PROCEEDINGS OF SCIENCE

Measurement of $e^+e^- \to$ hadrons cross sections at BABAR, and implication for the muon $g-2$

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The *BABAR* Collaboration has an intensive program of studying hadronic cross sections at low-energy e^+e^- collisions, accessible via initial-state radiation. Our measurements allow significant improvements in the precision of the predicted value of the muon anomalous magnetic moment, that shed light on the current ≈ 3.5 sigma difference between the predicted and the experimental values. We have published results on a number of processes with two to six hadrons in the final state. We report here the results of recent studies with the final states that constitute the main contribution to the hadronic cross section below 3 GeV, as $e^+e^- \to \pi^+\pi^-$, K^+K^-, $K_S^0K_L^0$ and $e^+e^- \to 4$ hadrons.

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The last decades have seen an increase of the precision of both the measurement and of the theoretical understanding of the magnetic moment of the muon g_μ, presently one of the most precisely known quantity in physics. The "anomalous" magnetic moment, i.e. the deviation $a_\mu \equiv (g_\mu - 2)/2$ of g_μ from the value of $g = 2$ for a pointlike Dirac particle is presently measured to $a_\mu(\text{expt}) = (11659208.0 \pm 5.4(\text{stat}) \pm 3.3(\text{syst})) \times 10^{-10}$ [1], while the prediction is close to $a_\mu(th) = (11659181 \pm 5(\text{stat}) \pm 1(\text{syst})) \times 10^{-10}$, where I dared compute an average of the predictions from various authors, [2, 3, 4], and I give as statistical uncertainty the typical uncertainty of each prediction and as systematics uncertainty an estimate of the variation amongst these three references. The measured value exceeds the prediction by $\Delta a_\mu = (27 \pm 8) \times 10^{-10}$, assuming Gaussian statistics, that is an ≈ 3.3 standard deviation effect.

The computation of a_μ involves a perturbative development and the QED contributions is obviously the main contribution to a_μ, but its precision is actually extremely small, equal to 8×10^{-13} from the recent 10th order calculation of Ref. [6]. The uncertainty of the present prediction of a_μ is actually dominated by the contribution of the hadronic vacuum polarization (VP). As is well known, QCD is not suited to low energy calculations. Therefore the VP contribution to a_μ is computed from the "dispersion integral" (see the detailed presentation at [5]) :

$$a_\mu^{\text{had}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int \frac{R_{\text{had}}(s) \times \hat{K}(s)}{s^2} \mathrm{d}s, \tag{1}$$

where $R_{\text{had}}(s)$ is the the cross section of e^+e^- to hadrons at center-of-mass (CMS) energy squared s, normalized to the pointlike cross section : $R_{\text{had}}(s) = \sigma_{e^+e^- \to hadrons}/\sigma_{e^+e^- \to \mu^+\mu^-}$, and $\hat{K}(s)$ is a known function that is of order unity on the s range $[(2m_\pi c^2)^2, \infty]$. Technically, the lowest energy part of the integral is obtained from experimental data (currently up to $E_{cut} = 1.8\,\text{GeV}$), while the high-energy part is computed from perturbative QCD. Due to the presence of the s^2 factor at the denominator of the integrand, the precision of the prediction of a_μ relies on precise measurements at the lowest energies, and the channels $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, $\pi^+\pi^-2\pi^0$, $\pi^+\pi^-\pi^+\pi^-$, KK are of particular importance.

The *BABAR* experiment has committed itself to the systematic measurement of the production of all hadronic final states within reach in the relevant energy range ($E < E_{cut}$) over the last decade, using the initial state radiation (ISR) process (Fig. 1, Tab. 1). The cross section of the e^+e^- production of a final state f at a CMS energy squared s' can be obtained from the differential cross section of the ISR production $e^+e^- \to f\gamma$ through the expression :

$$\frac{\mathrm{d}\sigma_{[e^+e^- \to f\gamma]}}{\mathrm{d}s'}(s') = \frac{2m}{s} W(s,x) \sigma_{[e^+e^- \to f]}(s'), \tag{2}$$

where $W(s,x)$, the density of probability to radiate a photon with energy $E_\gamma = x\sqrt{s}$ is a known "radiator" function [7], and s is here the CMS energy squared of the initial e^+e^- pair. In contrast with the energy scans that provided the earlier experimental information on the variations of R, this ISR method allows a consistent measurement on the full energy range with the same accelerator and detector conditions. The observation of the hadronic final state alone would allow the reconstruction of the event and the measurement of s', but in addition the observation of the ISR photon (γ-tagging) provides a powerful background rejection and a good signal purity.

