PROCEEDINGS OF SCIENCE

Total Hadronic Cross Sections and $\pi^{\mp}\pi^{+}$ Scattering

Keiji Igi*
Mathematical Physics Laboratory, RIKEN Nishina Ctr., Wako, Saitama 351-0198, Japan
E-mail: igi@phys.s.u-tokyo.ac.jp

Recent measurements of the inelastic and total proton-proton cross section at the LHC, and at cosmic ray energies by the Auger experiment, have quantitatively confirmed fits to lower energy data constrained by the assumption that the proton is asymptotically a black disk of gluons. We show that data on $\bar{p}(p)p, \pi^{\mp}p$, and $K^{\mp}p$ forward scattering support the related expectation that the asymptotic behavior of all cross sections is flavor independent. By using the most recent measurements from ATLAS, CMS, TOTEM and Auger, we predict $\sigma_{\rm tot}^{pp}(\sqrt{s}=7\ \text{TeV}) = 98.2\pm 2.7$ mb, $\sigma_{\rm tot}^{pp}(\sqrt{s}=8\ \text{TeV}) = 100.6\pm 2.9$ mb, $\sigma_{\rm tot}^{pp}(\sqrt{s}=10\ \text{TeV}) = 104.5\pm 3.1$ mb, and $\sigma_{\rm tot}^{pp}(\sqrt{s}=14\ \text{TeV}) = 110.8\pm 3.5$ mb, as well as refine the total cross section $\sigma_{\rm tot}^{pp}(\sqrt{s}=57\ \text{TeV}) = 139.6\pm 5.4$ mb. Our analysis also predicts the total $\pi^{\mp}\pi^{+}$ cross sections as a function of $\sqrt{s}$.

XV International Conference on Hadron Spectroscopy-Hadron 2013
4-8 November 2013
Nara, Japan

*Speaker.

PoS(Hadron 2013)146

1. Introduction

Recent high energy measurements of the inelastic proton-proton cross section, made possible by the Large Hadron Collider (LHC) and a new generation of cosmic ray experiments, have convincingly confirmed [1] indications [2, 3, 4, 5] in lower energy data that the total cross section $\sigma_{\rm tot}$ behaves asymptotically as the squared log of the center-of-mass energy $\sqrt{s}$, reminiscent of the energy dependence of Froissart's unitarity bound [6]. This energy dependence is now solidly anchored to all pp and $\bar{p}p$ total and inelastic cross section measurements, from threshold data averaged by finite energy sum rules, to the result at 57 TeV center-of-mass energy of the Auger cosmic ray array [7].

The COMPETE Collaboration already proposed that this asymptotic behavior $\sigma_{\rm tot} \simeq B\log^2(s/s_0)$ applies to all hadron total cross sections, with a universal value of the coefficient B [3, 7].

In order to empirically test this universality, the $\bar{p}(p)p, \pi^{\mp}p$, and $K^{\mp}p$ forward scattering amplitudes are analyzed, and the values of B, denoted respectively as $B_{pp}, B_{\pi p}$, and B_{Kp}, were estimated independently [8]. The analysis was refined [9] for B_{Kp}. The resulting values are consistent with the universality, $B_{pp} \simeq B_{\pi p} \simeq B_{Kp}$, and thus, the universality of B is suggested.

In this work we first update the analysis of the $\bar{p}(p)p, \pi^{\mp}p$, and $K^{\mp}p$ data by including newly measured LHC results as well as very high energy measurements based on cosmic ray data. We also fit the $\bar{p}(p)n$ data at the same time. Subsequently, assuming the universality of B, we calculate the $\pi^{\mp}\pi^{+}$ total cross section $\sigma_{\rm tot}^{\pi^{\mp}\pi^{+}}(s)$ at all energies.

2. Update of the fits to $\sigma_{\rm total}$

2.1 Analysis of Forward $\bar{p}(p)p, \pi^{\mp}p, K^{\mp}p, \bar{p}(p)n$ Amplitudes

The crossing-even forward scattering amplitude, $F_{ab}^{(+)}(\nu)$, is given by the sum of Pomeron and Reggeon (including P' trajectory) exchange terms, while the crossing-odd $F_{ab}^{(-)}(\nu)$ is given by a single contribution from Reggeon (corresponding to vector-meson trajectories) exchange contributions. Here the subscripts ab and $\bar{a}b$ represent $ab = pp, \pi^{+}p, K^{+}p, pn$ and $\bar{a}b = \bar{p}p, \pi^{-}p, K^{-}p, \bar{p}n$, respectively. We consider the exchange degenerate $f_2(1270)$-, $a_2(1320)$-trajectories for the crossing-even Reggeon (tensor-meson) term and the ρ-, ω-trajectories for the vector-meson term. Their imaginary parts are given explicitly by

$$Im\, F_{ab}^{(+)}(\nu) \;=\; \frac{\nu}{m^2}\left(c_2^{ab}\log^2\frac{\nu}{m} + c_1^{ab}\log\frac{\nu}{m} + c_0^{ab}\right) + \frac{\beta_T^{ab}}{m}\left(\frac{\nu}{m}\right)^{\alpha_T(0)} . \tag{2.1}$$

