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Complex 2D Matrix Model and Its Application to N_c-dependence of Hadron Structures

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We study the internal structure of resonance states in a complex 2D matrix model. We show that the geometry with "exceptional points" in the complex-parameter space can be useful to discuss parameter dependence of the structures within real-parameter subspace. By applying the model to hadron physics, we consider the N_c-dependence of hadron structures from the geometry on the complex-N_c plane.

XV International Conference on Hadron Spectroscopy-Hadron 2013
4-8 November 2013
Nara, Japan

*Speaker.

PoS(Hadron 2013)108

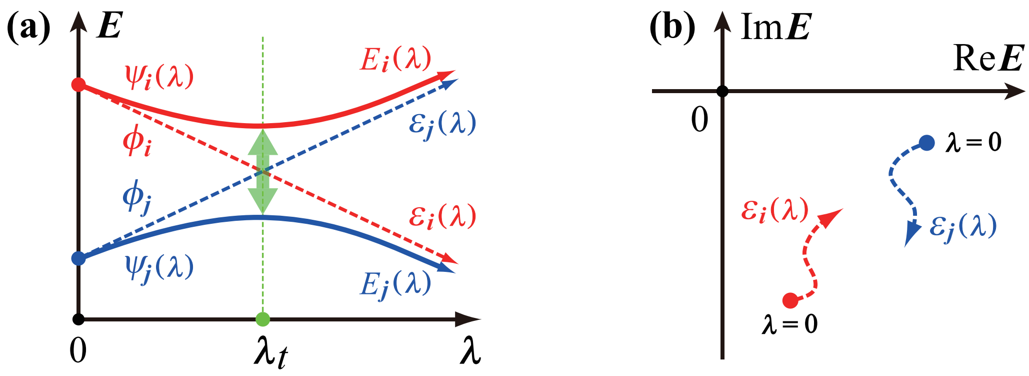

Figure 1: (Color) (a) Hermitian case with anticrossing between ith and jth eigenstates with variation of $\lambda \in \mathbf{R}$. Indices of lines are explained in the text. (b) Non-Hermitian case of $\varepsilon_i(\lambda)$ and $\varepsilon_j(\lambda)$ with variation of $\lambda \in \mathbf{R}$ on complex energy plane.

1. Introduction

The parameter dependence of quantum states has been extensively studied in various physics. An interesting example is the solar neutrino conversion: electron neutrinos produced in the center of the sun via nuclear fusion reaction can become muon neutrinos around the surface of the sun [1], so that less amount of electron neutrinos can be observed by a water Cherenkov detector on the earth, formerly known as "solar neutrino puzzle" [2]. To analyze such parameter dependence, a Hermite model Hamiltonian $\hat{H}(\lambda)$ with a real parameter λ is usually adopted. Let us assume that the eigenstates ϕ_i $(i = 1, 2, \cdots)$ of $\hat{H}(\lambda)$ at $\lambda = 0$ can have clear characters. The full eigenstates $\psi_i(\lambda)$ $(i = 1, 2, \cdots)$ of $\hat{H}(\lambda)$ coincide with ϕ_i at $\lambda = 0$, i.e., $\psi_i(0) = \phi_i$. The Neumann-Wigner non-crossing rule [3] tells us that, if the energy expectation values $\varepsilon_i(\lambda) \equiv \langle \phi_i | \hat{H}(\lambda) | \phi_i \rangle$ $(i = 1, 2, \cdots)$ cross with each other at $\lambda = \lambda_t \in \mathbf{R}$, the energy eigenvalues $E_i(\lambda)$ of $\psi_i(\lambda)$ can have anticrossing at λ_t (see Fig. 1(a)). At this point, the overlap of $\psi_i(\lambda)$ with ϕ_i is found to be exceeded by that with ϕ_j $(i \neq j)$ as $|\langle \phi_i | \psi_i \rangle|^2 \leq |\langle \phi_j | \psi_i \rangle|^2$. That is, due to orthogonality, $\psi_i(\lambda)$ and $\psi_j(\lambda)$ exchange their characters in terms of ϕ_i and ϕ_j at $\lambda = \lambda_t$. We call such phenomena as "nature transition". In fact, knowing *a priori* the critical value λ_t is very useful to inclusively discuss the parameter dependence of the internal structure of the quantum states.

