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First application of coupled-channel Complex Scaling Method to the $K^{bar}N$-πY system

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$\bar{K}N$-πY system is a key ingredient for the study of exotic $\bar{K}$-nuclear system which has been discussed for a long time. In this article, we have studied the $\bar{K}N$-πY system with a coupled-channel Complex Scaling Method (ccCSM). The ccCSM is expected to deal appropriately with important points in the study of $\bar{K}$-nuclear system: 1. coupled-channel problem and 2. resonant states. Here, we report the result of the first application of the ccCSM to the $\bar{K}N$-πY system, in which we have treated resonance and scattering problems in a single framework of the ccCSM with Gaussian base.
We have proposed a chiral SU(3)-based $\bar{K}N$-πY potential with a Gaussian form in r-space for both isospin 0 and 1 sectors, constrained by the $\bar{K}N$ scattering length. Using this potential, we have calculated the $I = 0$ scattering amplitudes with the ccCSM and then confirmed a resonance structure. A resonance pole is found around $(1418, -21)$ MeV ($(1420, -25)$ MeV) on the complex-energy plane for non-relativistic (semi-relativistic) kinematics. The meson-baryon mean distance of the resonance is found to be $1.3 - 0.3i$ $(1.2 - 0.5i)$ fm. Furthermore, using the complex-range Gaussian function as a basis function, we have found another pole in the lower-energy region involving large decay width. These resonances are considered to form the double pole of $\Lambda(1405)$ as reported in past studies with chiral unitary model. Similarly, we have investigated the $I = 1$ sector in which three channels of $\bar{K}N$, $\pi\Sigma$ and $\pi\Lambda$ are taken into account.

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In strange nuclear physics and hadron physics, nuclear system with anti-kaon ($\bar{K}$=K^-, $\bar{K}^0$) has been an interesting topic. Because of the strong attraction between anti-kaon and nucleon, kaonic nuclei are expected to have exotic nature such as formation of dense nuclear state [1, 2]. To clarify the detailed property of such the *possible* exotic system, many studies are devoted to the three-body system composed of two nucleons and an anti-kaon, $K^- pp$, which could be a prototype of kaonic nuclei. In spite of many studies from both theoretical and experimental sides, the conclusive result of $K^- pp$ has not been achieved yet. However, all theoretical studies suggest that $K^- pp$ should exist as a resonant state between $\bar{K}NN$ and $\pi\Sigma N$ thresholds [3]. Considering such a current situation of the $K^- pp$ study, we have started the investigation of kaonic nuclei with a coupled-channel Complex Scaling Method (ccCSM). The ccCSM can correctly deal with the important ingredients in the study of kaonic nuclei; 1. channel coupling of $\bar{K}N$ and πY, and 2. resonant states. As for the second point, CSM has been successfully applied in the field of unstable nuclei [4]. In this work, we have applied the ccCSM to the two-body system of $\bar{K}N$-πY coupled system. The content of this article is essentially based on our work [5].

We give a brief comment on the ccCSM. In our study, both of resonance and scattering problems are dealt with a single framework of ccCSM and are solved with Gaussian basis which is easy to handle. In the ccCSM, we perform a complex scaling in Hamiltonian and wave function; $\boldsymbol{r} \to \boldsymbol{r}e^{i\theta}$ and $\boldsymbol{p} \to \boldsymbol{p}e^{-i\theta}$. This transformation changes divergent (oscillating) behavior of resonance wave function (scattered part of wave function in a scattering wave) to a damping form. It is a key point that a non-square integrable function is transformed to be square integrable. Since the complex-scaled wave functions are square integrable, we can expand them with Gaussian base. Consequently, resonance and scattering problems are reduced to a complex-eigenvalue problem and linear-equation problem, respectively.

