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# Skyrmions with vector mesons revisited

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In order to develop a model that can describe both a single baryon and multi-baryon systems on the same footing, we re-investigate the Skyrme model in a chiral Lagrangian derived from the hidden local symmetry (HLS) up to $O(p^4)$ including the homogeneous Wess-Zumino terms. We use the master formulas that connect the parameters of the HLS Lagrangian and a class of holographic QCD models, which provides a controllable way to determine the low-energy constants of the Lagrangian once the pion decay constant and the vector meson mass are given. Therefore, this model allows us to study the role of vector mesons in the skyrmion structure. We find that the $\rho$ and $\omega$ vector mesons have different roles in the skyrmion structure and that the $\omega$ meson has an important role in the properties of the nucleon.

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## 1. Introduction

The Skyrme model is expected to provide a unified way to describe both a single baryon and multi-baryon systems as suggested by Refs. [1, 2]. Starting with a mesonic chiral Lagrangian, the single baryon emerges as a skyrmion and the multi-baryon systems are simulated, for example, by putting skyrmions on a crystal lattice.

However, the previous works along this line suffer from the ambiguities in determining the form and the low-energy constants of the effective chiral Lagrangian. In fact, when we consider higher-order chiral interactions or introduce more mesonic degrees of freedom in the Lagrangian, there are too many parameters and it is not possible to control them to understand the role of each meson in the skyrmion structure [3, 4].

In a series of publications [5, 6, 7, 8], we have investigated the properties of a single skyrmion and the role of vector mesons in the skyrmion structure and in the skyrmion crystal that simulates dense baryonic matter. In this article, we focus on the role of vector mesons in the single skyrmion structure and our study on nuclear matter in this approach is reported in Ref. [9]. The main idea of this approach is to start with holographic QCD models and integrate out infinite towers of mesons except a few low-lying mesons [10]. It then leads to a chiral Lagrangian whose low-energy constants are fixed by a few inputs through the master formula. Therefore, we can control the parameters of the Lagrangian, which makes it possible to study the skyrmion structure in a systematic way. In this work, we use the effective Lagrangian of pion and $\rho/\omega$ mesons up to $O(p^4)$ in the hidden local symmetry (HLS) scheme. Thanks to the master formula we have only three input parameters, namely, the pion decay constant, vector meson mass, and the HLS parameter $a$. However, the physical quantities we are considering here are independent of $a$, so the number of independent parameters is reduced to two.

Another point that should be addressed compared to other holographic QCD models [11] is the inclusion of the $\omega$ vector meson that can be introduced through the homogeneous Wess-Zumino terms. In holographic QCD, this is equivalent to include the five-dimensional Chern-Simons term, and there is no additional free parameters.

## 2. Soliton properties and the role of vector mesons

The basic building blocks of the HLS Lagrangian are the two 1-forms $\hat{\alpha}_{\parallel\mu}$ and $\hat{\alpha}_{\perp\mu}$ defined by

$$\hat{\alpha}_{\parallel\mu} = \frac{1}{2i}\left(D_\mu \xi_R \xi_R^\dagger + D_\mu \xi_L \xi_L^\dagger\right), \qquad \hat{\alpha}_{\perp\mu} = \frac{1}{2i}\left(D_\mu \xi_R \xi_R^\dagger - D_\mu \xi_L \xi_L^\dagger\right), \tag{2.1}$$

with the chiral fields $\xi_L$ and $\xi_R$, which are written in the unitary gauge as $\xi_L^\dagger = \xi_R \equiv \xi = e^{i\pi/2f_\pi}$. The vector mesons, which are the gauge bosons of the HLS, are introduced through

$$D_\mu \xi_{R,L} = (\partial_\mu - iV_\mu)\xi_{R,L}, \tag{2.2}$$

where $V_\mu = \frac{g}{2}(\omega_\mu + \rho_\mu)$ with $g$ being the gauge coupling constant.

Then the chiral Lagrangian up to $O(p^4)$ can be written as

$$\mathscr{L}_{\rm HLS} = \mathscr{L}_{(2)} + \mathscr{L}_{(4)} + \mathscr{L}_{\rm anom}, \tag{2.3}$$

where $\mathscr{L}_{(2)}$ and $\mathscr{L}_{(4)}$ are the terms of $O(p^2)$ and $O(p^4)$, respectively, and $\mathscr{L}_{\rm anom}$ is the homogeneous Wess-Zumino terms. The explicit expressions for the interactions and the master formula that determines the low-energy constants of this model can be found, for example, in Ref. [6]. In this work, we use $f_\pi = 92.4$ MeV and $m_\rho = 775.5$ MeV, and we use the Sakai-Sugimoto model [12].

