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Nucleon Σ Term in the Chiral Mixing Approach

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We calculate the $\Sigma_{\pi N}$ term in the chiral mixing approach to baryons, i.e., with $SU_L(3) \times SU_R(3)$ chiral multiplets $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$, $[\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ and $[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$, admixed in the baryons, using known constraints on the current quark masses m_u^0, m_d^0. We show that the $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$ multiplet makes a contribution enhanced by a factor of $\frac{57}{9} \simeq 6.33$, (purely due to $SU_L(2) \times SU_R(2)$ algebra) that leads to $\Sigma_{\pi N} \geq \left(1 + \frac{48}{9}\sin^2\theta\right)\frac{3}{2}\left(m_u^0 + m_d^0\right) = 60$ MeV, in general accord with "experimental" values of $\Sigma_{\pi N}$. The chiral mixing angle θ is given by $\sin^2\theta = \frac{3}{8}\left(g_A^{(0)} + g_A^{(3)}\right)$, where $g_A^{(0)} = 0.33 \pm 0.08$, or 0.28 ± 0.16, and $g_A^{(3)} = 1.267$, are the flavor singlet and third component of the octet axial couplings. These results show that there is no need for $q^4\bar{q}$ components, and in particular, no need for an $s\bar{s}$ component in the nucleon.

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PoS(Hadron 2013)120

1. Introduction

For more than a quarter-century any deviation of the nucleon $\Sigma_{\pi N}$ term extracted from the measured πN scattering partial wave analyses (in the following to be called "measured value", for brevity) from 25 MeV was interpreted as an increase of Zweig-rule-breaking in the nucleon, or equivalently to an increased $s\bar{s}$ content $y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{u}u|N\rangle}$ of the nucleon, Refs. [1, 2, 3]. As all "measurements" of $\Sigma_{\pi N}$ have yielded values ranging from 55 MeV to 75 MeV [4], that are substantially larger than the expected 25 MeV, it has consequently appeared that the $s\bar{s}$ content of the nucleon must be (very) large.

A number of experiments have measured the $s\bar{s}$ contributions to nucleon observables other than the $\Sigma_{\pi N}$ term [5]. Not one of them has found a result larger than a few % of the u (and/or $\bar{u}$) and d (and/or $\bar{d}$) contributions, [1] thus making the $s\bar{s}$ content of the nucleon effectively negligible $y \simeq 0$. Thus, the enigma has deepened: how is it possible to have such a large $\Sigma_{\pi N}$ term without any $s\bar{s}$ content in other observables? In the meantime the nucleon $\Sigma_{\pi N}$ term has been shown as an important ingredient in searches for (supersymmetric) cold dark matter, Ref. [7] and in the QCD phase diagram, thereby only increasing the stakes.

In this report we show explicitly an alternative mechanism of hadronic $\Sigma_{\pi N}$ term enhancement with strangeness content $y = 0$ and pin-point the source of the enhancement to the $(6,3) = [(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})] \to (1,\frac{1}{2}) = [(\mathbf{1},\frac{\mathbf{1}}{\mathbf{2}}) \oplus (\frac{\mathbf{1}}{\mathbf{2}},\mathbf{1})]$ chiral component (in the $SU_L(3) \times SU_R(3)$ and $SU_L(2) \times SU_R(2)$ notations, respectively) of the nucleon. This component contributes about three quarters of the enhanced value of $\Sigma_{\pi N} \geq 55$ MeV, which would otherwise be ≥ 14 MeV, while keeping a vanishing $s\bar{s}$ component in the nucleon. The same $(1,\frac{1}{2})$ chiral component is crucial for the proper description of the nucleon's isovector axial coupling $g_A^{(3)} = 1.267$.

