\delta &lt; V_{CM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>[3,3,0]</td>
<td>0</td>
<td>3+</td>
<td>$\Delta\Delta$</td>
<td>0</td>
</tr>
<tr>
<td>$-1$</td>
<td>[3,2,1]</td>
<td>8</td>
<td>2+</td>
<td>$\sqrt{1/5} \ [N\Sigma^* + 2\Delta\Sigma]$</td>
<td>$-\mathcal{M}_0$</td>
</tr>
<tr>
<td>$-2$</td>
<td>[2,2,2]</td>
<td>1</td>
<td>0+</td>
<td>$\sqrt{1/8} \ [\Lambda\Lambda + 2N\Xi - \sqrt{3}\Sigma\Xi]$</td>
<td>$-2\mathcal{M}_0$</td>
</tr>
<tr>
<td>$-3$</td>
<td>[3,2,1]</td>
<td>8</td>
<td>2+</td>
<td>$\sqrt{1/5} \ [\sqrt{2}N\Omega - (\Lambda\Xi^* - \Sigma^<em>\Xi + \Sigma\Xi^</em>)]$</td>
<td>$-3\mathcal{M}_0$</td>
</tr>
</tbody>
</table>

Table 1: Leading quark-based $L = 0$ dibaryon candidates, adapted from Ref. [2].

Let’s comment on the two extreme cases in the table: $\mathcal{F} = -2$ and $\mathcal{F} = 0$. For $\mathcal{F} = -2$, the table suggests that the listed $H$ dibaryon is deeply bound, located well below the $\Lambda\Lambda$ lowest particle-stability threshold, but in fact SU(3)$_t$ breaking effects abort its anticipated stability, as concluded recently from chiral extrapolations of lattice QCD calculations [3]; see also [4] for arguments based on hypernuclear phenomenology. For $\mathcal{F} = 0$, in contrast, the table suggests no outstanding nonstrange dibaryon candidate resulting from the quark-based CM interaction. However, $N\Delta$ and $\Delta\Delta$ $s$-wave dibaryon resonances $\mathcal{D}_{15}$ with isospin $I$ and spin $S$ were proposed as early as 1964, when quarks were still perceived as merely mathematical entities, by Dyson and Xuong [5] who focused on the lowest-dimension SU(6) multiplet in the $56 \times 56$ product that contains the SU(3) $\mathcal{T}^0$ and 27 multiplets in which the deuteron $\mathcal{D}_{01}$ and $NN$ virtual state $\mathcal{D}_{10}$ are classified. This yields two dibaryon candidates, $\mathcal{D}_{12}$ for $N\Delta$ and $\mathcal{D}_{03}$ for $\Delta\Delta$ listed in Table 2 with masses $M = A + B[I(I + 1) + S(S + 1) - 2]$ in terms of constants $A, B$. Identifying $A$ with the $NN$ threshold mass 1878 MeV, $B \approx 47$ MeV was determined by assigning $\mathcal{D}_{12}$ to the $pp \leftrightarrow \pi^+d$ reaction channels resonating at 2160 MeV near the $N\Delta$ threshold. This led to a predicted mass value of 2350 MeV for $\mathcal{D}_{03}$. The $\mathcal{D}_{03}$ dibaryon was the subject of many quark-based model calculations since 1980.

<table>
<thead>
<tr>
<th>$\mathcal{D}_{15}$</th>
<th>$\mathcal{D}_{01}$</th>
<th>$\mathcal{D}_{10}$</th>
<th>$\mathcal{D}_{12}$</th>
<th>$\mathcal{D}_{03}$</th>
<th>$\mathcal{D}_{30}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BB'</td>
<td>NN</td>
<td>NN</td>
<td>N\Delta</td>
<td>N\Delta</td>
<td>$\Delta\Delta$</td>
</tr>
<tr>
<td>SU(3)$_t$</td>
<td>$\mathcal{T}^0$</td>
<td>27</td>
<td>27</td>
<td>35</td>
<td>$\mathcal{T}^0$</td>
</tr>
</tbody>
</table>
| $M(\mathcal{D}_{15})$ | $A$            | $A$            | $A + 6B$        | $A + 6B$        | $A + 10B$       | $A + 10B$

Table 2: SU(6) predictions [5] for nonstrange $L = 0$ dibaryons $\mathcal{D}_{15}$ with isospin $I$ and spin $S$.

