PROCEEDINGS OF SCIENCE

Structure of the sigma meson from lattice QCD

SCALAR Collaboration : Teiji Kunihiro[a], Shin Muroya[b], Atsushi Nakamura[c], Chiho Nonaka[d,e], Motoo Sekiguchi[f], Hiroaki Wada[g] and Masayuki Wakayama[*] [d]

[a]*YITP, Kyoto University, Kyoto 606-8502, Japan*
[b]*Matsumoto University, Matsumoto 390-1295, Japan*
[c]*IMC, Hiroshima University, Higashi-Hiroshima 739-8521, Japan*
[d]*Department of Physics, Nagoya University, Nagoya 464-8602, Japan*
[e]*Kobayashi Maskawa Institute, Nagoya University, Nagoya 464-8602, Japan*
[f]*School of Science and Engineering, Kokushikan University, Tokyo 154-8515, Japan*
[g]*Faculty of Political Science and Economics, Kokushikan University, Tokyo 154-8515, Japan*

Our purpose is to obtain insights of structure of the sigma meson from the first principle calculation, lattice QCD. At present we do not reach a conclusive understanding of nature of the sigma meson. Currently it is considered as a usual two-quark state, four-quark states such as a tetra-quark and mesonic molecules or superposition of them. At present we do not reach a conclusive understanding of nature of the sigma meson. Besides, the mixing with glueballs is one of important and interesting ingredients for structure of the sigma meson. Furthermore, a disconnected diagram of the sigma meson plays an important role in the structure of the sigma meson. However, to evaluate the disconnected part of the propagator is not an easy task in lattice QCD calculation. To compute the disconnected part of the propagator, we use the Z_2 noise method and the time dilution for estimating the all-to-all quark propagators. Here, we focus on four-quark states in the sigma meson. From investigation of two-quark and four-quark states with the inclusion of disconnected diagrams, we will discuss the structure of the sigma meson and the mass of it.

XV International Conference on Hadron Spectroscopy-Hadron 2013
4-8 November 2013
Nara, Japan

[*]Speaker. E-mail: wakayama@hken.phys.nagoya-u.ac.jp (M. Wakayama)

PoS(Hadron 2013)106

1. Introduction

Since many light scalar mesons such as $\sigma(600)$, $\kappa(800)$, $f_0(980)$ and $a_0(980)$ were found in experiments [1], a lot of theoretical studies have been devoted to investigation of their states. For the structure of light scalar mesons, in addition to the conventional two quark state from the quark model, several possibilities are proposed [2]; four-quark, molecular and scattering states. Because the sigma meson is considered as a chiral partner of the pion in the mechanism of hadron mass generation, it is interesting to investigate a roll of four-quark states in the mechanism. The study of four-quark states in light scalar mesons gives us a chance to get an insight of important QCD feature.

Investigations for the light scalar mesons from lattice QCD were done in Refs. [3, 4, 5, 6, 7, 8, 9, 10]. Recently using tetra-quark interpolators, not only ground states but also resonance states of scalar mesons on the lattice were reported [8]. However there is no study including all possible structures of the sigma meson at the same time. Here we will show the first full QCD lattice calculation including two-quark, tetra-quark and molecular states in the sigma meson.

2. Propagators of the sigma meson

First we show the formulation of propagators of sigma meson in terms of two quarks. The two-quark propagators of the sigma meson are given by

$$G^{\text{two}}(t) \;=\; \left\langle \mathscr{O}^{\text{two}}(t)\mathscr{O}^{\text{two}\,\dagger}(0)\right\rangle \;=\; -G^{\text{conn}}(t) \;+\; 2G^{\text{disc}}(t)\;, \tag{2.1}$$

where $\mathscr{O}^{\text{two}}(t)$ are two-quark operators. The two-quark operators are given by

$$\mathscr{O}^{\text{two}}(t) \;=\; \frac{1}{\sqrt{2}}\sum_{\mathbf{x}a\alpha}\left[\bar{u}^a_\alpha(t,\mathbf{x})u^a_\alpha(t,\mathbf{x})+\bar{d}^a_\alpha(t,\mathbf{x})d^a_\alpha(t,\mathbf{x})\right]\;. \tag{2.2}$$

The two-quark propagators of sigma meson are composed of connected parts $G^{\text{conn}}(t)$ and disconnected ones $G^{\text{disc}}(t)$. To calculate the connected diagrams we use the conjugate gradient method, however we can not apply it to the calculation of the disconnected diagrams naively. To obtain the disconnected part efficiently, we use the Z_2 noise method in which we estimate the quark propagators stochastically with some noise vectors.