We investigate $\bar{K}N$ and $\pi\Sigma$ scattering amplitudes in the $I = 0$ channel by using the ccCSM. At first, calculating the scattering length, we determine parameters of our $\bar{K}N$-πY coupled potential which is based on the chiral SU(3) theory and has a local Gaussian form in the coordinate space. The range parameters of Gaussian functions are fixed so as to reproduce the $\bar{K}N$ scattering length obtained by Martin's analysis [6]. Fig. 1 shows the scattering amplitudes calculated with non-relativistic versions of the potential. A resonance structure appears at $\sim$ 1410 MeV in the $\bar{K}N$ scattering amplitude, which corresponds to the hyperon resonance $\Lambda(1405)$. We make a calculation also in semi-relativistic kinematics. The amplitudes are qualitatively the same as those of non-relativistic kinematics. As a result that we search for a resonance pole in the $I = 0$ channel by using the ccCSM, the pole is found on complex-energy plane around $(M, -\Gamma/2) = (1418, -21)$ MeV

Table 1: Essence of the complex scaling method for resonance problem and scattering problem. "wfnc." is the abbreviation of "wave function".

	For resonances	For scattering problem
Role of complex rotation	Resonance wfnc.: divergent $\to$ damping	Scattered part wfnc.: oscillating $\to$ damping
Additional help	"ABC theorem" [4] Resonance pole ... independent of θ	"Cauchy's theorem" Scattering amplitude ... from the wfnc. on $re^{i\theta}$ axis

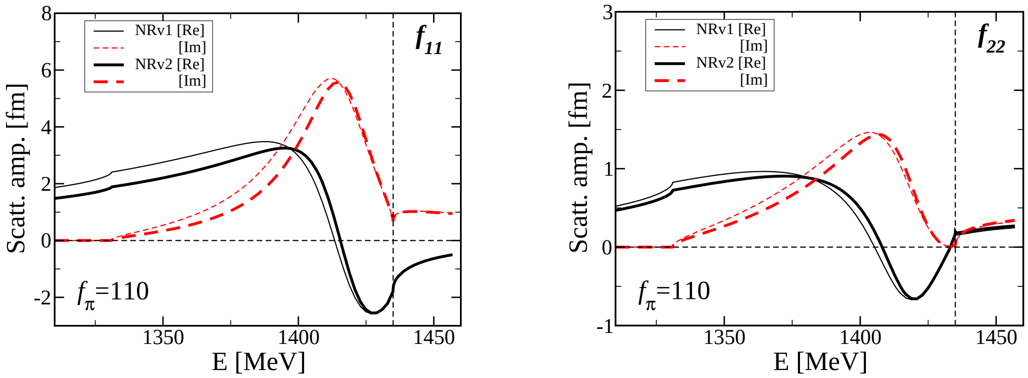

Figure 1: $I = 0$ scattering amplitudes calculated with coupled-channel Complex Scaling Method using non-relativistic potentials (NRv1, 2 with $f_\pi = 110$ MeV). The real (imaginary) part of scattering amplitude is drawn with a black-solid (red-dashed) line. Left (right) panel shows $\bar{K}N$ ($\pi\Sigma$) scattering amplitude. Vertical dashed line means the $\bar{K}N$ threshold.

($(1420, -25)$ MeV) in non-relativistic (semi-relativistic) case. It is an advantage of the ccCSM that we can obtain an explicit wave function of resonant state. We calculate the mean distance between meson and baryon which compose the resonant state, by using the complex-scaled wave function. The mean distance is found to be $1.3 - 0.3i$ ($1.2 - 0.5i$) fm in non-relativistic (semi-relativistic) case. Interestingly, these values are quite close to the mean distance which is deduced from the electro-magnetic form factor of $\Lambda(1405)$ investigated with a chiral unitary model, as Sekihara pointed out in his talk [7].

