Nucleon $\Sigma$ Term in the Chiral Mixing Approach

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We calculate the $\Sigma_{\pi N}$ term in the chiral mixing approach to baryons, i.e., with $SU_L(3) \times SU_R(3)$ chiral multiplets $[(6,3) \oplus (3,6)], [(3,\bar{3}) \oplus (\bar{3},3)]$ and $[(\bar{3},3) \oplus (3,\bar{3})]$, admixed in the baryons, using known constraints on the current quark masses $m^0_u, m^0_d$. We show that the $[(6,3) \oplus (3,6)]$ multiplet makes a contribution enhanced by a factor of $\frac{57}{9} \approx 6.33$, (purely due to $SU_L(2) \times SU_R(2)$ algebra) that leads to $\Sigma_{\pi N} \geq (1 + \frac{57}{9} \sin^2 \theta) \frac{3}{2} (m^0_u + m^0_d) = 60$ MeV, in general accord with “experimental” values of $\Sigma_{\pi N}$. The chiral mixing angle $\theta$ is given by $\sin^2 \theta = \frac{3}{5} \left( g_{A}^{(0)} + g_{A}^{(3)} \right)$, where $g_{A}^{(0)} = 0.33 \pm 0.08$, or $0.28 \pm 0.16$, and $g_{A}^{(3)} = 1.267$, are the flavor singlet and third component of the octet axial couplings. These results show that there is no need for $q^s \bar{q}$ components, and in particular, no need for an $s \bar{s}$ component in the nucleon.

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1. Introduction

For more than a quarter-century any deviation of the nucleon $\Sigma_{\pi N}$ term extracted from the measured $\pi N$ scattering partial wave analyses (in the following to be called “measured value”, for brevity) from 25 MeV was interpreted as an increase of Zweig-rule-breaking in the nucleon, or equivalently to an increased $s\bar{s}$ content $y = \frac{2\langle |N|s\bar{s}|N\rangle}{\langle N|d_{u}\bar{d}_{u}+s_{s}\bar{s}_{s}|N\rangle}$ of the nucleon, Refs. [123]. As all “measurements” of $\Sigma_{\pi N}$ have yielded values ranging from 55 MeV to 75 MeV [2], that are substantially larger than the expected 25 MeV, it has consequently appeared that the $s\bar{s}$ content of the nucleon must be (very) large.

A number of experiments have measured the $s\bar{s}$ contributions to nucleon observables other than the $\Sigma_{\pi N}$ term [3]. Not one of them has found a result larger than a few % of the $u$ (and/or $\bar{u}$) and $d$ (and/or $\bar{d}$) contributions, thus making the $s\bar{s}$ content of the nucleon effectively negligible $y \approx 0$. Thus, the enigma has deepened: how is it possible to have such a large $\Sigma_{\pi N}$ term without any $s\bar{s}$ content in other observables? In the meantime the nucleon $\Sigma_{\pi N}$ term has been shown as an important ingredient in searches for (supersymmetric) cold dark matter, Ref. [7] and in the QCD phase diagram, thereby only increasing the stakes.

In this report we show explicitly an alternative mechanism of hadronic $\Sigma_{\pi N}$ term enhancement with strangeness content $y = 0$ and pin-point the source of the enhancement to the $(6, 3) = [(6, 3) \oplus (3, 6)] \rightarrow (1, 1) = [(1, 1) \oplus (1, 1)]$ chiral component (in the $SU_{L}(3) \times SU_{R}(3)$ and $SU_{L}(2) \times SU_{R}(2)$ notations, respectively) of the nucleon. This component contributes about three quarters of the enhanced value of $\Sigma_{\pi N} \geq 55$ MeV, which would otherwise be $\geq 14$ MeV, while keeping a vanishing $s\bar{s}$ component in the nucleon. The same $(1, 1)$ chiral component is crucial for the proper description of the nucleon’s isovector axial coupling $g_{A}^{(3)} = 1.267$.