In the case of *BABAR* the e^+e^- initial state is strongly boosted so that the reconstruction efficiency is large down to threshold. Most of these measurements have used a leading-order (LO)

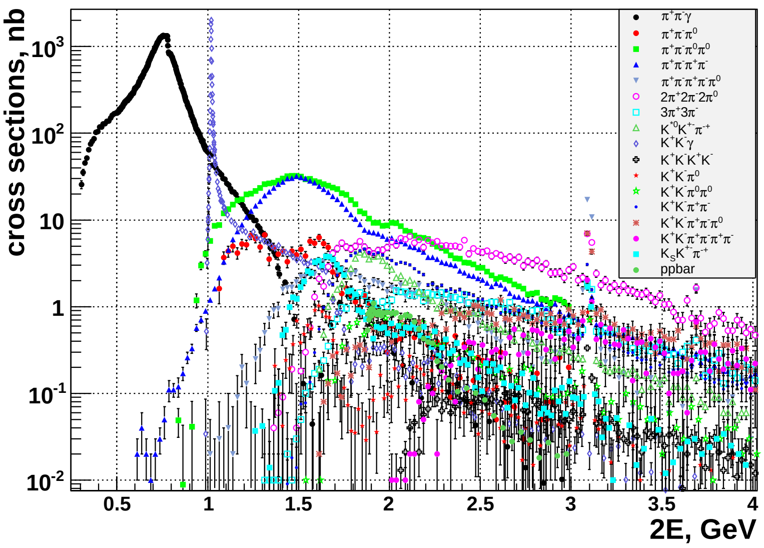

Figure 1: Summary of the *BABAR* measurements (The $\pi^+\pi^-\pi^0\pi^0$ entry is preliminary [21]). Thanks to Fedor V. Ignatov.

Channels	$\mathscr{L}$ fb^{-1}	method	reference
$K_S^0K_L^0$, $K_S^0K_L^0\pi^+\pi^-$, $K_S^0K_S^0\pi^+\pi^-$, $K_S^0K_S^0K^+K^-$	469	LO	preliminary
$\overline{p}p$	454	LO	[18]
$\overline{p}p$	469	no tag LO	[19]
K^+K^-	232	NLO	[15]
$2(\pi^+\pi^-)$	454	LO	[16]
$K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$, $K^+K^-K^+K^-$	454	LO	[17]
$\pi^+\pi^-$	232	NLO	[13] [14]
$K^+K^-\eta$, $K^+K^-\pi^0$, $K^0K^\pm\pi^\mp$	232	LO	[12]
$\pi^+\pi^-\pi^0\pi^0$		LO	[21] preliminary
$2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-)\eta$, $K^+K^-\pi^+\pi^-\pi^0$, $K^+K^-\pi^+\pi^-\eta$	232	LO	[11]
$\Lambda\overline{\Lambda}$, $\Lambda\Sigma^0$, $\Sigma^0\Sigma^0$	232	LO	[10]
$3(\pi^+\pi^-)$, $2(\pi^+\pi^-\pi^0)$, $K^+K^-2(\pi^+\pi^-)$	232	LO	[9]
$\pi^+\pi^-\pi^0$	89	LO	[8]

Table 1: Summary of the *BABAR* results on ISR production of exclusive hadronic final states (publications that have been superseded by updated results with a larger dataset are removed).

method, in which the final state f and the ISR photon are reconstructed regardless of the eventual presence of additional photons. The differential luminosity is obtained from the luminosity of the collider, known with a typical precision of 1%, and involves a computation of the detection efficiency that relies on Monte Carlo (MC) simulations [8, 9, 10, 11, 12]. This experimental campaign has lead *BABAR* to improve the precision of the contribution to a_μ of most of the relevant channels by a large factor, typically close to a factor of three. More recently *BABAR* has developped a new method that was applied to the dominant channel $\pi^+\pi^-$ [13, 14] and to the K^+K^- channel [15]. The control of the systematics below the % level made it necessary to perform the

	$\pi^+\pi^-$	$\pi^+\pi^-\pi^+\pi^-$	K^+K^-
BABAR	$514.1\pm2.2\pm3.1$ [13] [14]	$22.93\pm0.18\pm0.22\pm0.03$ [16]	$13.64\pm0.03\pm0.36$ [15]
Previous average [2]	503.5 ± 4.5	$21.63\pm0.27\pm0.68$	$13.35\pm0.10\pm0.43\pm0.29$
Their difference Δ	$+10.6\pm5.9$	$+1.30\pm0.79$	$+0.29\pm0.63$

Table 2: Contributions to a_μ for recent *BABAR* publications : comparison of the measured value to the previous world average on the energy range $< 1.8\,\mathrm{GeV}$ (units 10^{-10}).