In our work, we consider the open quantum systems having dissipation into decay channels outside of the model space. These systems can be effectively described by non-Hermite model Hamiltonian $\hat{H}(\lambda)$ accompanying with "complex energy eigenvalues" for "resonance states". In this case, $\varepsilon_i(\lambda)$ can move two dimensionally on the complex energy plane (see Fig. 1(b)) without having degeneracy at a certain value of λ. Therefore the criterion of nature transition between resonance states becomes completely uncertain. To solve this problem, we formulate the complex two-dimensional (2D) matrix model. (Two dimensions represent two levels of resonances.) We found that the geometry with "exceptional points" in the complex-parameter space can play a key role for such parameter dependence within real-parameter subspace [4].

We apply the model to the hadron physics with $1/N_c$ expansion. We discuss the typical N_c-dependence of the internal structures of hadrons from the geometry on the complex-N_c plane [4].

2. Complex 2D matrix model

First we consider a two-level problem in an open quantum system. We describe resonance states in *bi-orthogonal representation* as $|\phi_i)(i=1,2)$, where its bra-state is the complex conjugate of the Dirac bra-state $(\phi_i| \equiv \langle\phi_i^*|$ [5]. We assume that $|\phi_i)$, the eigenstates of $\hat{H}(\lambda)$ at $\lambda=0$, are an appropriate basis with clear characters. Then we consider the Hamilton matrix in this basis:

$$H(\lambda) = \begin{pmatrix} (\phi_1|\hat{H}(\lambda)|\phi_1) & (\phi_1|\hat{H}(\lambda)|\phi_2) \\ (\phi_2|\hat{H}(\lambda)|\phi_1) & (\phi_2|\hat{H}(\lambda)|\phi_2) \end{pmatrix} \equiv \begin{pmatrix} \varepsilon_1(\lambda) & V_{12}(\lambda) \\ V_{21}(\lambda) & \varepsilon_2(\lambda) \end{pmatrix}. \tag{2.1}$$

$\varepsilon_i(\in \mathbf{C})$ is the energy of $|\phi_i)$ and $V_{ij}(\in \mathbf{C})$ are the interaction satisfying $V_{ij}(0)=0$. $\lambda(\in \mathbf{R})$ is a parameter, controlling the development of the two eigenstates $|\psi_i(\lambda))$ which can be decomposed by $|\phi_i)$ as $|\psi_i(\lambda)) \equiv C_{i1}(\lambda)|\phi_1)+C_{i2}(\lambda)|\phi_2)$ $(i=1,2)$. $C_{ij}(\lambda)$ carry the information about the internal structure of the eigenstates $|\psi_i(\lambda))$ in terms of $|\phi_i)$. Due to bi-orthogonality, the norms $(\psi_i|\psi_i)=C_{i1}^2+C_{i2}^2$ can become complex, while we simply assume the module, $|C_{ij}(\lambda)|^2$, to be the *component weights*, being suitable for narrow resonances.

Now let us consider the condition of the nature transition, i.e., $|C_{i1}(\lambda)|^2 = |C_{i2}(\lambda)|^2$, where the two basis components $|\phi_1)$ and $|\phi_2)$ are equally mixed in each eigenstate $|\psi_i(\lambda))$ as a character exchanging point. We found that the *geometry on complex-λ plane* gives simple criterion of nature transition within the real-λ subspace as follows. The transition condition $|C_{i1}(\lambda)|^2 = |C_{i2}(\lambda)|^2$ is equal to the two conditions:

