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Roles of η-channels in $\pi\pi$, $\pi\eta$ and πK scatterings

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We propose the roles of η channels in $\pi\pi - K\bar{K} - \pi\eta - \eta\eta$ and $\pi K - \eta K$ interactions at low-energies ($\sqrt{s} < 1.5$ GeV) based on the $SU(3)$-symmetric one-meson-exchange model. In the model, we use both $s-$ and $t-$ channels exchange diagrams to reproduce the realistic meson-meson interactions. The coupling constants and cut-off masses in form factors are adjusted to reproduce the experimental phase shifts. Beside the monopole type of form factors, the Gaussian type is also considered. We find the resonances corresponding to $a_0(980)$, $f_0(980)$, $f_2(1270)$, $\kappa(1430)$, $\kappa(700)$, $\rho(770)$, $K^*(982)$, $\phi(1020)$ and $\sigma_1(600)$. Their masses and widths are determined by the S-matrix calculation on the complex E-plane.

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1. Introduction

To investigate the properties of meson-meson interactions, many theoretical models have been proposed such as quark models, chiral perturbation models [1, 2], LQCD [3], and meson-exchange models [4, 5]. Each model has their own validity. Meson-exchange should remain a valid concept for deriving the realistic hadron-hadron interactions in common, particularly meson-meson interactions. Until now, by using the one-meson-exchange potential we have already completed the calculations of two models. The first model is the one in which we did not include η meson (no-η model). In this first model, by reproducing the well-known $\pi\pi, \pi K$ interactions, we made the prediction of KK interactions [5] at low energies ($\sqrt{s} < 1.5$ GeV). To make the full treatment of meson-meson interactions, natuarally, we need to put the η meson in the model. The second model is in which we investigate $\pi\pi - K\bar{K} - \pi\eta - \eta\eta$ and $\pi K - \eta K$ scatterings. In this paper, we discuss the quantitative calculations of the second model and propose the phase shifts of $\pi\pi$, $\pi\eta$, $K\bar{K}$ and πK scatterings in the effect of η meson. We also determine the physical masses and widths of $a_0(980), f_0(980), f_2(1270), \kappa(1430), \kappa(700), \rho(770), K^*(982), \phi(1020), \sigma_1(600)$ through the S-matrix calculations on the complex-E plane.

2. One-meson exchange model

To construct the meson-meson interactions, we considered t- and s-channel exchange diagrams. As exchanged mesons, we introduce scalar- , vector- and tensor-mesons. Using time-ordered perturbation theory, we derive the potential for each diagram. For s-channel exchange potential, which has a pole at the meson mass on the real E axis, we need a renormalization calculation. The details of our model are shown in the Ref. [5]

3. Parameters in the potential model

At present, we have two sets of parameters. The first parameter set is used in the no η model, which is already shown in Ref. [5]. The second one is the ones in which we readjust some parameters when the $\pi\eta - \eta\eta$ and ηK channels are added in the model. From 18 parameters in the no η model, now the parameters are expanded into the set of 24 ones. Their values are listed in Table 1 and 2.

4. Results for meson-meson scattering

Before discussing the results, we notice that all phase shifts and reasonaces given in this paper are calculated with the second parameter set. With this set, we examine the effects of η channels by comparing the cases of without η(no-η model) and with η (η model).

For $\pi\pi - K\bar{K} - \eta\eta$ scatterings, we consider the phase shifts of δ_0^0, δ_1^1 and δ_2^0 and obtain the results shown in Fig.1 All of S-, P- and D- wave phase shifts are attractive. The $I = 0$ S-wave has a good description at the low energies ($\sqrt{s} < 1.0$GeV), while at higher energies phase shifts are somehow smaller than the experimental data. In this partial wave, the s-channel ε-meson exchange couples to not only π meson but also K and η mesons. Because of the large bare mass of ε,

Table 1: Coupling constants and Cut-off parameters Λ(MeV).