The soliton wave functions can be obtained by solving the equations of motion of the meson fields. For a single baryon that carries unit baryon number, the solitonic solution can be found by using the following configurations,

$$\xi(\boldsymbol{r}) = \exp\left[i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\frac{F(r)}{2}\right], \quad \omega_\mu = W(r)\,\delta_{0\mu}, \quad \rho_0 = 0, \quad \boldsymbol{\rho} = \frac{G(r)}{gr}\,(\hat{\boldsymbol{r}}\times\boldsymbol{\tau}) \tag{2.4}$$

with the boundary conditions

$$F(0) = \pi, \quad F(\infty) = 0, \quad G(0) = -2, \quad G(\infty) = 0, \quad W'(0) = 0, \quad W(\infty) = 0. \tag{2.5}$$

In order to describe a realistic baryon of definite spin and isospin quantum numbers, the classical configuration should be quantized. In this work, we follow the standard collective quantization method [13], which transforms the meson fields as

$$\begin{aligned} \xi(\boldsymbol{r}) &\to \xi(\boldsymbol{r},t) = A(t)\,\xi(\boldsymbol{r})A^\dagger(t), \\ V_\mu(\boldsymbol{r}) &\to V_\mu(\boldsymbol{r},t) = A(t)\,V_\mu(\boldsymbol{r})A^\dagger(t), \end{aligned} \tag{2.6}$$

where $A(t)$ is a time-dependent SU(2) matrix, which introduces the angular velocity $\boldsymbol{\Omega}$ of the collective coordinate rotation as

$$i\boldsymbol{\tau}\cdot\boldsymbol{\Omega} \equiv A^\dagger(t)\partial_0 A(t). \tag{2.7}$$

Under this rotation, the space component of the $\omega$ field and the time component of the $\rho$ field get excited and their most general forms are found as [14]

$$\begin{aligned} \rho^0(\boldsymbol{r},t) &= A(t)\frac{2}{g}\left[\boldsymbol{\tau}\cdot\boldsymbol{\Omega}\,\xi_1(r) + \hat{\boldsymbol{\tau}}\cdot\hat{\boldsymbol{r}}\,\boldsymbol{\Omega}\cdot\hat{\boldsymbol{r}}\,\xi_2(r)\right]A^\dagger(t), \\ \omega^i(\boldsymbol{r},t) &= \frac{\varphi(r)}{r}\,(\boldsymbol{\Omega}\times\hat{\boldsymbol{r}})^i, \end{aligned} \tag{2.8}$$

with the boundary conditions given by

$$\xi_1'(0) = \xi_1(\infty) = 0, \quad \xi_2'(0) = \xi_2(\infty) = 0, \quad \varphi(0) = \varphi(\infty) = 0, \tag{2.9}$$

In order to understand the role of vector mesons, we consider three models. The first is the full model in this approach that contains $\pi$, $\rho$, and $\omega$ mesons explicitly. We call this model $\mathrm{HLS}_1(\pi,\rho,\omega)$. To see the role of the $\omega$ meson, we decouple the $\omega$ meson from the full model. This can be achieved by neglecting the homogeneous Wess-Zumino terms. This is the model $\mathrm{HLS}_1(\pi,\rho)$. Finally, to see the role of the $\rho$ meson, we consider the model $\mathrm{HLS}_1(\pi)$ by integrating out the $\rho$ meson in $\mathrm{HLS}_1(\pi,\rho)$. Then the soliton wave functions can be obtained by solving the equations of motion. The obtained results are given in Fig. 1. (The solution for the $\omega$ meson wave function can be found in Ref. [6].)

When the wave functions are obtained, it is straightforward to calculate the mass and radius of a single baryon. In Table 1 we give the soliton mass, the mass difference between the nucleon and the $\Delta$, and the rms energy radius $\sqrt{\langle r^2\rangle_E}$ obtained in three models.

From these results, we can know the followings.