$$\mathrm{Re}[A(\lambda)^*\overline{V}(\lambda)] = 0, \tag{2.2}$$

$$|A(\lambda)|^2 \leq |\overline{V}(\lambda)|^2, \tag{2.3}$$

where $A(\lambda) \equiv \{\varepsilon_1(\lambda)-\varepsilon_2(\lambda)\}/2$ and $\overline{V}(\lambda)^2 \equiv V_{12}(\lambda)V_{21}(\lambda)$. The "transition line" is defined as the region satisfying $|C_{i1}(\lambda)|^2 = |C_{i2}(\lambda)|^2$, i.e., both conditions (2.2) and (2.3) on the complex-λ plane. The line (2.2) is named as "line 1", which includes the transition line. The boundary of the region (2.3) is named as "line 2", which selects the proper part for the transition line from line 1. If the transition line crosses the real-λ axis, the nature transition occurs at the crossing point $\lambda = \lambda_t \in \mathbf{R}$ (see Fig. 2).

We can show that *all* of the crossing points between line 1 and 2 correspond to the "exceptional points" [6], where two energy eigenvalues coincides with each other. The significance of the exceptional points has been investigated in the context of quantum chaos [7], where the dense exceptional points on the complex-λ plane indicates the development of quantum chaos in the energy-level statistics. In fact, the exceptional point corresponds to the phase singular point of the Berry phase [8]. In our study, we can newly show that line 1 and 2 cross with each other at all exceptional points, so that these points should always be the *end points* of the transition lines. In this way, the exceptional points tell us not only the global information like development of quantum chaos, but also the information about internal structure of each quantum level.

3. N_c-dependence of hadron structures

We employ the complex 2D matrix model (2.1) to the analysis of the N_c-dependence of the internal structure of hadrons. For a demonstration, we consider the admixed nature of the $a_1(1260)$ meson carrying $q\bar{q}$ and $\pi\rho$-molecule components.

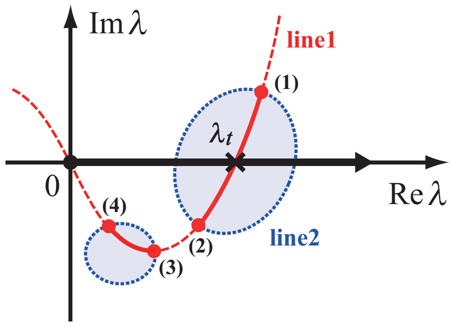

Figure 2: (Color) (a) Schematic figure of geometrical map with transition lines and exceptional points on complex λ plane. Roles of Line 1 and shaded area with boundary of line 2 are given in the text. Points (n) denote the exceptional points $\lambda_{\mathrm{EX}}^{(n)}$. Transition lines are shown by the solid curves, satisfying the transition condition: $|C_{i1}(\lambda)|^2 = |C_{i2}(\lambda)|^2$.

First, we analyze the scattering equation for the π-ρ propagator in the $J^P = 1^+$ channel. Then, by reducing the relativistic eigenvalue equation to the Schrödinger equation of the model (2.1) in a non-relativistic approximation, we can get the geometrical map on the complex-N_c plane for the a_1 meson [4]. The complex 2D matrix model for a_1 meson with N_c dependence factored out by λ becomes

$$H(\lambda) = \begin{pmatrix} \frac{1}{\lambda^2}\sqrt{s_p} & \frac{\lambda}{2\tilde{m}}\sqrt{Z}v_{a_1\pi\rho} \\ \frac{\lambda}{2\tilde{m}}\sqrt{Z}v_{a_1\pi\rho} & m_{a_1} \end{pmatrix}, \tag{3.1}$$

$$\lambda \equiv \sqrt[4]{3/N_c}. \tag{3.2}$$

s_p is the pole mass of $\pi\rho$-molecule state and Z is its pole residue. $v_{a_1\pi\rho}$ is a three-point coupling and $\tilde{m} \equiv \sqrt{\sqrt{s_p}m_{a_1}}$ is the typical mass scale of the problem. These constants are numerically estimated by perturbative resummation for the chiral Lagrangian induced by holographic QCD [9, 10] as $\sqrt{s_p} \simeq 1012 - 221i$, $\sqrt{Z} \simeq 84 - 21i$, $v_{a_1\pi\rho} \simeq -6493$, and $m_{a_1} = 1189$ in MeV unit.