$\mathcal{D}_{03}$ mass predictions are listed in Table 3 for several representative approaches. The table exhibits a broad range of calculated $\mathcal{D}_{03}$ masses. Except for the Dyson-Xuong pioneering prediction
Table 3: Quark-based model predictions of $D_{03}$ and $D_{12}$, except where denoted by asterisks: * denotes post-experiment $D_{03}$ calculation and ** denotes input from experiment. Experimental evidence for $D_{03}(2370)$ is shown in the figure below. The $N\Delta$ and $\Delta\Delta$ thresholds are at 2.17 and 2.46 GeV, respectively.

none of those confronting $D_{03}$ and $D_{12}$ succeeded to correctly reproduce both. Recent experimental evidence for $D_{03}$ is displayed in Fig. 1–left. Isospin $I = 0$ is uniquely fixed in this two-pion production reaction and a spin-parity $3^+$ assignment follows from the measured deuteron and pions angular distributions, assuming $s$-wave decaying $\Delta\Delta$ pair. The peak of the $M_{d\pi}$ distribution on the right panel at $\sqrt{s} \approx 2.13$ GeV, almost at the $D_{12} N\Delta$ dibaryon peak, suggests that $D_{12}$ plays a role in forming the $\Delta\Delta$ dibaryon $D_{03}$. It is shown below that the pion-assisted methodology applied by us recently [18, 19] couples the two dibaryons dynamically in a more natural way than appears in quark-based models. Our calculations emphasize the long-range physics aspects of nonstrange dibaryons, as described briefly in the next section.

Figure 1: $D_{03}(2370) \Delta\Delta$ dibaryon signal on the left panel, and its $M_{d\pi}^2$ Dalitz-plot projection on the right panel, from $pn \rightarrow d\pi^0\pi^0$ by WASA-at-COSY [15]. Figures courtesy of Heinz Clement.

2. Pion-assisted nonstrange dibaryons

2.1 $N\Delta$ dibaryons

The $D_{12}$ dibaryon shows up experimentally as $NN(1D_2) \leftrightarrow \pi d(3P_2)$ coupled-channel resonance corresponding to a quasibound $N\Delta$ with mass $M \approx 2.15$ GeV, near the $N\Delta$ threshold, and width $\Gamma \approx 0.12$ GeV [16, 17]. In our recent work [19] we have calculated this dibaryon and other $N\Delta$ dibaryon candidates such as $D_{21}$ (see Table 2) by solving Faddeev equations with relativistic kinematics for the $\pi NN$ three-body system, where the $\pi N$ subsystem is dominated by the $P_{33} \Delta(1232)$ resonance channel and the $NN$ subsystem is dominated by the $3S_1$ and $1S_0$ channels. The coupled Faddeev equations give rise then to an effective $N\Delta$ Lippmann-Schwinger (LS) equation for the three-body $S$-matrix pole, with energy-dependent kernels that incorporate spectator-hadron propagators, as shown diagrammatically in Fig. 2 where circles denote the $N\Delta T$ matrix.
Of the four possible $L = 0$ $N\Delta$ dibaryon candidates $D_{15}$ with $IS = 12, 21, 11, 22$, the latter two do not provide resonant solutions. For $D_{12}$, only $^3S_1$ contributes out of the two $NN$ interactions, while for $D_{21}$ only $^1S_0$ contributes. Since the $^3S_1$ interaction is the more attractive one, $D_{12}$ lies below $D_{21}$ as borne out by the calculated masses listed in Table 4 for two choices of the $P_{33}$ interaction form factor corresponding to spatial sizes of 1.35 fm and 0.9 fm of the $\Delta$ isobar. The two dibaryons are found to be degenerate to within less than 20 MeV. The mass values calculated for $D_{12}$ are reasonably close to the value $W = 2148 - i63$ MeV [16] and $W = 2144 - i55$ MeV [17] derived in coupled-channel phenomenological analyses.