We show in some detail how the $\Sigma_{\pi N}$ term enhancement emerges from the $SU_{L}(2) \times SU_{R}(2)$ chiral algebra. To that end we use a hadronic two-flavor $SU_{L}(2) \times SU_{R}(2)$ chiral mixing model, in which the $s\bar{s}$ content of the nucleon vanishes, $y = 0$, *per definitionem*. Baryons in the spontaneously broken symmetry phase may be effectively described by a few chiral components: it was shown in Refs. [89110], that several nucleon’s properties can be successfully described by mixing of three chiral multiplet components. Of the two viable chiral mixing scenarios, only the Harari one [210], described by

$$|N\rangle = \sin \theta |(6, 3)\rangle + \cos \theta (\cos \varphi |(3, \bar{3})\rangle + \sin \varphi |(\bar{3}, 3)\rangle),$$

(1.1)

has survived the inclusion of the baryons’ anomalous magnetic moments in the three-flavor case [110]. Here we use the original $SU_{L}(3) \times SU_{R}(3)$ notation, so as to distinguish between the two kinds of $(\frac{1}{2}, 0)$ multiplets in $SU_{L}(2) \times SU_{R}(2)$, though we shall need only the two-flavor version.

2. Calculation

To calculate the nucleon $\Sigma_{\pi N}$ term, we use the $\Sigma$ operator defined as the double commutator

$$\Sigma = \frac{1}{3} \delta^{ab}[Q^{a}_{b}, [Q^{b}_{c}, H_{XSB}]],$$

(2.1)

1This makes these effects compatible with the (much more) mundane isospin-violating corrections, from which they are indistinguishable [9].
of the axial charges \(Q^a_5\) and the chiral symmetry breaking Hamiltonian \(H_{\chi_{SB}}\). It was introduced by Dashen as a way of separating out the explicit chiral \(SU_L(2) \times SU_R(2)\) symmetry breaking part \(H_{\chi_{SB}}\) from the total Hamiltonian. Its (diagonal) nucleon matrix element \(\Sigma_{\chi_{SB}} = \langle N|\Sigma|N\rangle\) determines the shift of the nucleon mass due to the chiral symmetry breaking current quark masses. Ensuring that the (spontaneously broken) chiral symmetry is properly implemented is particularly important in a calculation at the hadron level. In a series of papers we have developed a (linear realization) chiral Lagrangian that reproduces the results of the phenomenological chiral mixing method.

We follow Ref. [12], and use an explicit \(\chi_{SB}\) “bare” nucleon mass and the corresponding \(\chi_{SB}\) Hamiltonian density

\[
\mathcal{H}^N_{\chi_{SB}} = \sum_{i=1}^{3} \tilde{N}_i M_\Delta^0 N_i + \tilde{\Delta}_{(1/2)} M_\Delta^0 \Delta_{(1/2)},
\]

where \(i\) stands for the three chiral multiplets \((1, 1/2), (1, 0), (0, 1/2)\). \textit{A priori}, we do not know the values of the “current” nucleon masses, except for a lower limit: they cannot be smaller than three isospin-averaged current quark masses: \(M_N^0 = 3m_q^0 = \frac{3}{2} (m_u^0 + m_d^0)\). For simplicity’s sake, we shall assume, as a first approximation, that all three chiral components have the same “current” nucleon mass \(M_N^0 = M_N^{0(6, 3)} = M_N^{0(1, 1)} = M_N^{0(3, 3)} = M_N^{0(3, 0)} = M_N^{0(0, 1)} = \frac{3}{2} (m_u^0 + m_d^0)\).

