Possible existence of $K^{\bar{b}ar}$-hyperon resonances and the origin of $K^{\bar{b}ar}$-hyperon attractions

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A flavor-$SU(3)$-symmetric one-hadron-exchange potential model which describes consistently baryon-baryon and meson-baryon interactions is proposed. The short-range part of the potential model is assumed to have a common range and the strengths based on lattice QCD calculations. This potential model is extended to meson-baryon potentials in strangeness $S=-2$ and $-3$ sectors. We find strongly attractive $K^{\bar{b}ar}$-hyperon $S$-wave interactions and possible $S$-wave resonances, corresponding to $\Xi^*(I = 1/2, J^P = 1/2^-)$ and $\Omega^*(I = 0, J^P = 1/2^-)$ at around 1.5GeV and 1.8GeV, respectively.
1. Introduction

Theoretical understanding of hadron-hadron (HH) interactions is a longstanding fundamental problem in particle and nuclear physics, since Yukawa’s meson-exchange theory. In recent decades, the HH interactions in the strange sectors ($S \neq 0$) have been intensively discussed both theoretically and experimentally.

In theoretical studies of HH interactions at low energies (up to 1GeV), three types of models, that is, hadron-exchange models, quark models, and the chiral perturbation models have been discussed. Recently, the first-principle calculations based on the lattice QCD (LQCD) have been performed and have provided quantitative results for not only the short-range but also the long-range part of HH potentials[1]. But the physical models, which must clarify the effective degrees of freedom in interaction mechanisms, have not been established yet. We need a theoretical model to understand the physical picture of the HH interactions, under the condition that the model is justified by QCD.

In this paper, firstly, we propose a one-hadron-exchange potential (OHEP) model which describes consistently baryon-baryon (NN, ΛN, ΣN and ΛΛ) interactions and meson-baryon (πN, KN and $K^\text{baru}N$) interactions. This model assumes the flavor-$SU(3)$ symmetric coupling constants and physical meson masses in OHEP[2][3]. For the short-range part of potentials, we introduce the $SU(3)$-symmetric Gaussian soft core potentials[3]. Their relative strengths are determined by LQCD calculations[1]. Secondly, we extend our potential model to meson-baryon $S = -2$ and $-3$ sectors ($\pi \Xi - K^\text{baru} \Lambda$ and $K^\text{baru} \Sigma$) and discuss the properties of their $S$-wave interactions[4]. Finally, we discuss resonance and bound state poles of the $S$-matrix on complex-$E$ plane and predict possible existence of exotic baryon states with $S = -2$ and $-3$[4].

2. One-hadron-exchange potential model

Our potential model is constructed based on the flavor-$SU(3)$ symmetry. The breaking of this symmetry comes from only exchanged hadron masses, for which we use physical masses.

Our baryon-baryon (BB) potentials consist of two parts, that is, the long-range part (one-boson-exchange potential, OBEP) and the short-range part with Gaussian form:

$$V_{BB} = V(\text{OBEP})[1 - \exp(-\frac{(r - r_c)^2}{r_Q^2})] + V_{\text{short}} \exp(-\frac{(r - r_Q)^2}{r_Q^2})$$

$V(\text{OBEP})$ is smoothly cutoff by the cutoff function with a common range, $r_c = 0.4$fm for all BB pairs. As exchanged bosons, we consider scalar-, pseudoscalar- and vector-meson nonets with physical masses. Among the $SU(3)$ coupling parameters, $g_{\text{ps}}$, $\theta_{\text{ps}}$, $\alpha_{\text{ps}}$, $\alpha$, and $\theta$, are fixed by established or theoretical values. As the range of the short-range part, $r_Q$, we use a common value 0.475fm for all BB pairs. The strengths $V_{\text{short}}$ are determined as follows: In even ($L = \text{even}$) states, relative strengths of $V_{\text{short}}$ for six $SU(3)$ representations are determined by $L = 0$ results of the LQCD calculations[1]. The remaining one absolute strength in even states and six strengths in the odd states are treated as free parameters determined by phenomenological fit. Number of parameters in our BB potentials are only 17 ($10 \ SU(3)$ parameters + 7 $V_{\text{short}}$ strengths). They are determined by phenomenological fitting to NN phase shifts, YN cross sections and ΛΛ data.
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
\( \pi N \) & pot I & pot II & exp & \( KN \) & pot I & pot II & exp \\
\hline
\( S_{11} \) & 0.2458 & 0.2482 & 0.2473\pm0.0043 & \( S_{01} \) & -0.008 & -0.013 & 0.00\pm0.02 \\
\( S_{31} \) & -0.1496 & -0.1466 & -0.1444\pm0.0057 & \( S_{11} \) & -0.365 & -0.369 & -0.33\pm0.02 \\
\( P_{11} \) & -0.2359 & -0.2340 & -0.2368\pm0.0058 & \( P_{01} \) & 0.166 & 0.179 & 0.08\pm0.02 \\
\( P_{31} \) & -0.1375 & -0.1290 & -0.1310\pm0.0058 & \( P_{11} \) & -0.106 & -0.103 & -0.16\pm0.02 \\
\( P_{13} \) & -0.0862 & -0.0894 & -0.0877\pm0.0058 & \( P_{03} \) & -0.058 & -0.071 & -0.13\pm0.02 \\
\( P_{33} \) & 0.6238 & 0.6235 & 0.6257\pm0.0058 & \( P_{13} \) & 0.047 & 0.040 & 0.07\pm0.02 \\
\hline
\end{tabular}