We show in some detail how the $\Sigma_{\pi N}$ term enhancement emerges from the $SU_L(2) \times SU_R(2)$ chiral algebra. To that end we use a hadronic two-flavor $SU_L(2) \times SU_R(2)$ chiral mixing model, in which the $s\bar{s}$ content of the nucleon vanishes, $y = 0$, *per definitionem*. Baryons in the spontaneously broken symmetry phase may be effectively described by a few chiral components: it was shown in Refs. [8, 9, 10], that several nucleon's properties can be successfully described by mixing of three chiral multiplet components. Of the two viable chiral mixing scenarios, only the Harari one [9, 10], described by

$$|N\rangle \; = \; \sin\theta|(6,3)\rangle + \cos\theta(\cos\varphi|(3,\bar{3})\rangle + \sin\varphi|(\bar{3},3)\rangle)\,, \tag{1.1}$$

has survived the inclusion of the baryons' anomalous magnetic moments in the three-flavor case [11]. Here we use the original $SU_L(3) \times SU_R(3)$ notation, so as to distinguish between the two kinds of $(\frac{1}{2},0)$ multiplets in $SU_L(2) \times SU_R(2)$, though we shall need only the two-flavor version.

2. Calculation

To calculate the nucleon $\Sigma_{\pi N}$ term, we use the Σ operator defined as the double commutator

$$\Sigma \; = \; \frac{1}{3}\delta^{ab}[Q_5^a,[Q_5^b,H_{\chi\mathrm{SB}}]]\,, \tag{2.1}$$

[1] This makes these effects compatible with the (much more) mundane isospin-violating corrections, from which they are indistinguishable [6].

of the axial charges Q_5^a and the chiral symmetry breaking Hamiltonian $H_{\chi SB}$ [2]. It was introduced by Dashen [13] as a way of separating out the explicit chiral $SU_L(2) \times SU_R(2)$ symmetry breaking part $H_{\chi SB}$ from the total Hamiltonian. Its (diagonal) nucleon matrix element $\Sigma_{\pi N} = \langle N|\Sigma|N\rangle$ determines the shift of the nucleon mass due to the chiral symmetry breaking current quark masses [13, 14]. Ensuring that the (spontaneously broken) chiral symmetry is properly implemented is particularly important in a calculation at the hadron level. In a series of papers [11, 15, 16, 17, 18, 19, 20, 21] we have developed a (linear realization) chiral Lagrangian that reproduces the results of the phenomenological chiral mixing method.

We follow Ref. [12], and use an explicit χSB "bare" nucleon mass and the corresponding χSB Hamiltonian density

$$\mathscr{H}_{\chi SB}^{\rm N} = \sum_{i=1}^{3} \bar{N}_i M_{N_i}^0 N_i + \bar{\Delta}_{(1,\frac{1}{2})} M^0_{\Delta(1,\frac{1}{2})} \Delta_{(1,\frac{1}{2})} ,$$

where i stands for the three chiral multiplets $(1,\frac{1}{2})$, $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$. *A priori*, we do not know the values of the "current" nucleon masses, except for a lower limit: they cannot be smaller than three isospin-averaged current quark masses: $M_{N_i}^0 \geq 3\bar{m}_q^0 = \frac{3}{2}\left(m_u^0 + m_d^0\right)$. For simplicity's sake, we shall assume, as a first approximation, that all three chiral components have the same "current" nucleon mass $M_N^0 = M^0_{N(6,3)} = M^0_{N(1,\frac{1}{2})} = M^0_{\Delta(1,\frac{1}{2})} = M^0_{(3,\bar{3})} = M^0_{(\frac{1}{2},0)} = M^0_{(\bar{3},3)} = M^0_{(0,\frac{1}{2})} = \frac{3}{2}\left(m_u^0 + m_d^0\right)$.