The sigma meson channel may have also overlap with four-quark states. As four-quark states, we calculate molecular and tetra-quark propagators. Molecular operators consist of two color singlet two-quark operators, whereas tetra-quark operators consist of a color anti-triplet diquark operator and a color triplet anti-diquark operator. The molecular propagators of the sigma meson are defined by

$$G^{\text{molec}}(t) \;=\; \left\langle \mathscr{O}^{\text{molec}}(t)\mathscr{O}^{\text{molec}\,\dagger}(0)\right\rangle\;, \tag{2.3}$$

where $\mathscr{O}^{\text{molec}}(t)$ are the molecular operators. The molecular operators are given by

$$\mathscr{O}^{\text{molec}}(t) \;=\; \frac{1}{\sqrt{3}}\left[\mathscr{O}^{\pi^+}(t)\mathscr{O}^{\pi^-}(t)-\mathscr{O}^{\pi^0}(t)\mathscr{O}^{\pi^0}(t)+\mathscr{O}^{\pi^-}(t)\mathscr{O}^{\pi^+}(t)\right]\;, \tag{2.4}$$

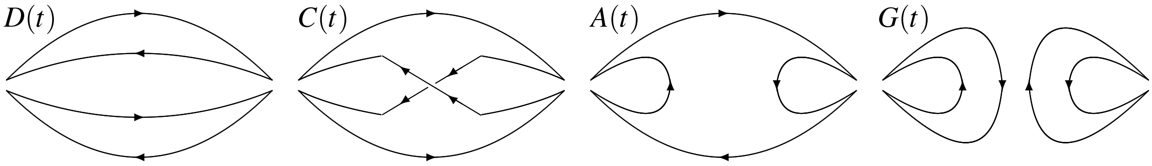

Figure 1: The diagrams for the propagator from the molecular state to the molecular state $G^{\text{molec}}(t)$.

where $\mathcal{O}^{\pi}(t)$ are two-quark pion operators written as

$$\begin{aligned}
\mathcal{O}^{\pi^+}(t) &= -\sum_{\mathbf{x}a} \bar{d}^a(t,\mathbf{x})\gamma_5 u^a(t,\mathbf{x}) \ , \quad \mathcal{O}^{\pi^-}(t) \ = \ \sum_{\mathbf{x}a} \bar{u}^a(t,\mathbf{x})\gamma_5 d^a(t,\mathbf{x}) \ , \\
\mathcal{O}^{\pi^0}(t) &= \frac{1}{\sqrt{2}} \sum_{\mathbf{x}a} \left[\bar{u}^a(t,\mathbf{x})\gamma_5 u^a(t,\mathbf{x}) - \bar{d}^a(t,\mathbf{x})\gamma_5 d^a(t,\mathbf{x}) \right] \ .
\end{aligned} \tag{2.5}$$

We define the tetra-quark propagators of the sigma meson as

$$G^{\text{tetra}}(t) = \left\langle \mathcal{O}^{\text{tetra}}(t)\mathcal{O}^{\text{tetra}\,\dagger}(0) \right\rangle \ , \tag{2.6}$$

where $\mathcal{O}^{\text{tetra}}(t)$ are the tetra-quark operators. The tetra-quark operators are given by

$$\mathcal{O}^{\text{tetra}}(t) = \sum_{a} [ud]^a(t)[\bar{u}\bar{d}]^a(t) \ , \tag{2.7}$$

where $[ud]^a(t)$ and $[\bar{u}\bar{d}]^a(t)$ are the diquark and anti-diquark operators written as

