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Size measurement of dynamically generated hadronic resonances with finite volume effect

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The structures of the hyperon resonance $\Lambda(1405)$ and the scalar mesons σ, $f_0(980)$, and $a_0(980)$ are investigated based on the coupled-channels chiral dynamics with finite volume effect. The finite volume effect is utilized to extract the coupling constant, compositeness, and mean squared distance between two constituents of a Feshbach resonance state as well as a stable bound state. In this framework, the real-valued size of the resonance can be defined from the downward shift of the resonance pole according to the decreasing finite box size L on a given closed channel. As a result, we observe that, when putting the $\bar{K}N$ and $K\bar{K}$ channels into a finite box while other channels being unchanged, the poles of the higher $\Lambda(1405)$ and $f_0(980)$ move to lower energies while other poles do not show downward mass shift, which implies large $\bar{K}N$ and $K\bar{K}$ components inside higher $\Lambda(1405)$ and $f_0(980)$, respectively. Extracting structures of $\Lambda(1405)$ and $f_0(980)$ in our method, we find that the compositeness of $\bar{K}N$ ($K\bar{K}$) inside $\Lambda(1405)$ [$f_0(980)$] is 0.82 – 1.03 (0.73 – 0.97) and the mean distance between two constituents is evaluated as 1.7–1.9 fm (2.6–3.0 fm).

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1. Introduction

Establishing exotic hadrons, which have different quark configurations from the ordinary $q\bar{q}$ (mesons) or qqq (baryons) state, is one of the most important issues in the physics of strong interaction. This is because there is no clear experimental evidence on the existence of the exotic hadrons while quantum chromodynamics, the fundamental theory of strong interaction, does not prohibit their existence. A classical example of the exotic hadron candidates is $\Lambda(1405)$, which has been considered as an s-wave $\bar{K}N$ quasi-bound state rather than an uds three-quark state [1]. The lightest scalar mesons in a nonet state, $\sigma = f_0(500)$, $\kappa = K_0^*(800)$, $f_0(980)$, and $a_0(980)$ are also candidates of the exotic hadrons, and they may be multi-quark hadrons [2] or $K\bar{K}$ quasi-bound states for $f_0(980)$ and $a_0(980)$ [3]. Recently both $\Lambda(1405)$ and the lightest scalar mesons have been well described phenomenologically in the so-called chiral unitary approach in the meson-baryon [4] and meson-meson [5] scatterings, respectively. To pin down the structure of these hadrons, we here focus on their spatial size.

If a resonance state is a quasi-bound state with a small binding energy, one can expect that the resonance state has a larger spatial size than typical size of the constituents. Motivated by this expectation, the spatial structures of $\Lambda(1405)$ and σ are theoretically measured in the meson-baryon [6] and meson-meson [7] scatterings, respectively, and it is found that the baryonic and strangeness mean squared radii of $\Lambda(1405)$ are $\langle r^2\rangle_{\mathrm{B}} = 0.783 - 0.186i\ \mathrm{fm}^2$ and $\langle r^2\rangle_{\mathrm{S}} = -1.097 + 0.662i\ \mathrm{fm}^2$, respectively [6], while σ is a compact object with the mean squared scalar radius $\langle r^2\rangle = (0.19\pm 0.02) - (0.06\pm 0.02)i\ \mathrm{fm}^2$ [7]. However, due to the decay process, the mean squared radius of the resonance state is evaluated as a complex number, whose interpretation is not straightforward. In Ref. [8] we have overcome this difficulty by using the finite volume effect on the quasi-bound state. Intuitively, hadrons with a large (small) spatial size are sensitive (insensitive) to the finite volume effect. In the following, we quantitatively formulate this picture using the mass shift in the finite volume.

2. Structure of dynamically generated states with finite volume effect

Here we investigate structures of dynamically generated hadronic resonances obtained from the Bethe-Salpeter equation for the two-body scattering amplitude T_{ij} in an algebraic form:

$$T_{ij}(s) = V_{ij}(s) + \sum_k V_{ik}(s) G_k(s) T_{kj}(s) = \sum_k (1 - VG)^{-1}{}_{ik} V_{kj}, \tag{2.1}$$

where i, j, and k are channel indices, s is the Mandelstam variable, V is the interaction kernel to be fixed later, and G is the two-body loop integral. The resonances appear as poles in the scattering amplitude as follows:

$$T_{ij}(s) = \frac{g_i g_j}{s - s_{\mathrm{pole}}} + T_{ij}^{\mathrm{BG}}(s). \tag{2.2}$$

Here g_i is coupling constant of the resonances to the two-body state in channel i and $T_{ij}^{\mathrm{BG}}(s)$ represents a background contribution. The coupling constant contains information on the structure of the resonances, and it is discussed in Ref. [9] that the coupling constant g_i is related to the compositeness for the resonance with respect to the channel i, X_i, which is the amount of two-body states composing the resonance, in the following manner:

$$X_i = -g_i^2 \frac{dG_i}{ds}(s_{\rm pole}). \tag{2.3}$$

The compositeness X_i approaches unity if the state is dominated by the two-body component in channel i, while it becomes zero if the state does not contain the two-body component in channel i.