The chiral \(SU_L(2) \times SU_R(2)\) generators \(Q^a_5\) and their commutators with fields were worked out in Refs. [12, 13, 14, 15, 16, 17, 18, 19, 20]:

\[
\begin{align*}
[Q^a_5, N_{(1, 1/2)}] &= \gamma_5 \left( \frac{5}{3} \tau^a N_{(1, 1/2)} + \frac{2}{\sqrt{3}} T^{a} \Delta_{(1/2)} \right), \\
[Q^a_5, \Delta_{(1/2)}] &= \gamma_5 \left( \frac{2}{\sqrt{3}} T^{a} N_{(1, 1/2)} + \frac{1}{3} \delta^{a} \Delta_{(1/2)} \right), \\
[Q^a_5, N_{(0, 1/2)}] &= \gamma_5 \frac{\tau^a}{2} N_{(0, 1/2)}, \\
[Q^a_5, N_{(1, 0)}] &= -\gamma_5 \frac{\tau^a}{2} N_{(1, 0)},
\end{align*}
\]

where \(a = 1, 2, 3, \tau_{(1/2)}^i\) are the isospin-\(\frac{1}{2}\) generators of the \(SU(2)\) group and \(T^{i}\) are the so-called iso-superion \((2 \times 4)\) matrices, with the following properties (see Appendix B of Ref. [13])

\[
\begin{align*}
T^{ik} T^k &= \frac{3}{4} \delta^{ij} - \frac{1}{6} \left\{ t^i_{(1/2)}, t^j_{(1/2)} \right\} + \frac{i}{3} \varepsilon^{ijk} t^k_{(1/2)}, \\
T^{i\dagger} T^{k\dagger} &= P^{ik}_{\frac{1}{2}},
\end{align*}
\]

The chiral \(SU_L(2) \times SU_R(2)\) double commutators for the \((1, 1/2)\) and \((1, 0)\) chiral multiplets are

\[
[Q^a_5, [Q^b_5, N_{(1, 1/2)}] N_{(1, 1/2)}] = \frac{41}{9} \delta^{ab} \tilde{N}_{(1, 1/2)} + \tilde{\Delta}_{(1, 1/2)} \left( 2 \delta^{ab} - \frac{4}{9} \left\{ t^a_{(1/2)}, t^b_{(1/2)} \right\} \right) \Delta_{(1, 1/2)} + \ldots
\]

where \(\ldots\) stand for the off-diagonal terms, such as \(\tilde{N}_{(1, 1/2)} \ldots \Delta_{(1, 1/2)}\), and their Hermitian conjugates.

\[2\] For normalization and notational conventions see Ref. [12].
We contract Eq. (2.7) with \( \frac{1}{3} \delta^{ab} \) (where summation over repeated indices is understood) to find:

\[
\frac{1}{3} \delta^{ab} [Q^b_5, [Q^a_5, \bar{N}(1, \frac{1}{2}) N(1, \frac{1}{2})]] = \frac{41}{9} \bar{N}(1, \frac{1}{2}) N(1, \frac{1}{2}) + \frac{8}{9} \bar{\Delta}(1, \frac{1}{2}) \Delta(1, \frac{1}{2}) + \ldots
\]  

(2.5)

where we have used the identity \( r^a_{(\frac{1}{2})} r^a_{(\frac{1}{2})} = \frac{15}{4} 1_{4 \times 4} \), and similarly for the \( \Delta \)-field contribution

\[
\frac{1}{3} \delta^{ab} [Q^b_5, [Q^a_5, \bar{\Delta}(1, \frac{1}{2}) \Delta(1, \frac{1}{2})]] = \frac{16}{9} \bar{N}(1, \frac{1}{2}) N(1, \frac{1}{2}) + \frac{13}{9} \bar{\Delta}(1, \frac{1}{2}) \Delta(1, \frac{1}{2}) + \ldots
\]  

(2.6)

This finally leads to

\[
\Sigma_{\pi N} = \sin^2 \theta \left( \frac{41}{9} M^0_{N(1, \frac{1}{2})} + \frac{16}{9} M^0_{\Delta(1, \frac{1}{2})} \right) + \cos^2 \theta \left( \cos^2 \varphi M^0_{N(1, \frac{1}{2})} + \sin^2 \varphi M^0_{\Delta(1, \frac{1}{2})} \right),
\]  

(2.7)

which is our basic result here.