The chiral $SU_L(2) \times SU_R(2)$ generators Q_5^a and their commutators with fields were worked out in Refs. [16, 17, 18, 19]:

$$\begin{aligned}
[Q_5^a, N_{(1,\frac{1}{2})}] &= \gamma_5 \left(\frac{5}{3}\frac{\tau^a}{2} N_{(1,\frac{1}{2})} + \frac{2}{\sqrt{3}} T^a \Delta_{(1,\frac{1}{2})} \right) , \\
[Q_5^a, \Delta_{(1,\frac{1}{2})}] &= \gamma_5 \left(\frac{2}{\sqrt{3}} T^{\dagger a} N_{(1,\frac{1}{2})} + \frac{1}{3} t^a_{(3/2)} \Delta_{(1,\frac{1}{2})} \right) , \\
[Q_5^a, N_{(\frac{1}{2},0)}] &= \gamma_5 \frac{\tau^a}{2} N_{(\frac{1}{2},0)} , \\
[Q_5^a, N_{(0,\frac{1}{2})}] &= -\gamma_5 \frac{\tau^a}{2} N_{(0,\frac{1}{2})} ,
\end{aligned} \tag{2.2}$$

where $a = 1,2,3$, $t^i_{(\frac{3}{2})}$ are the isospin-$\frac{3}{2}$ generators of the $SU(2)$ group and T^i are the so-called iso-spurion (2×4) matrices, with the following properties (see Appendix B of Ref. [18])

$$\begin{aligned}
T^{i\dagger} T^k &= \frac{3}{4}\delta^{ik} - \frac{1}{6}\left\{ t^i_{(\frac{3}{2})}, t^j_{(\frac{3}{2})} \right\} + \frac{i}{3}\varepsilon^{ijk} t^k_{(\frac{3}{2})} , \\
T^i T^{k\dagger} &= P^{ik}_{\frac{3}{2}} .
\end{aligned} \tag{2.3}$$

The chiral $SU_L(2) \times SU_R(2)$ double commutators for the $(1,\frac{1}{2})$ and $(\frac{1}{2},1)$ chiral multiplet are

$$[Q_5^b, [Q_5^a, \bar{N}_{(1,\frac{1}{2})} N_{(1,\frac{1}{2})}]] = \frac{41}{9}\delta^{ab} \bar{N}_{(1,\frac{1}{2})} N_{(1,\frac{1}{2})} + \bar{\Delta}_{(1,\frac{1}{2})} \left(2\delta^{ab} - \frac{4}{9}\left\{ t^a_{(\frac{3}{2})}, t^b_{(\frac{3}{2})} \right\} \right) \Delta_{(1,\frac{1}{2})} + \ldots \tag{2.4}$$

where $\ldots$ stand for the off-diagonal terms, such as $\bar{N}_{(1,\frac{1}{2})}(\ldots)\Delta_{(1,\frac{1}{2})}$, and their Hermitian conjugates.

[2]For normalization and notational conventions see Ref. [12].

We contract Eq. (2.4) with $\frac{1}{3}\delta^{ab}$ (where summation over repeated indices is understood) to find:

$$\frac{1}{3}\delta^{ab}[Q_5^b,[Q_5^a,\bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})}]] = \frac{41}{9}\bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})} + \frac{8}{9}\bar{\Delta}_{(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})} + \dots \tag{2.5}$$

where we have used the identity $t^a_{(\frac{3}{2})}t^a_{(\frac{3}{2})} = \frac{15}{4}\mathbf{1}_{4\times4}$, and similarly for the Δ-field contribution

$$\frac{1}{3}\delta^{ab}[Q_5^b,[Q_5^a,\bar{\Delta}_{(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})}]] = \frac{16}{9}\bar{N}_{(1,\frac{1}{2})}N_{(1,\frac{1}{2})} + \frac{13}{9}\bar{\Delta}_{(1,\frac{1}{2})}\Delta_{(1,\frac{1}{2})} + \dots \tag{2.6}$$

This finally leads to

$$\Sigma_{\pi N} = \sin^2\theta\left(\frac{41}{9}M^0_{N(1,\frac{1}{2})} + \frac{16}{9}M^0_{\Delta(1,\frac{1}{2})}\right) + \cos^2\theta\left(\cos^2\varphi M^0_{N(\frac{1}{2},0)} + \sin^2\varphi M^0_{N(\frac{1}{2},0)}\right), \tag{2.7}$$

which is our basic result here.