$$\begin{aligned}
[ud]^a(t) &= \frac{1}{2} \sum_{\mathbf{x}b,c} \varepsilon^{abc} \left[u^{Tb}(t,\mathbf{x})\, C\gamma_5\, d^c(t,\mathbf{x}) - d^{Tb}(t,\mathbf{x})\, C\gamma_5\, u^c(t,\mathbf{x}) \right] \ , \\
[\bar{u}\bar{d}]^a(t) &= \frac{1}{2} \sum_{\mathbf{x}b,c} \varepsilon^{abc} \left[\bar{u}^b(t,\mathbf{x})\, C\gamma_5\, \bar{d}^{Tc}(t,\mathbf{x}) - \bar{d}^b(t,\mathbf{x})\, C\gamma_5\, \bar{u}^{Tc}(t,\mathbf{x}) \right] \ ,
\end{aligned} \tag{2.8}$$

where C is the charge conjugate matrix. These operators have a color index. The tetra-quark operator is defined with the diquark and anti-diquark operators by constructing the color index.

The molecular and tetra-quark propagators are composed of the following diagrams,

$$G^{\text{molec}}(t) = 2\left[D(t) + \frac{1}{2}C(t) - 3A(t) + \frac{3}{2}G(t) \right] \ , \tag{2.9}$$

$$\begin{aligned}
G^{\text{tetra}}(t) = 2\Big[& 2\left(D_1'(t) + D_2'(t)\right) - 2\left(A_1'(t) + A_2'(t) + A_3'(t) + A_4'(t)\right) \\
& + \left(G_1'(t) + G_2'(t) + G_3'(t) + G_4'(t)\right) \Big] \ ,
\end{aligned} \tag{2.10}$$

where D, C, A and G diagrams are shown in Fig. 1. The difference between D and D' stands for the difference way of the contraction in combination of color. We calculate the disconnected diagrams with the Z_2 noise method. We neglect the doubly disconnected diagrams G or G' which are suppressed by $1/N_c$ compared to singly disconnected diagrams [11].

PoS(Hadron 2013)106

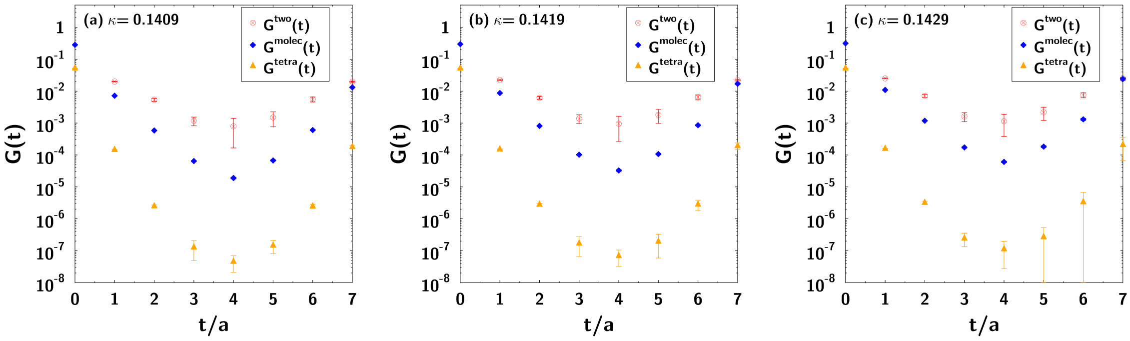

Figure 2: The time dependence of the two-quark (red open circles), molecular (blue solid diamonds) and tetra-quark (orange solid triangles) propagators for $\kappa = 0.1409$ (left panel (a)), 0.1419 (center panel (b)) and 0.1429 (right panel (c)).

3. Results

The sigma propagators are calculated on the $N_f = 2$ full QCD gauge configurations which are generated with the Wilson gauge and the clover quark actions. The lattice size is $4^3 \times 8$. The lattice coupling β, the clover coefficient c_{SW} and the hopping parameter κ are set to be $\beta = 1.8$, giving an inverse lattice spacing $a^{-1} \sim 1$ GeV [12], $c_{SW} = 1.6$ and $\kappa = 0.1409$, 0.1419 and 0.1429, respectively. In the calculation of disconnected diagrams, we perform the Z_2 noise method with 960 noise vectors per a quark propagator and the time dilution [13] in the Z_2 noise method. We measure observables on 8080 gauge configurations for the three different hopping parameters, $\kappa =$ 0.1409, 0.1419 and 0.1429.