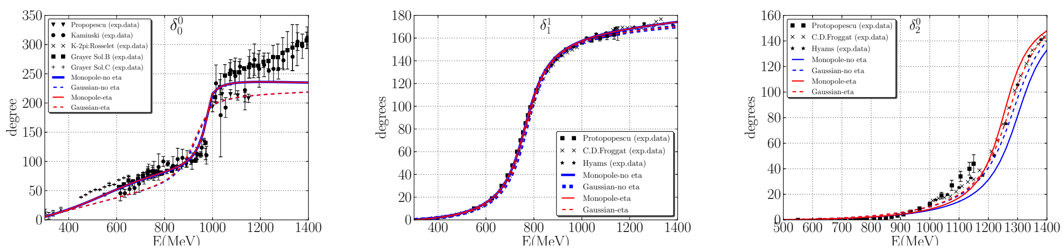
Parameters	Monopole	Gaussian
$g_{\pi\pi\rho}$	0.52247	0.47320
$g_{\pi\pi f_2}$	0.029255	0.0578475
$g_{\pi\pi\varepsilon}$	0.030139	0.390161×10^{-2}
$g_{\pi\pi\kappa}$	0.102299×10^{-3}	0.464903×10^{-3}
$g_{K\bar{K}a_0}$	0.04	0.1
$\Lambda_{\pi\pi\rho}$	2704.325	1589.421
$\Lambda_{\pi KK^*}$	2405.654	2996.391
$\Lambda_{KK\rho}$	4191.844	2674.091
$\Lambda_{KK\omega,\phi}$	4562.535	4609.568
$\Lambda_{\pi\pi\rho,s}$	2753.454	6000.000
$\Lambda_{\pi\pi f_2,s}$	3186.415	1479.935
$\Lambda_{\pi\pi\varepsilon,s}$	1416.148	2679.790
$\Lambda_{\pi KK^*,s}$	3141.034	3445.081
$\Lambda_{\pi K\kappa,s}$	3453.687	4910.219
$\Lambda_{\eta KK^*}$	757.09	700.00
$\Lambda_{K\bar{K}a_0}$	1000.00	1000.00
$\Lambda_{K\bar{K}\phi}$	2000.00	4200.00

Table 2: Bare masses.

Mesons	Monopole	Gaussian
ε	6375.511	6500.000
κ	1451.646	1516.317
ρ	1135.000	2187.112
K^*	1238.183	2187.112
f_2	2680.000	1501.877
a_0	1120	1200
ϕ	1145	1402

the present result of δ_0^0 does not fit the experimental data. The different form factors give us the different characters of the scalar contributions and $\eta\eta$ channel effects.

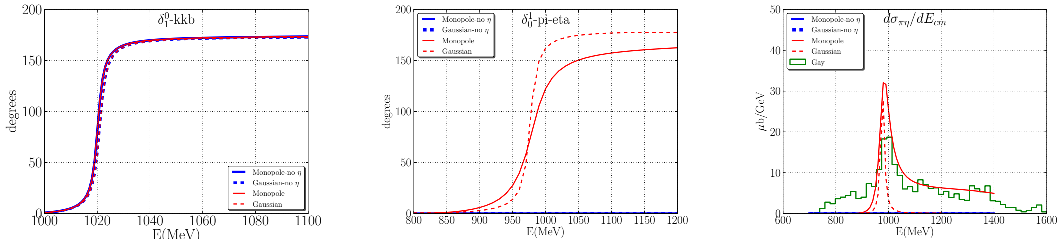
In the $I = 1$ P-wave, both s- anf t-channel ρ-exchange diagrams give important contributions in the attractive phase shifts shown in the Fig. 1(b). This phase shifts easily fit with the experimental data. We obtain the resonance of ρ meson shown in the Table 3. The pole positions in no-η and η models are unchanged in both two types of form factors. This means that the η channel does not play any role in this partial wave. When we examine the $I = 0$ D-wave scattering, we also obtain the attractive phase shift, which is shown in the Fig. 1(c). The s-channel f_2-exchange contribution makes the $\pi\pi$ phase shifts more attractive and produces a pole around the $f_2(1270)$ in both two types of form factors. The pole position for f_2 resonance is shown in the Table 3.



(a) δ_0^0. (b) δ_1^1. (c) δ_2^0.

Figure 1: The phase shifts of $\pi\pi - K\bar{K} - \eta\eta$ scatterings. The solid and dashes lines show respectively the results of monopole and Gaussian form factors.

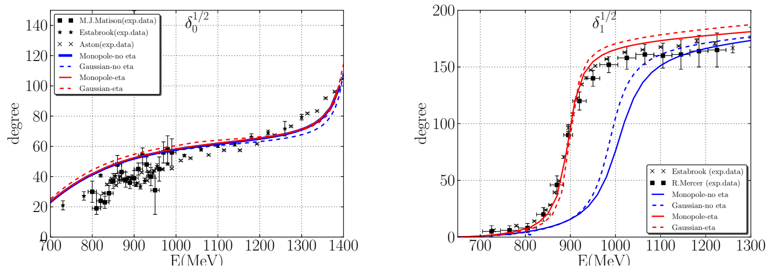
Naturally, when we investigate the $\pi\pi - K\bar{K} - \pi\eta - \eta\eta$ scatterings, we want to understand quantitatively the $K\bar{K}$ phase shifts with the s-channel ω- and ϕ-exchange and the t-channel K^*-exchange. The understanding of $K\bar{K}$ interaction help us to know deeply the large effect of $K\bar{K}$ in the $\pi\pi$ scattering, espeacially in the δ_0^0 phase shift. In $K\bar{K}$ interaction, the $I = 0$ P-wave phase shifts, which are shown in Fig. 2(a), are attractive in the energy region from 1.0 GeV to 1.1 GeV. The η channel has no effect to the $K\bar{K}$ interaction, so the phase shifts in no-η and η models are the same for both two types of form factors. We find a pole corresponding to the $\phi(1020)$ meson, as shown in the Table 3, with mass and width very near to those reported in the PDG data. Then, the $\pi\eta$ S-wave phase shifts, which are shown in the Fig. 2(b) are also attractive. In this interaction, there is an appreciable mixture of the $K\bar{K}$ and $\pi\eta$. We observe that the cross sections, which are shown in the Figure 2(c), are in a rather good agreement with the experimental data in CERN. We also find a pole (a_0 pole) on the complex-E plane. Its position is also shown in the Table 3.