3. Result and Discussion

Inserting our simplifying assumption that all the "current nucleon" masses are equal, one finds the final result

$$\Sigma_{\pi N} = \left(1 + \frac{16}{3}\sin^2\theta\right)M^0_N. \tag{3.1}$$

Note that the enhancement term $\frac{16}{3}\sin^2\theta$ is due to the factor $\frac{41+16}{9} = \frac{19}{3} \approx 6.33$ appearing in Eq. (2.7) of the $[(\mathbf{1},\frac{\mathbf{1}}{\mathbf{2}})\oplus(\frac{\mathbf{1}}{\mathbf{2}},\mathbf{1})]$ chiral multiplet, which, in turn, is due to the presence of the iso-spurion (2×4) matrices T^i, that are equivalent to the SU(2) Clebsch-Gordan coefficients $\langle\frac{3}{2}I_3(\Delta)|1I_3(i)\frac{1}{2}I_3(N)\rangle$. Thus, the enhancement factor $\frac{19}{3}$ in Eq. (2.7) and consequently also the $\frac{16}{3}\sin^2\theta$ in Eq. (3.1) , are of $SU_L(2)\times SU_R(2)$ algebraic origin. This shows that, irrespective of the specific value of the chiral mixing angle θ, there is room for improvement of the $\Sigma_{\pi N}$ predictions within the chiral $SU_L(2)\times SU_R(2)$ algebra approach.

The relevant chiral mixing angle θ has been extracted in Refs. [15, 16, 17, 18] as $\frac{8}{3}\sin^2\theta = g_A^{(0)} + g_A^{(3)}$, a function of $g_A^{(3)}$, and the flavor-singlet axial coupling $g_A^{(0)} = 0.28\pm0.16$ [22], or $g_A^{(0)} = 0.33\pm0.03\pm0.05$ [23]. Here we have taken the values of current quark masses from PDG2012 [24]: $m_u^0 = 2.3\times1.35$ MeV and $m_d^0 = 4.8\times1.35$ MeV, yielding $\frac{1}{2}\left(m_u^0+m_d^0\right)\approx 4.73$ MeV, substantially lower than before (cf. 7.6 MeV in Ref. [25]), and inserted them into the current nucleon mass to find $M_N^0 = \frac{3}{2}\left(m_u^0+m_d^0\right)\approx 14.2$ MeV and $\Sigma_{\pi N} = 59.5\pm2.3$ MeV, with $g_A^{(0)} = 0.33\pm0.03\pm0.05$ [23], or $\Sigma_{\pi N} = 58.0\pm4.5$ MeV, with $g_A^{(0)} = 0.28\pm0.16$ [22], in fair agreement with the "observed" $\Sigma_{\pi N}$ value range (55 - 75) MeV, see Ref. [4].

The above result Eq. (3.1) ought to be viewed as a lower bound on the "true" $\Sigma_{\pi N}$ value, as we have assumed that all current nucleon masses $M^0_{N_i}$ equal three times the isospin-averaged current quark mass $\bar{m}_q^0$, which is appropriate only when all chiral components of the nucleon consist of three-quark fields. That, however, condition is not necessary (because there are $q^4\bar{q}$ baryon fields that belong to the same chiral multiplets [21], and such fields would have a larger current mass ($M^0_{N'} = 5\bar{m}_q^0$, that would consequently lead to a higher value of $\Sigma_{\pi N}$), but merely a sufficient one, as all of these chiral multiplets exist as bi-local three-quark fields [20]).

In summary, the "observed" $\Sigma_{\pi N}$ term values ($\geq$ 55 MeV) have often been interpreted as a sign of a substantial $s\bar{s}$ content of the nucleon. Here we have shown that values of $\Sigma_{\pi N} \geq 55$ MeV are readily obtained in the chiral-mixing approach without any strangeness degrees of freedom in the nucleon, as a natural consequence of the substantial chiral $(6,3) = [(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})] \to (1,\frac{1}{2})$ multiplet component. The precise value of $\Sigma_{\pi N}$ is a linear function of the sum of the flavor-singlet $g_A^{(0)}$, and the isovector $g_A^{(3)}$ axial coupling of the nucleon.

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PoS(Hadron 2013)120