Here, we mention to the double-pole structure of $\Lambda(1405)$. According to former studies with chiral unitary model [8, 9], it is pointed out that there are two poles in the $I = 0$ channel of $\bar{K}N$-$\pi\Sigma$ system. One pole (called higher pole) gives a shallow $\bar{K}N$ binding with narrow width, whereas the other one (called lower pole) gives a deep $\bar{K}N$ binding with broad width. Since our potential is also based on chiral SU(3) theory, such the double-pole structure is expected. Then, we have a question, "Where is the other pole? ". It is generally difficult to apply the basis expansion method to describing the resonances with broad decay widths, even if with the complex scaling method. In particular, the continuum state, whose wave function oscillates originally, can't be correctly described with the Gaussian base, As a result of limitation of the description of the continuum states, it becomes difficult to separate broad resonant states from continuum states. Recently, a method to overcome this difficulty has been proposed by Kamimura *et al.* [10]. Following their idea, we use Gaussian functions with *complex* range parameters as base functions:

$$G_n^{(\pm)}(r) \; = \; N_n^{(\pm)} \, r^l \, \exp[-(a_n \pm i\omega)\, r^2], \tag{1}$$

where $N_n^{(\pm)}$ means a normalization factor, $\{a_n\}$ is a real range parameter, and $\pm i\omega$ is introduced to make the range parameters complex values. Taking the linear combination of $G_n^{(\pm)}(r)$, it is easily found that the complex-range Gaussian involves trigonometric functions; $G_n^{(+)}(r) + G_n^{(-)}(r) \propto r^l \exp[-a_n r^2] \cos \omega r^2$ and $G_n^{(+)}(r) - G_n^{(-)}(r) \propto r^l \exp[-a_n r^2] \sin \omega r^2$. Owing to the oscillatory behavior of the trigonometric functions, wave functions of continuum states are described better, compared to the usual calculation with the real-range Gaussian base. Consequently, the energy eigenvalues of continuum states appear more clearly along the 2θ line on the complex energy

Table 2: Complex energies $(M, -\Gamma/2)$ of lower and higher poles for each f_π value. The scaling angle for the lower pole and higher pole is taken to $\theta = 40^\circ$ and 30°, respectively. The lower pole is calculated with the complex-range Gaussian basis ($\omega = 2.0$ [fm^{-2}]), whereas the higher pole is calculated with the real-range Gaussian basis. The potential NRv2 is used. All values are given in unit of MeV.

	f_π	90	100	110	120
Lower pole	M	$\sim$1350	1368.8	1395.2	1424.7
	$-\Gamma/2$	~ -66	-109.1	-137.8	-163.2
Higher pole	M	1419.7	1418.3	1417.5	1418.7
	$-\Gamma/2$	-23.3	-19.8	-16.4	-14.6

Table 3: $I = 1$ scattering length. In the column "Condition", "Re fitted" ("Im fitted") indicates that the parameters are tuned to reproduce the real (imaginary) part of the Martin's value. All values are given in unit of fm. "SR-A" means a semi-relativistic potential.

Potential	NRv2		SR-A		Martin [6]
Condition	Re fitted	Im fitted	Re fitted	Im fitted	
Re $a_{KN(I=1)}$	0.372	0.657	0.371	0.659	0.37
Im $a_{KN(I=1)}$	1.504	0.599	1.493	0.600	0.60

plane (indicating the line of $\tan^{-1}(\mathrm{Im}E\,/\,\mathrm{Re}E) = -2\theta$), and then we can distinguish resonance poles from continuum states, even if resonances have a broad width. Certainly, the poles found newly involve large decay width as summarized in Table 2. They correspond to the lower pole of the double pole, and the poles which have already found should be the higher pole. As shown in the table, the higher-pole position is well determined independently of the f_π value which is a parameter in our potential, whereas the lower-pole position depends strongly on f_π value. We consider the reason of this result as follows: As mentioned before, our potential is constrained by the $\bar{K}N$ scattering length. Therefore, the higher pole is much affected by this constraint because it appears near the $\bar{K}N$ threshold. On the other hand, the lower-pole nature can't be well constrained by our potential because they are far from the $\bar{K}N$ threshold. Anyway, thus, the double-pole structure is quite well confirmed in our chiral SU(3)-based potentials by means of the ccCSM.