Next we consider the finite volume effect in a spatial box with the periodic boundary condition of size L. In the finite volume, the momentum in the loop integral is discretized as $\boldsymbol{q} = 2\pi \boldsymbol{n}/L$ ($\boldsymbol{n} \in \mathbb{Z}^3$) [10]. Then, if a stable bound state is put into a finite box, the mass of the bound state $M_{\rm B}$ is changed due to the finite volume effect, and its mass shift is predicted as [10]:

$$\Delta M_{\rm B}(L) = -\frac{3g^2}{8\pi M_{\rm B}^2 L}\exp[-\gamma L] + \mathscr{O}(e^{-\sqrt{2}\gamma L}), \quad \gamma \equiv \frac{\sqrt{-\lambda(M_{\rm B}^2, m^2, M^2)}}{2M_{\rm B}}. \tag{2.4}$$

with the masses of two constituents m and M. Since this mass shift formula contains the coupling constant squared g^2, which is equivalent to the residue in Eq. (2.2), the mass shift formula brings a possibility to determine the structure of the bound state with the finite volume effect. Namely, putting a bound state generated in Eq. (2.1) into a finite box with the momentum-discretized loop integral and measuring the mass shift $\Delta M_{\rm B}(L)$, one can deduce the coupling constant from the finite volume effect as:

$$g_{\rm FV} = \sqrt{\frac{\Delta M_{\rm B}(L)}{-3/(8\pi M_{\rm B}^2 L)\exp[-\gamma L]}}, \tag{2.5}$$

and further deduce the compositeness from the finite volume effect as:

$$X_{\rm FV} = -g_{\rm FV}^2 \frac{dG}{ds}(M_{\rm B}^2). \tag{2.6}$$

Then, since the mean squared distance between two constituents for the bound state is evaluated as $X/(4\mu B_{\rm E})$ with the reduced mass μ for the small binding energy $B_{\rm E}$, we define the mean squared distance from the finite volume effect as:

$$\langle r^2 \rangle_{\rm FV} = \frac{X_{\rm FV}}{4\mu B_{\rm E}}. \tag{2.7}$$

We emphasize that our procedure can be easily applied to Feshbach resonances with finite widths, which is a quasi-bound state of a higher energy channel embedded in the continuum of a lower channel. For Feshbach resonances, by putting the higher channel into a finite box while the lower channel being unchanged and identifying the real part of the resonance pole as the bound state mass $M_{\rm B} = {\rm Re}[W_{\rm pole}]$, we can deduce the coupling constant, compositeness, and mean squared distance with respect to the higher channel. Furthermore, it is important that we define the real-valued distance between constituents for the resonance states with respect to the closed channel.

Let us now utilize our procedure for physical resonances in the chiral unitary approach. Here we discuss $\Lambda(1405)$ and the scalar mesons σ, $f_0(980)$, and $a_0(980)$ in coupled-channels problems with the the lowest-order s-wave chiral interaction. Details of the model setup and the model parameters are given in Ref. [8]. Our model in infinite volume dynamically generates resonance poles of two $\Lambda(1405)$ and the scalar mesons, which are shown as the filled symbols in Fig. 1. Then,

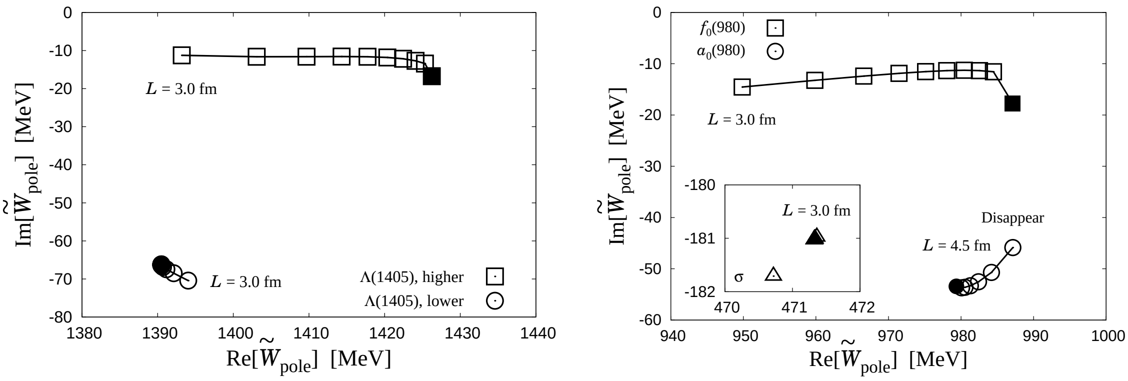

Figure 1: Behavior of the resonance pole positions for two $\Lambda(1405)$ (left) and three scalar mesons (right) in the complex energy plane [8]. Here filled symbols indicate the pole positions in infinite volume and open symbols are plotted in interval 0.5 fm with respect to the box size L from $L = 3.0$ fm to $L = 7.0$ fm.