<table>
<thead>
<tr>
<th>$W^&gt; (D_{12})$</th>
<th>$W^&gt; (D_{21})$</th>
<th>$W^&lt; (D_{12})$</th>
<th>$W^&lt; (D_{21})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2147–i60</td>
<td>2165–i64</td>
<td>2159–i70</td>
<td>2169–i69</td>
</tr>
</tbody>
</table>

Table 4: $N\Delta$ dibaryon $S$-matrix poles (in MeV) for $D_{12}$ and $D_{21}$, obtained by solving $\pi NN$ Faddeev equations for two choices of the $\pi N P_{33}$ form factor, with large (small) spatial size denoted $>$ ($<$).

### 2.2 $\Delta\Delta$ dibaryons

Four-body $\pi \pi NN$ calculations are required, strictly speaking, to discuss $\Delta\Delta$ dibaryons. In Ref. [18] we studied the $D_{03}$ dibaryon by solving a $\pi NN'$ three-body model, where $\Delta'$ is a stable $\Delta(1232)$ and the $N\Delta'$ interaction is dominated by the $D_{12}$ dibaryon. The $I(J^P) = 1(2^+) \ N\Delta'$ interaction was not assumed to resonate but, rather, it was fitted within a $NN-\pi NN-N\Delta'$ coupled-channel caricature model to the $NN \ ^1D_2 \ T$-matrix, requiring that the resulting $N\Delta'$ separable-interaction form factor is representative of long-range physics, with momentum-space soft cutoff $\Lambda \lesssim 3$ fm$^{-1}$. The $\pi N$ interaction was again assumed to be dominated by the $P_{33} \ \Delta$ resonance. The Faddeev equations of this three-body model give rise, as before, to an effective LS equation for the $\Delta\Delta'$ $S$-matrix pole corresponding to $D_{03}$. This LS equation is shown diagrammatically in Fig. 3, where $D$ stands for the $D_{12}$ dibaryon. In Ref. [19] we have extended the calculation of $D_{03}$ to other $D_{15} \ \Delta\Delta$
dibaryon candidates, with $D$ now standing for both $N\Delta$ dibaryons $\mathcal{D}_{12}$ and $\mathcal{D}_{21}$. Since $\mathcal{D}_{21}$ is almost degenerate with $\mathcal{D}_{12}$, and with no $NN$ observables to constrain the input $(I,S)=(2,1)$ $N\Delta'$ interaction, the latter was taken the same as for $(I,S)=(1,2)$. The model dependence of this assumption is under study at present. The lowest and also narrowest $\Delta\Delta$ dibaryons found are $\mathcal{D}_{03}$ and $\mathcal{D}_{30}$.

<table>
<thead>
<tr>
<th>$W(\Delta')$</th>
<th>$W^{&gt;}(\mathcal{D}_{03})$</th>
<th>$W^{&gt;}(\mathcal{D}_{30})$</th>
<th>$W^{&lt;}(\mathcal{D}_{03})$</th>
<th>$W^{&lt;}(\mathcal{D}_{30})$</th>
<th>$W_{av}(\mathcal{D}_{03})$</th>
<th>$W_{av}(\mathcal{D}_{30})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1211−i49.5</td>
<td>2383−i47</td>
<td>2412−i49</td>
<td>2342−i31</td>
<td>2370−i30</td>
<td>2363−i39</td>
<td>2391−i39</td>
</tr>
<tr>
<td>1211−i(2/3)49.5</td>
<td>2383−i41</td>
<td>2411−i41</td>
<td>2343−i24</td>
<td>2370−i22</td>
<td>2363−i33</td>
<td>2390−i32</td>
</tr>
</tbody>
</table>

Table 5: $\Delta\Delta$ dibaryon $S$-matrix poles (in MeV) obtained in Refs. [18, 19] by using a spectator-$\Delta'$ complex mass $W(\Delta')$ (first column) in the propagator of the LS equation depicted in Fig. 3. The last two columns give calculated mass and width values averaged over those from the $>$ and $<$ columns, where $>$ and $<$ are defined in the caption of Table 4.