3. Result and Discussion

Inserting our simplifying assumption that all the “current nucleon” masses are equal, one finds the final result

\[
\Sigma_{\pi N} = \left( 1 + \frac{16}{3} \sin^2 \theta \right) M^0_N.
\]  

(3.1)

Note that the enhancement term \( \frac{16}{3} \sin^2 \theta \) is due to the factor \( \frac{41 + 16}{9} = \frac{19}{3} \approx 6.33 \) appearing in Eq. (2.20) of the \( [(1, \frac{1}{2}) \oplus (\frac{1}{2}, 1)] \) chiral multiplet, which, in turn, is due to the presence of the iso-spinor \((2 \times 4)\) matrices \( T^i \), that are equivalent to the SU(2) Clebsch-Gordan coefficients \( \langle \frac{3}{2} I_3(\Delta) | I_3(i) \frac{1}{2} I_3(N) \rangle \). Thus, the enhancement factor \( \frac{19}{3} \) in Eq. (2.7) and consequently also the \( \frac{16}{3} \sin^2 \theta \) in Eq. (3.1), are of \( SU_L(2) \times SU_R(2) \) algebraic origin. This shows that, irrespective of the specific value of the chiral mixing angle \( \theta \), there is room for improvement of the \( \Sigma_{\pi N} \) predictions within the chiral \( SU_L(2) \times SU_R(2) \) algebra approach.

The relevant chiral mixing angle \( \theta \) has been extracted in Refs. [15, 16, 17, 18] as \( \frac{8}{3} \sin^2 \theta = g_A^{(0)} + g_A^{(3)} \), a function of \( g_A^{(3)} \), and the flavor-singlet axial coupling \( g_A^{(0)} = 0.28 \pm 0.16 \) [22], or \( g_A^{(0)} = 0.33 \pm 0.03 \pm 0.05 \) [23]. Here we have taken the values of current quark masses from PDG2012 [24]: \( m_u^0 = 2.3 \times 1.35 \) MeV and \( m_d^0 = 4.8 \times 1.35 \) MeV, yielding \( \frac{1}{2} (m_u^0 + m_d^0) \approx 4.73 \) MeV, substantially lower than before (cf. 7.6 MeV in Ref. [25]), and inserted them into the current nucleon mass to find \( M^0_N = \frac{3}{2} (m_u^0 + m_d^0) \approx 14.2 \) MeV and \( \Sigma_{\pi N} = 59.5 \pm 2.3 \) MeV, with \( g_A^{(0)} = 0.33 \pm 0.03 \pm 0.05 \) [23], or \( \Sigma_{\pi N} = 58.0 \pm 4.5 \) MeV, with \( g_A^{(0)} = 0.28 \pm 0.16 \) [22], in fair agreement with the “observed” \( \Sigma_{\pi N} \) value range (55 - 75) MeV, see Ref. [1].

The above result Eq. (3.1) ought to be viewed as a lower bound on the “true” \( \Sigma_{\pi N} \) value, as we have assumed that all current nucleon masses \( M^0_N \) equal three times the isospin-averaged current quark mass \( \bar{m}_q \), which is appropriate only when all chiral components of the nucleon consist of three-quark fields. That, however, condition is not necessary (because there are \( q^i \bar{q} \) baryon fields that belong to the same chiral multiplets [24], and such fields would have a larger current mass \( \langle M^0_N \rangle = 5 \bar{m}_q \), that would consequently lead to a higher value of \( \Sigma_{\pi N} \), but merely a sufficient one, as all of these chiral multiplets exist as bi-local three-quark fields [24]).
In summary, the “observed” $\Sigma_{P_N}$ term values ($\geq 55$ MeV) have often been interpreted as a sign of a substantial $s\bar{s}$ content of the nucleon. Here we have shown that values of $\Sigma_{P_N} \geq 55$ MeV are readily obtained in the chiral-mixing approach without any strangeness degrees of freedom in the nucleon, as a natural consequence of the substantial chiral $(6,3) = [(6,3) \oplus (3,6)] \rightarrow (1, \frac{1}{2})$ multiplet component. The precise value of $\Sigma_{P_N}$ is a linear function of the sum of the flavor-singlet $g_A^{(0)}$, and the isovector $g_A^{(3)}$ axial coupling of the nucleon.

References