We study the $\rho K \bar{K}$ system with an aim to describe the $\rho(1700)$ resonance. The chiral unitary approach has achieved success in a description of systems of the light hadron sector. With this method, the $K\bar{K}$ system in the isospin sector $I = 0$, is found to be a dominant component of the $f_0(980)$ resonance. Therefore, by regarding the $K\bar{K}$ system as a cluster, the $f_0(980)$ resonance, we evaluate the $\rho K \bar{K}$ system applying the fixed center approximation to the Faddeev equations. We construct the $\rho K$ unitarized amplitude using the chiral unitary approach. As a result, we find a peak in the three-body amplitude around 1739 MeV and a width of about 227 MeV. The effect of the width of $\rho$ and $f_0(980)$ is also discussed. We associate this peak to the $\rho(1700)$ which has a mass of $1720 \pm 20$ MeV and a width of $250 \pm 100$ MeV.
We study the $\rho K\bar{K}$ system in the sector $I(J^P) = 1(1^-)$ within the fixed center approximation (FCA) to obtain the $\rho(1700)$ resonance. A pair of $K\bar{K}$ is assumed to form the scalar cluster, the $f_0(980)$ resonance because the $K\bar{K}$ component in $f_0(980)$ is found to be dominant [1]. In order to obtain the unitarized $\rho K$ amplitude, we follow the schemes given by refs. [2, 3] and extend them to the isospin $I = 3/2$ sector. Basically we follow the formalism given by refs. [4, 5, 6, 7].

To implement the Faddeev equation within the fixed center approximation, we need the $\rho K (\rho \bar{K})$ unitarized amplitude. Here we utilize the amplitude given in the previous work [2, 3] as to the vector-pseudoscalar interaction in the sector with strangeness $S = 1$ and isospin $I = 1/2$. Following the Bethe-Salpeter approach, we have the $VP$ two-body scattering amplitude as

$$T = [1 + V\hat{G}]^{-1}(-V)\vec{e} \cdot \vec{e}'$$

where $V$ is an interaction kernel which will be discussed later, $\hat{G}$ is $(1 + \frac{q_l}{\sqrt{s}})G$ being a diagonal matrix and $\vec{e} (\vec{e}')$ represents a polarization vector of the incoming (outgoing) vector-meson. Thanks to the on-shell factorization and the dimensional regularization, a loop function of pseudoscalar and vector mesons $G_I$ can be expressed as a function of the energy $\sqrt{s}$

$$G_I(\sqrt{s}) = \frac{1}{16\pi^2} \left\{ a(\mu) + \ln \frac{M^2_I}{\mu^2} + \frac{m^2_I - M^2_I}{2s} \ln \frac{m^2_I}{M^2_I} \right.$$

$$\left. + \frac{q_l}{\sqrt{s}} \left[ \ln(s - (M^2_I - m^2_I) + 2q_l\sqrt{s}) + \ln(s + (M^2_I - m^2_I) + 2q_l\sqrt{s}) \right. \ln(-s + (M^2_I - m^2_I) + 2q_l\sqrt{s}) - \ln(-s - (M^2_I - m^2_I) + 2q_l\sqrt{s}) \right\},$$

where a momentum $q_l$ is determined at the center of mass frame and $\mu$ is a scale parameter in this scheme. Furthermore in accordance with ref. [3], we take into account the effect of the propagation of unstable particles in terms of the Lehmann representation. Instead of the original loop function eq. (2), we use

$$\hat{G}_I(\sqrt{s}) = \frac{1}{C_I} \int_{(M_I - 2\Gamma_I)^2}^{(M_I + 2\Gamma_I)^2} d\nu G_I(\sqrt{s}, \sqrt{s\nu}, m_I) \times \left(-\frac{1}{\pi}\right) \text{Im} \left\{ \frac{1}{s\nu - M^2_I + iM_I\Gamma_I} \right\},$$