Representative results for $\mathcal{D}_{03}$ and $\mathcal{D}_{30}$ are assembled in Table 5, where the calculated mass and width values listed in each row correspond to the value listed there of the spectator-$\Delta'$ complex mass $W(\Delta')$ used in the propagator of the LS equation shown in Fig. 3. The value of $W(\Delta')$ in the first row is that of the $\Delta(1232)$ $S$-matrix pole. It is implicitly assumed thereby that the decay $\Delta' \rightarrow N\pi$ proceeds independently of the $\Delta \rightarrow N\pi$ isobar decay. However, as pointed out in Ref. [18], care must be exercised to ensure that the decay nucleons and pions satisfy Fermi-Dirac and Bose-Einstein statistics requirements, respectively. Assuming $L=0$ for the decay-nucleon pair, this leads to the suppression factor $2/3$ depicted in the value of $W(\Delta')$ listed in the second row. It is seen that the widths obtained upon applying this width-suppression are only moderately smaller, by less than 15 MeV, than those calculating disregarding this quantum-statistics correlation.

The mass and width values calculated for $\mathcal{D}_{03}$ [18] agree very well with those determined by the WASA-at-COSY Collaboration [15], reproducing in particular the reported width value $\Gamma(\mathcal{D}_{03}) = 68$ MeV which is extremely low with respect to the expectation $\Gamma_\Lambda \leq \Gamma(\mathcal{D}_{03}) \leq 2\Gamma_\Lambda$, with $\Gamma_\Lambda \approx 118$ MeV. No other calculation has succeeded so far to do that. Similar small widths according to Table 5 hold for $\mathcal{D}_{30}$ which is located about 30 MeV above $\mathcal{D}_{03}$. This is about half of the spacing found very recently in the quark-based calculations of Ref. [13]. Note, however, that the widths calculated there are considerably larger than ours. A more complete discussion of these and other $\mathcal{D}_{1S}$ $\Delta\Delta$ dibaryon candidates is found in Ref. [19].

3. Conclusion

It was shown how the 1964 Dyson-Xuong SU(6)-based classification and predictions of non-strange dibaryons [5] are confirmed in our hadronic model of pion-assisted $N\Delta$ and $\Delta\Delta$ dibaryons [18, 19]. The input for dibaryon calculations in this model consists of nucleons, pions and $\Delta$'s, interacting via long-range pairwise interactions. These calculations reproduce the two nonstrange dibaryons established experimentally and phenomenologically so far, the $N\Delta$ dibaryon $\mathcal{D}_{12}$ [16, 17] and the $\Delta\Delta$ dibaryon $\mathcal{D}_{03}$ reported by the WASA-at-COSY Collaboration [15], and also predict an exotic $I=2$ $N\Delta$ dibaryon $\mathcal{D}_{21}$ nearly degenerate with $\mathcal{D}_{12}$. We note that $\mathcal{D}_{12}$ provides in our $\pi N\Delta$ three-body model of $\mathcal{D}_{03}$ a two-body decay channel $\pi \mathcal{D}_{12}$ with threshold lower than $\Delta\Delta$. Our calculations are capable of dealing with other $\Delta\Delta$ dibaryon candidates, in particular the $I=3$ exotic.
$\mathcal{D}_{30}$ highlighted recently by Bashkanov, Brodsky and Clement [20]. These authors emphasized the dominant role that 6q hidden-color configurations might play in binding $\mathcal{D}_{03}$ and $\mathcal{D}_{30}$, but recent explicit quark-based calculations [13] find these configurations to play a marginal role, enhancing dibaryon binding by merely $15\pm5$ MeV. Hidden-color considerations are of course outside the scope of hadronic models and it is gratifying that the results presented here in the hadronic basis are independent of such poorly understood configurations.

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**References**