In Fig. 2, the two-quark, molecular and tetra-quark propagators for $\kappa = 0.1409$, 0.1419 and 0.1429 are shown. From analyses of the effective masses from the propagators, we can not find a plateau in behavior of effective masses as a function of time, which suggests that the contamination from the excited states is not removed completely. In particular, to remove the effect from the excited states in four-quark states effectively we need a sufficient large lattice size. In spite of the calculation on such a small lattice, we extract the masses of two-quark, molecular and tetra-quark states from the propagators, and estimate the quark mass dependence of them. To suppress the contamination from excited state as much as possible, we evaluate the masses $m_{\rm two}$, $m_{\rm molec}$ and $m_{\rm tetra}$ at $t/a = 3,\ 4$ in Fig. 2. At each kappa we obtain the following mass relation: $m_{\rm two} < m_{\rm tetra} < m_{\rm molec} \sim 2m_\pi$. We find that the masses $m_{\rm two}$, $m_{\rm tetra}$ and $m_{\rm molec}$ decrease with the hopping parameter. Interestingly we observe that the mass reduction speed to the hopping parameter of tetra-quark state is larger compared to other states, two-quark and molecular states. This suggests that at the chiral limit the tetra-quark state could be the dominant grand state of the sigma meson. For the conclusive results, calculations on a efficient larger lattice with light quark mass are indispensable.

4. Summary

We investigated the structure of the sigma meson with full QCD simulation. Here we estimate the dominant ground state of the sigma meson from two-quark, tetra-quark and molecular states. In the diagrams for the propagators of each state we calculated not only the connected diagrams but also the disconnected ones which are evaluated with the Z_2 noise method. From propagators of two-quark, tetra-quark and molecular states, we observed the following mass relation, $m_{\rm two} < m_{\rm tetra} < m_{\rm molec} \sim 2m_\pi$ at κ = 0.1409, 0.1419 and 0.1429. Furthermore we observed that mass reduction speed to the hopping parameter of tetra-quark state is the fastest among the three states, which suggests that at the chiral limit the dominant ground state of the sigma is dominated by the tetra-quark state. This is the first full QCD lattice calculation for the sigma meson including two-quark and four-quark states, though it was done on the small lattice with heavy quark mass. Currently further analyses on a larger lattice with lighter quark mass are on going.

Acknowledgements

This work is supported in part by Nagoya University Program for Leading Graduate Schools "Leadership Development Program for Space Exploration and Research", Grant-in-Aid for Scientific Research (S) (22224003), the Kurata Memorial Hitachi Science and Technology Foundation and Daiko Foundation. Numerical calculations were performed on the super computers of the Research Center for Nuclear Physics (RCNP), Osaka University.

References

[1] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86**, 010001 (2012).

[2] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); R. L. Jaffe, Phys. Rev. D **15**, 281 (1977).

[3] M. G. Alford and R. L. Jaffe, Nucl. Phys. B **578**, 367-382 (2000) [hep-lat/0001023].

[4] M. Wakayama and C. Nonaka, [hep-lat/1211.2072].

[5] T. Kunihiro *et al.* [SCALAR Collaboration], Phys. Rev. D **70**, 034504 (2004) [hep-ph/0310312].

[6] H. Wada *et al.* [SCALAR Collaboration], Phys. Let. B **652**, 250 (2007).

[7] A. Hart *et al.* [UKQCD Collaboration], Phys. Rev. D **74**, 114504 (2006).

[8] G. P. Engel *et al.* [BGR Collaboration], Phys. Rev. D **85**, 034508 (2012).

[9] S. Prelovsek and D. Mohler, Phys. Rev. D **79**, 014503 (2009).

[10] C. Alexandrou *et al.* [ETM Collaboration], JHEP **1304**, 137 (2013).

[11] Feng-K. Guo, L. Liu, Ulf-G. Meißner and P. Wang, Phys. Rev. D **88**, 074506 (2013).

[12] A. Ali Khan *et al.* [CP-PACS Collaboration], Phys. Rev. D **63**, 034502 (2000).

[13] J. Foley *et al.* [TrinLat Collaboration], Comp. Phys. Comm. **172**, 145 (2005).