with the normalization for the $l$th component

$$C_I = \int_{(M_I - 2\Gamma_I)^2}^{(M_I + 2\Gamma_I)^2} d\nu \times \left(-\frac{1}{\pi}\right) \text{Im} \left\{ \frac{1}{s\nu - M^2_I + iM_I\Gamma_I} \right\},$$

with $m_I, M_I, \Gamma_I$ the mass of the pseudoscalar meson, mass of the vector and width of the vector respectively. We can obtain the interaction kernel $V$ by the use of the WCCWZ approach [8, 9, 10] where the interaction Lagrangian stems from a nonlinear realization of chiral symmetry. By projecting over $s$-wave, we have the $VP$ potential

$$V_{ij}(s) = -\frac{\vec{e} \cdot \vec{e}'}{8f^2} C_{ij} \left[ 3s - (M^2_I + m^2_i + M^2_j + m^2_j) - \frac{1}{s}(M^2_I - m^2_i)(M^2_I - m^2_j) \right],$$

where the index $i (j)$ represents the $VP$ channel of the incoming (outgoing) particles. The coefficients $C_{ij}$ in eq. (5) for the $I = 1/2$ and $I = 3/2$ sector are tabulated in tables 1 and 2 respectively.
Description of $\rho(1700)$ with the FCA method

T. Uchino

In order to have an appropriate unitarized amplitude, we use the following parameter set chosen to reproduce $K_1(1270)$ in ref. [3] as $\mu = 900$ MeV, $a(\mu) = -1.85$, $f = 115$ MeV.

Under the fixed center approximation [4, 5, 6, 7], it is assumed that $K$ and $\bar{K}$ cluster together and the structure of the cluster is kept against the collision of $\rho$. This idea leads the three-body scattering amplitude $T$ which can read a summation of the two following partition functions $T_1$ and $T_2$

$$\begin{align*}
T_1 &= t_1 + t_1 G_0 T_2, \\
T_2 &= t_2 + t_2 G_0 T_1, \\
T &= T_1 + T_2,
\end{align*}$$

where the subscripts $i = 1(2)$ of $T_i$ and $t_i$ represent the component particle $K(\bar{K})$ in the cluster and the diagrammatic sketches are depicted in fig. 1. In the present work, we can rewrite $t = t_1 = t_2$ and $t$ is given as a mixture of the different isospin states of $I = 1/2$ and $I = 3/2$, $t = \left(2t_1^{I=3/2} + t_1^{I=1/2}\right)/3$, where $t_1^{I=3/2}$ is the $\rho K$ unitarized scattering amplitude given by eq. (1). By adopting the field normalization used in refs. [5, 6, 7], $G_0$ of eq. (6) reads as a function of the energy $\sqrt{s}$

$$G_0(\sqrt{s}) = \frac{1}{2M_{f_0}} \int \frac{d^3q}{(2\pi)^3} F_{f_0}(q) \frac{1}{q^{02}(s) - q^2 - m_{f_0}^2 + im_p \Gamma_{\rho}}.$$  

where $M_{f_0}$ is the mass of the $f_0(980)$ resonance and the width of the $\rho$ is taken into account in the above propagator. The energy of the propagator $q^0$ is determined at the three-body rest frame, $q^0(\sqrt{s}) = (s + m_{f_0}^2 - M_{f_0}^2)/2\sqrt{s}$. Here we utilize the form factor $F_{f_0}$ to give the momentum distribution of $f_0$ to the $G_0$ function by the use of the formalism developed in refs. [11, 12, 13]

$$F_{f_0}(q) = \frac{1}{\mathcal{A}} \int_{p_{\text{min}}}^{p_{\text{max}}} \frac{d^3 p}{\sqrt{p^* - m_{f_0}^2}} \frac{1}{2\omega_K(p)} \frac{1}{2M_{f_0} - 2\omega_K(p)}.$$
Description of \( \rho(1700) \) with the FCA method

T. Uchino

\[ \rho K \bar{K} + + (a) (b) (c) (d) + + (e) (f) + + (g) (h) + + \ldots \]

Figure 1: Diagrammatic representation of the fixed center approximation for the \( \rho K \bar{K} \) system.