**Table 1:** \( \pi N \) and \( KN \) scattering lengths

\begin{tabular}{|c|c|c|c|}
\hline
 & pot I & pot II & exp \\
\hline
\( \gamma \) & 2.35 & 2.36 & 2.36\pm0.04 \\
\( R_c \) & 0.660 & 0.700 & 0.664\pm0.011 \\
\( R_n \) & 0.189 & 0.172 & 0.189\pm0.015 \\
Re(\( a_{K^- p} \)) & -0.666 & -1.019 & \text{See Fig.1} \\
Im(\( a_{K^- p} \)) & 0.462 & 0.398 & \text{See Fig.1} \\
\hline
\end{tabular}

**Table 2:** \( K^- p \) threshold quantities and scattering length

Our meson-baryon(\( MB \)) potentials are defined in momentum space and have a form given by

\[
V_{MB} = V(OHEP) \exp(-q^2/\Lambda_Q^2) + V_{\text{short}} \exp(-q^2/\Lambda_Q^3)
\]

For one-hadron-exchange part, \( V(OHEP) \), we consider \( t \)-channel meson-exchange, \( u \)-channel baryon-exchange and \( s \)-channel baryon-exchange potentials. As exchanged baryons, we employ octet baryons and decuplet baryons. \( S_{11} \) and \( N(1440) \) are added to get a better fit to \( \pi N \) data. All meson-meson-(octet) baryon coupling constants are predetermined by \( BB \) potentials given above. For short-range part, we use a common range \( \Lambda_Q = 2/r_G \) for all \( MB \) pairs. At present stage, we try two cases, \( r_G = 0.40 \) (pot I) and 0.45fm (pot II). The strength \( V_{\text{short}} \) for each \( SU(3) \) representation is treated as a free parameter. But, in near future they will be constrained by LQCD results.

The results for \( \pi N \) and \( KN \) scattering lengths are given in Table 1. We find a very good agreement with experimental data. For \( S \)- and \( P \)-wave phase shifts, we obtain again a good fitting below \( T_{lab} \sim 300 \text{MeV} \). But, above this energy we find some deviations from data in \( \pi N \) \( P_{11}^- \), \( P_{13}^- \)-waves and in \( KN \) \( P_{01}^- \), \( P_{11}^- \)-waves. For \( \pi\Lambda-\pi\Sigma-K^{\text{bar}} N \) coupled-channel potentials, we have only two adjustable parameters, \( V_{\text{short}} \) in \{8a\} and \{1\} (other parameters are determined by fitting to \( \pi N \) and \( KN \) data). In Table 2 and Fig. 1, \( K^- p \) threshold quantities calculated with our potentials are given in comparison with experimental data. We find both pot I and II give a good result. Quantitatively, pot I makes a better fit to data than pot II. Using our potentials (pot I and II), we obtain \( K^- p \) cross sections in a good agreement with experimental one, but both potentials give some over(under)estimations in \( K^- p \to K^0 n(\pi^+ \Sigma^-) \) cross sections at the high (low) energy region. Pot I and II give a pole at \( \sqrt{s} = 1393 - 16i \) and 1406 - 6iMeV, respectively, which corresponds to \( \Lambda(1405) \). Our model supports the single-pole picture by Akaishi et al.\[5\].
\[ K^{\text{bar}} B \quad I \quad \rho \quad \omega \quad \phi \quad \text{scalar} \quad \text{baryon} \quad \text{short} \quad \text{total} \]

| \( K^{\text{bar}} N \) | 0 | -42.4 | -91.7 | 20.0 | -25.0 | 22.4 | -44.9 | -161.5 |
| \( K^{\text{bar}} N \) | 1 | 14.1 | -91.7 | 20.0 | -25.3 | 156.5 | 11.5 | 85.2 |
| \( K^{\text{bar}} \Lambda \) | 1/2 | 0 | -84.7 | 49.6 | -28.7 | 6.5 | -12.1 | -69.4 |
| \( K^{\text{bar}} \Sigma \) | 1/2 | -78.4 | -87.5 | 55.9 | -27.1 | 30.7 | 18.5 | -87.9 |
| \( K^{\text{bar}} \Sigma \) | 3/2 | 39.2 | -87.5 | 55.9 | -30.0 | 0 | -2.2 | -24.6 |
| \( K^{\text{bar}} \Xi \) | 0 | -69.4 | -70.3 | 90.7 | -28.7 | 14.2 | 7.1 | -56.4 |
| \( K^{\text{bar}} \Xi \) | 1 | 23.1 | -70.3 | 90.7 | -32.7 | 5.2 | 14.2 | 30.4 |