The same analysis is performed in the $I = 1$ sector which involves three channels of $\bar{K}N$, $\pi\Sigma$ and $\pi\Lambda$. In this case, there are too many range parameters in the potential, compared to the number of constraint condition; the complex-valued $\bar{K}N$ scattering length in $I = 1$ channel, $a_{KN(I=1)}$. Therefore, we determine some of range parameters in an iso-symmetric way; the range parameters determined in the $I = 0$ sector are used also in the $I = 1$ sector. With the iso-symmetric choice, only a range parameter, $d_{KN,\pi\Lambda}$, remains unknown. Searching for $d_{KN,\pi\Lambda}$, we can't find such the parameter that satisfies the complex-valued $a_{KN(I=1)}$. We examined to find $d_{KN,\pi\Lambda}$ which reproduces either real or imaginary part of $a_{KN(I=1)}$. It is found that Im $a_{KN(I=1)}$ deviates largely from the Martin's value when Re $a_{KN(I=1)}$ is reproduced. On the other hand, when Im $a_{KN(I=1)}$ is reproduced, the deviation of Re $a_{KN(I=1)}$ from the Martin's value is rather small. (Table 3) When we remove the restriction of the iso-symmetric choice, we can find some parameter sets which reproduce completely the Martin's value. However, the scattering amplitudes calculated with such range parameters show strange behavior as depicted in Fig. 2: In a non-relativistic case, there is a

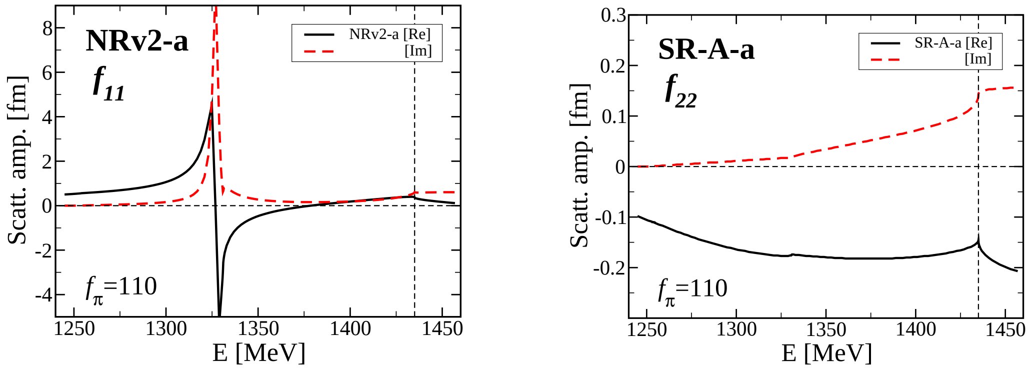

Figure 2: $I = 1$ scattering amplitudes when the complex value of Martin's scattering length is reproduced. (Read the text.) Left and right panels are $\bar{K}N$ scattering amplitude obtained with NRv2 potential and $\pi\Sigma$ one obtained with SR-A potential, respectively.

resonant state slightly below the $\pi\Sigma$ threshold, although such a resonance is not found experimentally and is not predicted by other theoretical studies. In a semi-relativistic case, the $\pi\Sigma$ scattering amplitude indicates a repulsive nature in spite of attractive $\pi\Sigma$-$\pi\Sigma$ potential.

By the present study, it is confirmed that the coupled-channel Complex Scaling Method is a powerful tool to investigate resonant states of hadronic systems as well as nuclear system. As a merit of the complex scaling method, it is easily applied to many-body systems. In future we will study various kinds of hadronic many-body systems with the ccCSM, since the channel coupling is in general important there. As the first priority we investigate the K^-pp system which motivated us originally, because the final result of new data on K^-pp will soon be reported by J-PARC experimental groups [11].

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