The intercepts are fixed with $\alpha_T(0) = 0.542,\ \alpha_V(0) = 0.455$, which is taken to be the same as the Particle Data Group [7]. In our analysis, $\rho^{\bar{a}b,ab}(s)$, the ratios of real to imaginary parts of forward amplitudes, are fitted simultaneously with the data on $\sigma_{\rm tot}^{\bar{a}b,ab}$. Real parts of the crossing-even/odd amplitudes are directly obtained from crossing symmetry $F^{(\pm)}(e^{i\pi}\nu) = \pm F^{(\pm)}(\nu)^*$ as

$$Re\, F_{ab}^{(+)}(\nu) \;=\; \frac{\pi\nu}{2m^2}\left(c_1^{ab} + 2c_2^{ab}\log\frac{\nu}{m}\right) - \frac{\beta_T^{ab}}{m}\left(\frac{\nu}{m}\right)^{\alpha_T(0)}\cot\frac{\pi\alpha_T(0)}{2} + F_{ab}^{(+)}(0) . \tag{2.2}$$

We introduce $F_{ab}^{(+)}(0)$ as a subtraction constant in the dispersion relation [10].

PoS(Hadron 2013)146

$\sqrt{s}$(TeV)	7	8	10	14	57
$\sigma_{\rm tot}^{pp}$(mb)	98.2(2.7)	100.6(2.9)	104.5(3.1)	110.8(3.5)	139.6(5.4)

Table 1: Predicted pp total cross sections.

2.2 Updated Analysis Including LHC and Very High Energy Cosmic-Ray Data

For detail please see ref.[11]. We estimate the value of the universal slope B as

$$B = 0.293 \pm 0.004_{stat} \pm 0.026_{syst} \text{ mb} . \tag{2.3}$$

By using this value, we predict the pp total cross sections at $\sqrt{s} = 7, 8, 10, 14, 57$ TeV in Table 1.

3. The $\pi\pi$ Total Cross Section

3.1 Theoretical Predictions of $\sigma_{\rm tot}^{\pi^\mp\pi^+}$

We infer the $\sigma_{\rm tot}^{\pi^\mp\pi^+}(s)$ based on the analyses of forward $\bar{p}(p)p$, $\pi^\mp p$, and $K^\mp p$ scattering amplitudes. Based on the result of the previous section, we can predict $\sigma_{\rm tot}^{\pi^\mp\pi^+}$ at high energy. By using the relation $s \simeq 2M\nu$ for $ab = ap = pp, Kp, \pi p$ in high-energies, $\sigma_{\rm tot}^{ap,ap}(s)$ can be written in the form

$$\sigma_{\rm tot}^{\bar{a}p,ap}(s) = B\log^2\frac{s}{s_0} + Z_{ap} + \tilde{\beta}_T^{ap}\left(\frac{s}{s_1}\right)^{\alpha_T(0)-1} \pm \tilde{\beta}_V^{ap}\left(\frac{s}{s_1}\right)^{\alpha_V(0)-1} . \tag{3.1}$$

The $\pi^\mp\pi^+$ total cross sections $\sigma_{\rm tot}^{\pi^\mp\pi^+}$ are expected to take the form

$$\sigma_{\rm tot}^{\pi^\mp\pi^+}(s) = B\log^2\frac{s}{s_0} + Z_{\pi\pi} + \tilde{\beta}_T^{\pi\pi}\left(\frac{s}{s_1}\right)^{\alpha_T(0)-1} \pm \tilde{\beta}_V^{\pi\pi}\left(\frac{s}{s_1}\right)^{\alpha_V(0)-1} , \tag{3.2}$$

where B and s_0 are given by Eq. (3.1).

In summary, $\sigma_{\rm tot}^{\pi^\mp\pi^+}(s)$ are predicted by Eq. (3.2) with the parameters B.