By applying the conditions (2.2) and (2.3) to the Hamiltonian (3.1), we can figure out the geometrical map on the complex-N_c plane for the a_1 meson in Fig. 3. The transition line as the solid curve is found to cross the real λ axis between $\lambda = 0$ ($N_c = \infty$) and $\lambda = 1$ ($N_c = 3$). A critical color number of the nature transition can be calculated by the crossing point as $\lambda_t = \sqrt[4]{3/N_c} \sim 0.93$, i.e., $N_c \sim 4.0$. That is, with continuous variation of N_c from ∞ to 3, the internal structures of two hadronic states are exchanged in terms of appropriate basis $q\bar{q}$ and $\pi\rho$-molecule at the critical color number: $N_c \sim 4.0$. Such a critical color number with character exchange for the a_1 meson is also suggested by analyzing the pole residues in Ref. [11]. In this way, by taking into account the existence of nature transition from the geometry on the complex-N_c plane, we can discuss the typical N_c-dependence of the hadron structures from $N_c = \infty$ to 3.

4. Summary

In this work, we discuss the parameter-dependence of the internal structure of resonances from the geometry of a complex-parameter space. By applying the model to hadron physics with $1/N_c$ expansion, we consider the typical N_c-dependence of hadrons from the geometry on the complex-N_c plane. Wide applications of the model to resonance physics are expected in near future.

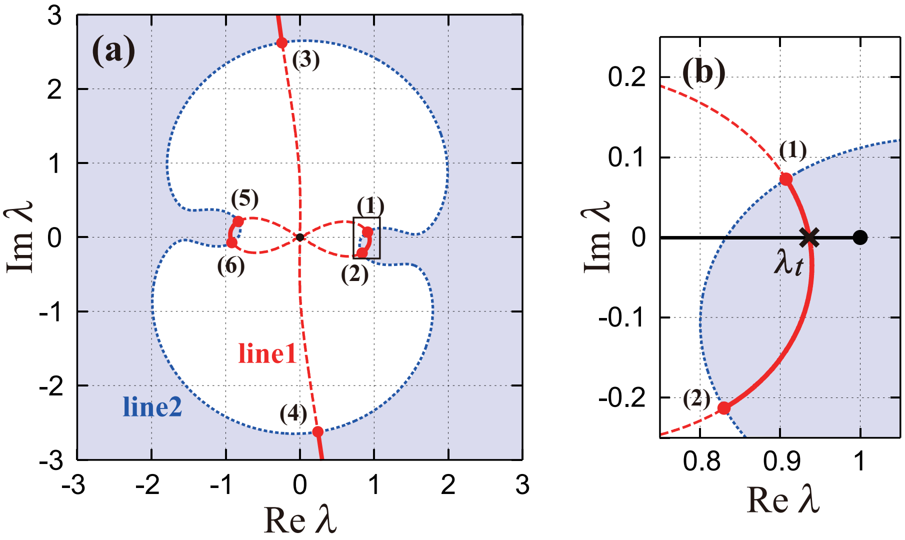

Figure 3: (Color) (a) Geometrical map on the complex-N_c plane with $\lambda = \sqrt[4]{3/N_c}$. (b) Close-up figure around a blank square in (a).

Acknowledgment

This work is supported by Grant-in-Aid for Scientific Research on Innovative Areas "Elucidation of New Hadrons with a Variety of Flavors" (Nos. 22105509 (K. N.), 22105510 (H. N.), 24105706 (D. J.) and E01:21105006 (A. H.)) from the Ministry of Education, Culture, Sports, Science, and Technology(MEXT) of Japan. K. N. is supported by the Special Postdoctoral Research Program of RIKEN.

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