(a) $\delta_1^0 - K\bar{K}$. (b) δ_0^1. (c) $\frac{d\sigma_{\pi\eta}}{dE_{cm}}$.

Figure 2: The phase shift of $K\bar{K} - \pi\eta$ scatterings. The solid and dashes lines show respectively the results of monopole and Gaussian form factors

In the πK scattering, the $I = \frac{1}{2}$ S-wave phase shifts are attractive as shown in the Fig. 3(a). In the low-energy region ($\sqrt{s} < 1$GeV), the calculated phase shifts are somewhat larger than the experimental values. The $\kappa(1430)$ resonance is reproduced rather well in both of the models with monopole and Gaussian form factors. We also find another pole (maybe the $\kappa(700)$ pole) in this $I = \frac{1}{2}$ S-wave state. You can see the details of $\kappa(1430)$ and $\kappa(700)$ in the Table 3. When extending the model to the coupled-channel $\pi K - \eta K$ scatterings, we find a large effect of ηK channel as shown in Fig. 3(b). The s-channel K^*-exchange diagram plays an important role in πK-ηK coupling potential. The physical mass of K^* resonance is somewhat larger than the value of PDG data as shown in the Table 3.



(a) $\delta_0^{\frac{1}{2}}$. (b) $\delta_1^{\frac{1}{2}}$.

Figure 3: The phase shifts of $\pi K - \eta K$ scatterings. The solid and dashes lines show respectively the results of monopole and Gaussian form factors

Table 3: Resonances of $\pi\pi - K\bar{K} - \pi\eta - \eta\eta$ and $\pi K - \eta K$ scatterings. The values in the square bracket mean $[M, \frac{\Gamma}{2}]$, M is the physical mass, Γ is the width.

	Exp. Data (PDG)	Monopole no η	Monopole$_\eta$	Gaussian no η	Gaussian$_\eta$
$a_0(980)$	$[984, -(25 \div 50)]$	$[857, -28]$	$[893, -52]$	$[843, -15]$	$[860, -20]$
$f_0(980)$	$[980, -(20 \div 50)]$	$[975, -36]$	$[975, -36]$	$[925, -60]$	$[935, -60]$
$f_2(1270)$	$[1275(1.2), -92(2)]$	$[1244, -78]$	$[1280, -90]$	$[1260, -89]$	$[1270, -100]$
$\kappa(1430)$	$[1412(6), -147(12)]$	$[1420, -23]$	$[1420, -25]$	$[1410, -17]$	$[1420, -16]$
$\kappa(700)$		$[650, -200]$	$[650, -203]$	$[649, -190]$	$[649, -200]$
$K^*(892)$	$[892, -25]$	$[905, -20]$	$[1006, -44]$	$[910, -18]$	$[998, -34]$
$\phi(1020)$	$[1019, -2]$	$[1020, -2]$	$[1020, -2]$	$[1021, -2]$	$[1021, -2]$
$\rho(770)$	$[771, -75]$	$[800, -60]$	$[800, -60]$	$[800, -60]$	$[800, -60]$
$\sigma_1(600)$	$[400 \div 1200, -(300 \div 500)]$	$[410, -540]$	$[410, -540]$	$[360, -510]$	$[360, -510]$
σ_2		$[530, -382]$	$[530, -372]$		

5. Summary

The η channel plays very important and necessary roles in the full treatment of the meson-meson interaction by one-meson-exchange mechanisms. In the the $\pi\pi$ scattering, the $\eta\eta$ channel has a small effect, while the $K\bar{K}$ effect is large. The η channel contributes to the appearance of a_0 resonance in the $\eta\pi$ interaction. In the P-wave $I = \frac{1}{2}$, η channel also has a part in proceducing the well-fitted phase shifts. Beside the well-reproduced poles of $f_0(980)$, $\rho(770)$, $f_2(1270)$, $\kappa(1430)$, $K^*(892)$, we also found the existence of the $a_0(980)$, $\phi(1020)$, $\sigma_1(600)$ and $\kappa(700)$ poles. In future, we will refine our model by making the well-fitted parameter set. After that, we advance forward by solving the three-body problems to study the existences and properties of the resonances in the hadronic systems.

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