3.2 Comparison with Indirect Experiments

There are no direct measurements of $\sigma_{\rm tot}^{\pi\pi}$ at present, however, indirect data at low- and intermediate-energy have been extracted in many papers which was recently updated in ref.[12] The result is given in Fig. 1.

4. Discussion and Conclusion

Our predictions for $\sigma_{\rm tot}^{\pi^\mp\pi^+}$ are shown along with the results of our best fit for $\sigma_{\rm tot}^{\pi^\mp p}$ and $\sigma_{\rm tot}^{\bar{p}(p)p}$ in Fig. 2. The difference in normalization of these curves is determined by the Z_{ab}, $Z_{pp} > Z_{\pi p} > Z_{\pi\pi}$, while their increase with energy is described by the universal value of B.

Although challenging, the data on $\pi^\mp\pi^+$ collisions could be extended to higher energies exploiting high intensity proton beam accelerator beams planned worldwide, such as Project X [13] of FNAL and J-PARC in Japan [14]. At a later stage these may develop into muon colliders. As

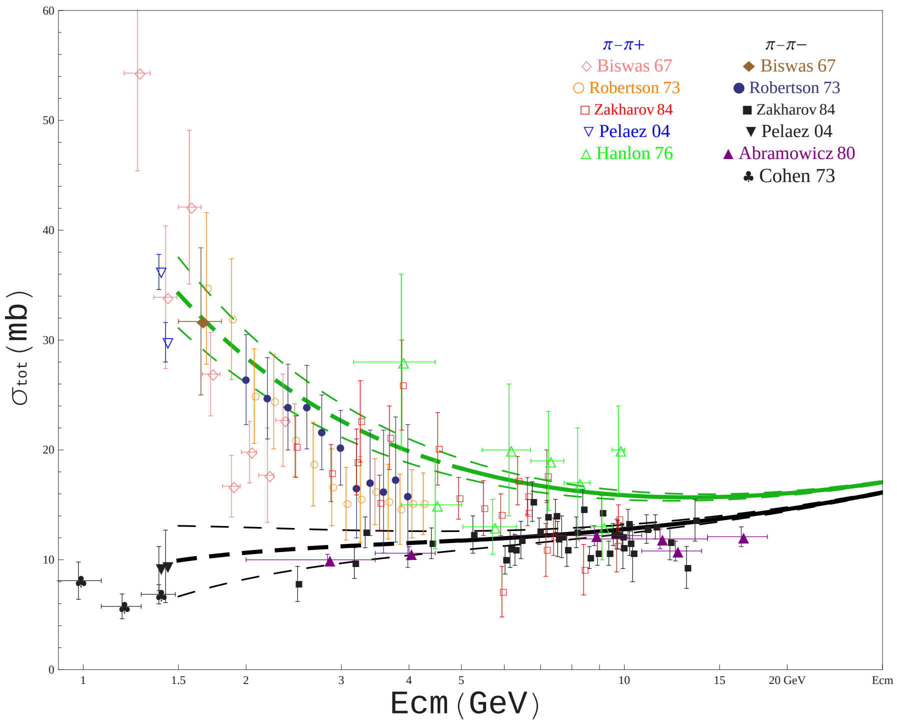

Figure 1: $\pi^{\mp}\pi^{+}$ total cross section (mb) versus $\sqrt{s}$. $\sigma_{\rm tot}^{\pi^-\pi^+}$(thick solid green) and $\sigma_{\rm tot}^{\pi^+\pi^+}$(thick solid black). Thin dashed lines represent the uncertainty from $\tilde{\beta}_V^{\pi\pi}$, which is the largest in the relevant energy region.

an example, Project X, a high intensity proton source proposed at Fermilab, would deliver proton beams at energies ranging from 2.5 to 120 GeV [13] and secondary pion beams with $E(\pi) \approx 2-15$ GeV. A muon collider with Project-X-intensity pion beams would represent a $\pi^+\pi^-$ collider with $\sqrt{s} = 1$ TeV and a luminosity of $10^{22} cm^{-2}/sec$ [15], not quite sufficient, even for measuring the large cross sections discussed here. Some manipulation of the secondary beams would be required.

References

[1] M. M. Block, F. Halzen, [arXiv:1109.2041 [hep-ph]].

[2] K. Igi, M. Ishida, Phys. Rev. D **66**, 034023 (2002).

[3] J.R.Cudell et al. (COMPETE Collab.), Phys. Rev. D **65**, 074024 (2002); Phys. Rev. Lett. **89**, 201801 (2002).

[4] M. M. Block, F. Halzen, Phys.Rev. D **72**,036006 (2005).