\[
\left( \frac{1}{2\omega_K(|\vec{p} - \vec{q}|)} \right)^2 \frac{1}{M_{f_0} - 2\omega_K(|\vec{p} - \vec{q}|)}.
\]

(8)

where the normalization \( N \) is given by

\[
N = \int_{p < k_{\text{max}}} d^3 p \left[ \left( \frac{1}{2\omega_K(|\vec{p}|)} \right)^2 \frac{1}{M_{f_0} - 2\omega_K(|\vec{p}|)} \right]^2.
\]

(9)

From ref. [1], we take \( k_{\text{max}} = \sqrt{\Lambda^2 - m_K^2} \) and \( \Lambda = 1030 \text{ MeV} \) for getting the \( f_0(980) \) from the \( K\bar{K} \) cluster. Considering the \( S \)-matrix, we have a simple expression of the three-body scattering amplitude

\[
T(s) = 2 \left[ \tilde{t}(s') + \tilde{t}(s') G_0(s) \tilde{t}(s') + \cdots \right] = 2 \frac{\tilde{t}(s')}{1 - \tilde{t}(s') G_0(s)},
\]

(10)

where \( \tilde{t} = (2m_{f_0}/2m_K)t \) coming from the normalization of the fields and note that the unitarized amplitude \( \tilde{t} \) is a function of \( s' = 1/2(s + M_\rho^2 + 2m_K^2 - M_{f_0}^2) \). Finally we consider the width of \( f_0 \) by replacing the mass of the cluster \( M_{f_0} \) in eqs. (8) and (9) with \( M_{f_0} - i\Gamma_{f_0}/2 \). The amplitude with the \( f_0(980) \) and \( \rho \) width effect is shown in fig. 2 and the masses and widths of the dynamically generated state are listed in table 3. It is shown that the inclusion of the \( f_0(980) \) width induces a suppression of the magnitude of the peak and the peak becomes broader as the width of the \( f_0(980) \) increases. Furthermore it is also a remarkable feature that the peak position is not so affected by this prescription.

Through this work, we construct the \( \rho K\bar{K} \) three-body amplitude by means of the fixed center approximation. In our framework, a pair of \( K\bar{K} \) is considered to form a scalar meson cluster \( f_0(980) \), based on ref. [1]. We use the \( \rho K \) unitarized amplitude provided by refs. [2, 3] in a manner giving a respect to chiral symmetry. In the three-body amplitude, we have a peak at the energy around 1748 MeV rather independent of the width of the \( f_0(980) \). Besides, it is seen that the inclusion of the \( f_0(980) \) width makes the peak wider and gives a good agreement with the
Description of $\rho(1700)$ with the FCA method

T. Uchino

Figure 2: The $\rho K\bar{K}$ amplitude with the width effect for $\Gamma_{f_0} = 0, 40, 70, 100$ MeV, respectively.

<table>
<thead>
<tr>
<th>$\Gamma_{f_0} = 0$</th>
<th>$\Gamma_{f_0} = 40$</th>
<th>$\Gamma_{f_0} = 70$</th>
<th>$\Gamma_{f_0} = 100$</th>
<th>PDG [14]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>1748.0</td>
<td>1743.6</td>
<td>1739.2</td>
<td>1734.8</td>
</tr>
<tr>
<td>Width</td>
<td>160.8</td>
<td>216.4</td>
<td>227.2</td>
<td>224.6</td>
</tr>
</tbody>
</table>

Table 3: The masses and widths of dynamically generated states with the width effects. (in MeV)

experimental data of the $\rho(1700)$, both for the position and the width. Since the $\rho$ decays into $\pi\pi$ mostly, the above results might be related to the dominant decay mode of the $\rho(1700)$, $\rho\pi\pi$ and $4\pi$. Our approach to the $\rho K\bar{K}$ system provides the description of the $\rho(1700)$ as a dynamically generated state and then we conclude that the building block of the $\rho(1700)$ resonance are the $\rho$ and $f_0(980)$.

References