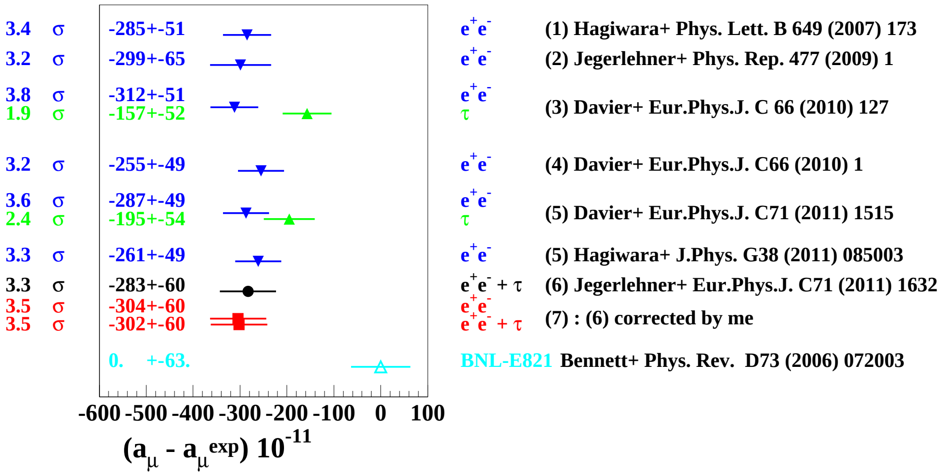

Figure 2: Recent predictions of the value of a_μ in chronological order, after the experimental value is subtracted. Blue : e^+e^--based; Green : τ spectral function-based; Black : e^+e^- and τ combinations; Red : "corrected" by me, see text.

analysis at the NLO level, that is to take into account the possible radiation of an additional photon ($e^+e^- \to f\gamma_{ISR}(\gamma)$). The impossibility to control the global differential luminosity with the desired precision, in particular the MC-based efficiency, lead us to derive the value of R from the ratio of the ISR production of the final state f to the ISR production of a pair of muons, $\mu^+\mu^-$. Most of the systematics, including those related to the absolute luminosity, of the ISR photon reconstruction, of additional ISR radiation, cancel in the ratio. *BABAR* also performed updates of former works, using the full data set for the $\pi^+\pi^-\pi^+\pi^-$ [16], $p\bar{p}$ [18], $K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$ and $K^+K^-K^+K^-$ [17] channels, and channels with two neutral kaons ($K^0_SK^0_L$, $K^0_SK^0_L\pi^+\pi^-$, $K^0_SK^0_S\pi^+\pi^-$, $K^0_SK^0_SK^+K^-$) [20] using the LO method. The $p\bar{p}$ measurement has also been extended up to 6.5 GeV by an untagged analysis [19]. The *BABAR* results for the contributions to a_μ for $\pi^+\pi^-$, $\pi^+\pi^-\pi^+\pi^-$ and K^+K^- are larger than, have a similar to or better precision than, and are compatible within less than two standard deviation with, the world combination of previous results (Table 2).

It is interesting to compare the evolution of the prediction of a_μ with the availability of experimental results of increasing precision and with the development of combination techniques. In Fig. 2 I show the value of the most recent predictions, after subtraction of the experimental value (units 10^{-11}). The most recent works of the three main groups [2, 3, 4] make use of our results up to and including $\pi^+\pi^-$ [13, 14], but are not yet using our recent [18, 15, 16, 17]. Given the absence of contribution of the $\bar{p}p$ [18] and $K^+K^-K^+K^-$ [17] channel below 1.8 GeV, and given the smallness of the difference Δ for the K^+K^- [15] and $2(\pi^+\pi^-)$ [16] channels, we see that including our recent results [18, 15, 16, 17] will barely change the prediction central value. It is reassuring to note that :

- these three predictions performed independently to a large extent, as far as the VP is concerned, provide results compatible with each other [1] within a couple of 10^{-10};
- after $\rho - \gamma$ mixing is taken into account, the discrepancy between the combinations based on e^+e^- results and those based on the τ decay spectral functions (see Ref. [22]) is solved [4].

The discrepancy between the prediction and the measurement still sits close to 3.3 – 3.6 standard deviations. Given that the precision of most measurements is now dominated by the systematics contribution there is most likely no hope of major improvements from a super *B* factory like Belle 2. Indeed, new measurements of a_μ at Fermilab [23] and at J-PARC [24] are eagerly awaited.

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[1] In (7) of Fig. 1 I dared correct the prediction (6) from [4] to align the μ_μ/μ_p value and the calculation of light-by-light scattering on that of the two other groups [2] and [3].