[5] K. Igi, M. Ishida, Phys. Lett. B **622** (2005) 286.

[6] M. Froissart, Phys. Rev. **123**, 1053 (1961).
A. Martin, Nuovo Cimento **42**, 930 (1966).

[7] K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010).

[8] M. Ishida, K. Igi, Phys. Rev. D 79, 096003 (2009).
M. Ishida, K. Igi, Phys. Lett. B 670, 397 (2009).

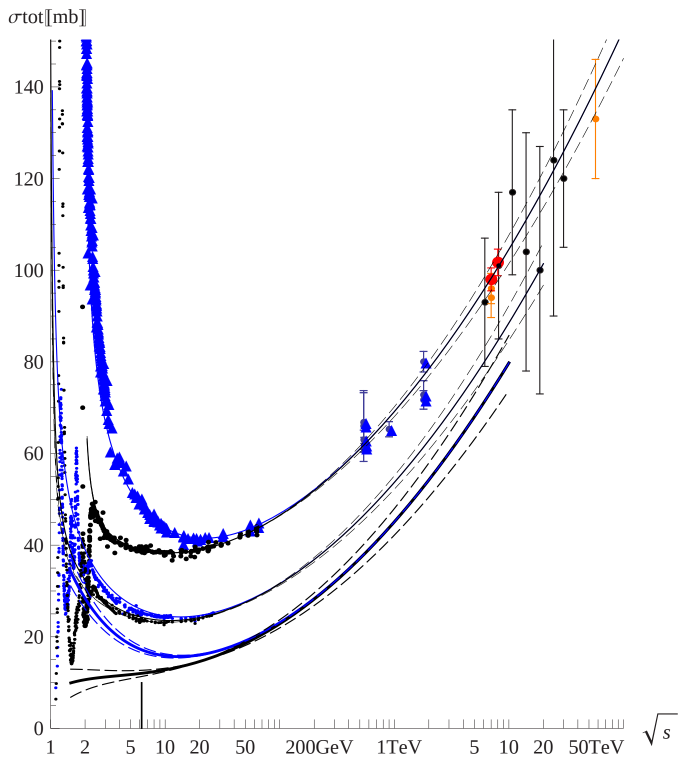

Figure 2: Total cross sections(mb) versus $\sqrt{s}$. Solid lines are $\bar{p}p, pp$, $\pi^- p, \pi^+ p$, $\pi^-\pi^+, \pi^+\pi^+$ from up-to-down. $\bar{p}p, pp$ and $\pi^- p, \pi^+ p$ are our best fit, while $\pi^-\pi^+, \pi^+\pi^+$ are our predictions. The dot-dashed lines represent the upper(lower)-limit of our predictions of $\sigma_{\rm tot}^{\pi^-\pi^+}(\sigma_{\rm tot}^{\pi^+\pi^+})$. Dashed lines for $\bar{p}(p)p$ and $\pi^\mp p$ represent the uncertainties of our predictions which are obtained from the errors of B and $Z_{\pi\pi}$. For pp we include the latest pp total cross sections, 98.0(2.5) mb at $\sqrt{s} = 7$ TeV[16] and 101.7(2.9) mb at $\sqrt{s} = 8$ TeV[17] as big red circles; $133 \pm 13_{\rm stat}\ {}^{+17}_{-20}{}_{\rm syst} \pm 16_{\rm Glauber}$ mb[18] as an orange triangle (where only statistical error is shown in the figure).

[9] M. Ishida and V. Barger, Phys. Rev. D **65**, 116005 (2011).

[10] M. M. Block, Phys. Rev. D **65**, 116005 (2002).

[11] F. Halzen, K. Igi, M. Ishida, and C. S. Kim, Phys. Rev. D **85**, 074020 (2012).

[12] R. Garcia-Martin, R. Kaminski, J. R. Pelaez, J. R. de Elvira, and F. J. Yundrain, Phys. Rev. D **83**, 074004 (2011).

[13] “A white paper based on the Project X Physics Workshop", November 2009.

[14] Japan Proton Accelerator Research Complex, http://j-parc.jp/index-e.html.

[15] Private communication with S. Geer.

[16] G. Antchev et al. (The TOTEM Collaboration), Europhys. Lett. **101**, 21003 (2013).

[17] G. Antchev et al. (The TOTEM Collaboration), Phys. Rev. Lett. **111**, 012001 (2013).

[18] P. Abreu et al. (The Pierre Auger Collaboration), Phys. Rev. Lett. **109**, 062002 (2012).