Table 1: Properties of $\Lambda(1405)$ and $f_0(980)$ with finite volume effect [8].

$\Lambda(1405)$, higher pole		$f_0(980)$	
$g_{\bar{K}N,\mathrm{FV}}$	4.6 – 5.2 GeV	$g_{K\bar{K},\mathrm{FV}}$	2.7 – 3.1 GeV
$X_{\bar{K}N,\mathrm{FV}}$	0.82 – 1.03	$X_{K\bar{K},\mathrm{FV}}$	0.73 – 0.97
$\sqrt{\langle r^2 \rangle_{\bar{K}N,\mathrm{FV}}}$	1.7 – 1.9 fm	$\sqrt{\langle r^2 \rangle_{K\bar{K},\mathrm{FV}}}$	2.6 – 3.0 fm
$\sqrt{\langle R^2 \rangle_{\mathrm{size,FV}}}$	1.1 – 1.2 fm	$\sqrt{\langle R^2 \rangle_{\mathrm{size,FV}}}$	1.4 – 1.6 fm

we put the $\bar{K}N$ and $K\bar{K}$ channels into finite boxes with other channels being unchanged to extract the spatial size of these channels. Turning on the finite volume effect on the $\bar{K}N$ and $K\bar{K}$ channels, we observe shifts of the resonance pole positions according to the box size L, which are plotted as open symbols in interval 0.5 fm from $L = 3.0$ fm to $L = 7.0$ fm in Fig. 1. From the figure, we can find that the poles of higher $\Lambda(1405)$ and $f_0(980)$ move to lower energies while other poles do not show downward mass shift. This result implies that higher $\Lambda(1405)$ and $f_0(980)$ have large $\bar{K}N$ and $K\bar{K}$ components, respectively, but lower $\Lambda(1405)$, σ, and $a_0(980)$ are not dominated by the $\bar{K}N$ nor $K\bar{K}$ component. From the mass shift of higher $\Lambda(1405)$ and $f_0(980)$ due to the finite volume effect, we can extract the coupling constant via Eq. (2.5), and further the compositeness and the mean squared distance between two constituents for the resonances as real values via Eqs. (2.6) and (2.7), respectively. The results are listed in Table 1, and one can see that these resonances have nearly-unity $\bar{K}N$ and $K\bar{K}$ compositeness with large spatial extent between two constituents beyond the typical hadronic size $\lesssim 0.8$ fm.

Furthermore, with spatial structures of constituents taken into account, we can estimate the root mean squared radii of the resonances $\sqrt{\langle R^2 \rangle_{\mathrm{size,FV}}}$ from a kinematical consideration, which results in 1.1–1.2 fm for higher $\Lambda(1405)$ and 1.4–1.6 fm for $f_0(980)$. Therefore, the root mean squared radii of higher $\Lambda(1405)$ and $f_0(980)$ are larger than the typical hadronic scale $\lesssim 0.8$ fm.

Finally we mention that the resonance mass may have uncertainties of the half-width, which is a subtle problem because the mean distance of the state is sensitive to the binding energy as in Eq. (2.7). This point has been also discussed in Ref. [8] by changing the mass $M_{\mathrm{B}} = \mathrm{Re}[W_{\mathrm{pole}}] \to$

$\mathrm{Re}[W_{\mathrm{pole}}] - \Gamma/2$, and it is found that, although the mean squared distance decreases with the scaling $\propto 1/B_{\mathrm{E}}$, the compositeness stays similar values regardless of the mass uncertainties.

3. Summary

We have investigated the structures of the dynamically generated hadronic resonances in the chiral unitary approach from the finite volume effect. For this purpose we have established the relation between the mass shift of a Feshbach resonance coming from the finite volume effect and the mean squared distance between two constituents of the resonance. Especially we can define the real-valued size of the resonance in a given closed channel from response to the finite volume effect on the channel. Utilizing this relation to the physical hadrons, we investigate the structure of $\Lambda(1405)$ and scalar mesons σ, $f_0(980)$, and $a_0(980)$ with respect to the $\bar{K}N$ and $K\bar{K}$ components. We have found that the poles of the higher $\Lambda(1405)$ and $f_0(980)$ move to lower energies while other poles do not show downward mass shift, which implies large $\bar{K}N$ and $K\bar{K}$ components inside higher $\Lambda(1405)$ and $f_0(980)$, respectively. The compositeness of $\bar{K}N$ ($K\bar{K}$) inside $\Lambda(1405)$ [$f_0(980)$] is evaluated as 0.82 – 1.03 (0.73 – 0.97) and the mean distance between two constituents is 1.7–1.9 fm (2.6–3.0 fm). Furthermore, from a kinematical consideration the root mean squared radii of the resonances are evaluated as 1.1–1.2 fm for higher $\Lambda(1405)$ and 1.4–1.6 fm for $f_0(980)$. As a consequence, both the root mean squared distances and radii for $\Lambda(1405)$ and $f_0(980)$ are larger than the typical hadronic scale $\lesssim 0.8$ fm.

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