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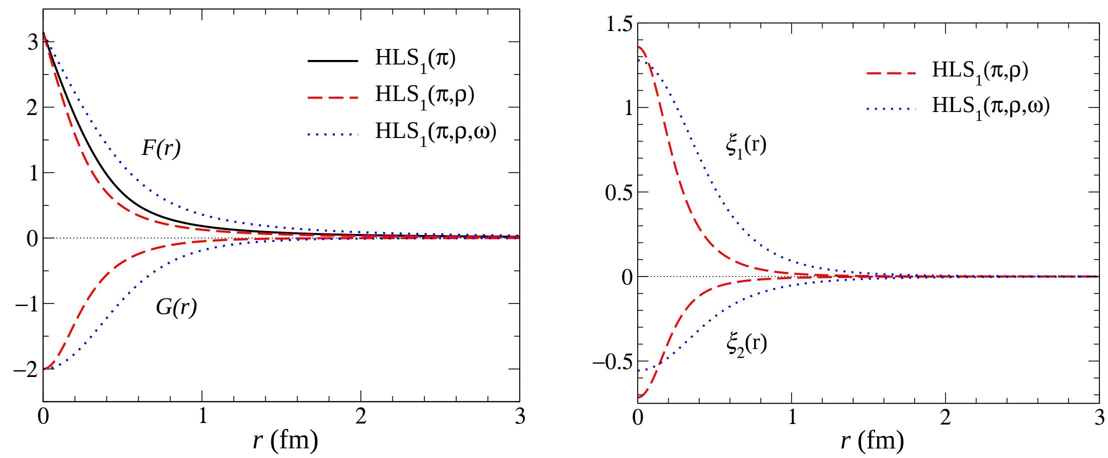


**Figure 1:** Comparison of the soliton wave functions $F(r)$, $G(r)$ $\xi_1(r)$, and $\xi_2(r)$ in the models of $\mathrm{HLS}_1(\pi)$, $\mathrm{HLS}_1(\pi,\rho)$, and $\mathrm{HLS}_1(\pi,\rho,\omega)$, which are represented by the solid line, dashed lines, and dotted lines, respectively.

- The inclusion of the $\rho$ meson reduces the soliton mass, which is consistent with the claim made in Refs. [11, 16] that the inclusion of isovector vector mesons makes the skyrmion closer to the BPS soliton. However, we find that the inclusion of the isoscalar $\omega$ vector meson increases the soliton mass. The different role of these mesons can be seen in Fig. 1 which shows that the $\rho$ meson shrinks the soliton wave functions, while the $\omega$ meson has the opposite effects.
- In the moment of inertia, which determines the $\Delta$-$N$ mass difference $\Delta_M$ in the standard collective quantization, the $\rho$ and $\omega$ vector mesons have the opposite role again, namely, the $\rho$ meson increases $\Delta_M$, while the $\omega$ meson decreases it. As a result, in the absence of the $\omega$ meson, $\Delta_M$ that is the quantity of $O(1/N_c)$ becomes even larger than the soliton mass that is of $O(N_c)$, which then causes a serious problem in the validity of the standard collective quantization method. Therefore, the inclusion of the $\omega$ meson is important not only in phenomenology but also to justify the standard collective quantization method.

## 3. Discussion

In summary, we have investigated the role of vector mesons in the skyrmion structure using

| | $\mathrm{HLS}_1(\pi,\rho,\omega)$ | $\mathrm{HLS}_1(\pi,\rho)$ | $\mathrm{HLS}_1(\pi)$ | $O(p^2)+\omega_\mu B^\mu$ [14] | $O(p^2)$ [15] |
|---|---|---|---|---|---|
| $M_{\mathrm{sol}}$ | 1184 | 834 | 922 | 1407 | 1026 |
| $\Delta_M$ | 448 | 1707 | 1014 | 259 | 1131 |
| $\sqrt{\langle r^2\rangle_E}$ | 0.608 | 0.371 | 0.417 | 0.725 | 0.422 |

**Table 1:** Skyrmion mass and size calculated in the HLS within the Sakai-Sugimoto model with $a=2$. The soliton mass $M_{\mathrm{sol}}$ and the $\Delta$-N mass difference $\Delta_M$ are in unit of MeV while $\sqrt{\langle r^2\rangle_E}$ is in unit of fm. The column of $O(p^2)+\omega_\mu B^\mu$ is "the minimal model" of Ref. [14] and that of $O(p^2)$ corresponds to the model of Ref. [15].

the HLS Lagrangian up to $O(p^4)$, which is matched by holographic QCD models by integrating out the vector mesons other than the lowest $\rho$ and $\omega$ vector mesons. The parameters of the effective Lagrangian are determined by the master formula except the pion decay constant and the vector meson mass. In particular, we have studied the role of the $\omega$ meson in this work by including the anomalous parity terms. We find that the inclusion of the $\omega$ meson has an important role not only in the properties of a single baryon but also in the justification of the use of the standard collective quantization method. The crucial role of the $\omega$ meson is also seen in the properties of baryonic matter in the skyrmion crystal calculation as reported in Ref. [9]. However, within a chiral Lagrangian of pion only, it was shown in Ref. [17] that the one-loop corrections are important to get the correct nucleon properties with the physical input parameters. Thus the work in this direction is desired to understand the soliton structure more rigorously.

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