**Table 3:** Contributions to \( K^{\text{bar}} \)-baryon S-wave potentials \((I:\text{isospin})\)

3. **Meson-baryon potentials and S-wave resonances in \( S = -2 \) and \(-3 \) sectors**

We can extend our potential model (pot I and II) to \( S = -2 \) and \(-3 \) sectors without any additional parameter, except baryon-exchange mechanisms with excited baryons, \( \Xi^* \) or \( \Omega^* \), etc, which will be discussed in near future.

In the \( S = -2 \) sector, we have two problems, \( \pi \Xi + K^{\text{bar}} \Lambda + K^{\text{bar}} \Sigma + \eta \Xi (I = 1/2) \) and \( \pi \Xi + K^{\text{bar}} \Sigma (I = 3/2) \). In the present calculations, we ignore the \( \eta \Xi \) channel because of its high threshold energy. In \( I = 1/2 \) S-wave phase shifts, we find a broad resonance at around \( \sqrt{s} = 1.50-1.55 \text{MeV} \) both with pot I and II. We obtain strongly attractive S-wave interaction for all of \( \pi \Xi, K^{\text{bar}} \Lambda \) and \( K^{\text{bar}} \Sigma \) pairs in \( I = 1/2 \) state. In \( I = 3/2 \) state, we find weak (attractive) \( K^{\text{bar}} \Sigma \) and (repulsive) \( \pi \Xi \) interactions, and no resonant behavior in phase shifts.

In the \( S = -3 \) sector, we have a single channel \( K^{\text{bar}} \Xi \) problem both in \( I = 0 \) and \( I = 1 \). In \( I = 0 \) state, we find a S-wave bound state with pot I. With pot II, we find strongly attractive potential, but its strength is insufficient to make a bound state. In Table 3, the contributions from various exchange mechanisms in \( K^{\text{bar}} B \) S-wave potentials are listed. The values are calculated at \( E_{\text{cm}} = 50 \text{MeV} \) above each \( K^{\text{bar}} B \) threshold. We find that the strong attractions in \( K^{\text{bar}} N (I = 0), K^{\text{bar}} \Lambda, K^{\text{bar}} \Sigma (I = 1/2) \) and \( K^{\text{bar}} \Xi (I = 0) \) come commonly from the isospin-dependent \( \rho \)-exchange and isospin-independent \( \omega \)-exchange. In \( K^{\text{bar}} N \) case, \( V_{\text{short}} \) provides an additional attraction, which is important to make the \( \Lambda(1405) \) pole. But, our model cannot say anything about origins of attractive \( V_{\text{short}} \).

To determine the pole positions for the resonant behavior and a bound state mentioned above, we calculate the \( S \)-matrix on the complex \( E \)-plane by extending our potentials to the complex \( p \)-plane. As a result, we obtain the \( S \)-matrix poles shown in Fig. 2, where \( |S| \) are displayed as
functions of the complex $E$. The pole positions are

$$\sqrt{s} = 1510 - 73i(1495 - 84i)\text{MeV with pot I (potII)}$$

for $S = -2, I = 1/2$ and

$$\sqrt{s} = 1796(1802)\text{MeV with pot I (pot II)}$$

for $S = -3, I = 0$. The pole at 1796MeV with pot I is located on the physical plane, but the pole at 1802MeV with pot II is on the unphysical plane, so the former (latter) is a bound (virtual) state.

4. Summary

We proposed a potential model describing consistently baryon-baryon and meson-baryon interactions at low energies. Our model consists of two parts, that is, the long-range part given by one-hadron-exchange mechanisms and the short-range part constrained by lattice QCD (if available). Our model satisfies the flavor-$SU(3)$ symmetry, except exchanged hadron masses. Using our potentials, pot I and II, we discussed the properties of meson-baryon $S$-wave interactions in $S = -2$ and $-3$ sectors. We find attractive interactions in $K^{\bar{b}}$-hyperon pairs with $I = I_-$ (the smaller case). The origin of this attraction is common to $K^{\bar{b}}N(I = 0)$ case, that is, $\rho$- and $\omega$-exchange. As a result, we predicted two resonance (bound) states: $\Xi^+(J^P = 1/2^-)$ with $M \sim 1500\text{MeV}, \Gamma \sim 160\text{MeV}$ and $\Omega^+(J^P = 1/2^-)$ with $M \sim 1800\text{MeV}$. Our model should be refined by several points. (1) The decuplet baryons should be fully considered. (2) Improvement of fitting to experimental data (phase shifts above $T_{lab} = 300\text{MeV}, K^-p$ cross sections) is needed.

References