

Ground State of Holographic Mean Field Theory and its Properties in Baryonic Matter

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We propose a minimal phenomenological extension of the Sakai-Sugimoto model to describe systems of finite baryon density. We introduce two classical Dirac fields with opposite signs of mass terms on the flavor branes so that the theory contains a rotationally invariant state. A preliminary result on the ground state of the model at finite baryon density is also presented.

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1. Introduction

Quark-hadron physics at finite baryon density is one of the central subjects in modern physics, in connection to studies on neutron stars, quark-gluon plasma and the QCD phase diagram. The AdS/CFT correspondence [1] is a useful method for non-perturbative computations in strongly coupled gauge theories, and the Sakai-Sugimoto model [2] reproduces the properties of low-energy physics of massless large- N_c QCD. Recently, there are several proposals to study the finite baryon density system through the holographic QCD models which does not suffer from the sign problem as opposed to the lattice QCD.

However, incorporation of baryons into a holographic model is not straightforward (see, for example, [3]). One simple attempt toward a full-fledged model is to describe baryons by a fermionic baryon field on the dual geometry. This idea has been employed in the holographic mean field (HMF) approach in [3] to construct a holographic model of finite baryon density systems.

In the present work, we find that the original model of [3] does not contain rotationally invariant states at finite density. We also point out that the original model is not parity invariant. Then we extend the model in such a way that the model has parity invariance. This can be achieved by introducing two baryon fields into the bulk with opposite signatures of the mass terms. We shall show that the new model contains a rotationally invariant state at finite baryon density. This observation suggests that the idea of parity-doublet model [4] is naturally requested if we follow the holographic mean field approach.

2. Holographic Mean Field theory and Its Ground State

In the Holographic Mean Field (HMF) theory proposed by Harada-Nakamura-Takemoto (HNT) in [3], a five dimensional baryonic field is introduced to study the baryonic matters. For demonstrating the idea of HMF approach, HNT used the model given in [5], where the baryon field is introduced into the Sakai-Sugimoto model [2]. The action is given by

$$\begin{aligned}
 S &= S_\Psi + S_A, \\
 S_\Psi &= \int dx^4 dw \left[\frac{i}{2} \bar{\Psi} \Gamma^M D_M \Psi - \frac{i}{2} \overline{(D_M \Psi)} \Gamma^M \Psi - m_5(w) \bar{\Psi} \Psi \right], \\
 S_A &= \int d^4 x dw \mathcal{L}_A = -C \int d^4 x dw U(w)^{1/4} \sqrt{\det \left(g_{MN} + \frac{2\pi\alpha'}{N_c} F_{MN} \right)}, \quad (2.1)
 \end{aligned}$$

where we have employed a conformally flat metric given by $ds_{5d}^2 = H(w) (\eta_{\mu\nu} dx^\mu dx^\nu - dw^2)$. Here $\eta_{\mu\nu} = \text{diag}(+, -, -, -)$ and w is the radial coordinate on which the boundaries are located at $w = \pm w_{\text{max}}$. The explicit form of $H(w)$ and $U(w)$ are given in [5]. $D_M = \partial_M - iA_M$ ($M = 0, 1, 2, 3, w$) is a covariant derivative, $m_5(w)$ denotes the 5d mass (see [5] for the explicit form). Here $C = \frac{1}{(2\pi)^7} \frac{4\pi^2}{3} \frac{1}{l_s^{11/2}} \left(\frac{\lambda}{2M_{KK}} \right)^{1/4} N_f N_c$, and the $A_\nu(w)$ here is N_c times $A_\nu(w)$ in the standard convention. M_{KK} is the energy scale of the theory, λ is the 't Hooft coupling, N_f is the number of flavors, and l_s is the string length which does not show up in the end of the calculation.

The holographic mean fields are introduced by the following replacement in the above action: $\Psi(x, w) \rightarrow \Psi(w)$ and $A_M(x, w) \rightarrow A_M(w)$. Equations of motion for the mean fields are

$$i\Gamma^w \partial_w \Psi(w) + \Gamma^M A_M(w) \Psi(w) - m_5(w) \Psi(w) = 0, \quad (2.2)$$

$$\partial_w \frac{\partial \mathcal{L}_A}{\partial (\partial_w A_N(w))} - J^N(w) = 0, \quad J^N(w) = \bar{\Psi}(w) \Gamma^N \Psi(w). \quad (2.3)$$

It should be noted, although the gauge transformation depending on the 4d coordinates has been fixed by the above choice of the mean fields, we still have a residual gauge symmetry in which the gauge transformation depends only on the w -coordinate. By using this gauge transformation we take $A_w(w) = 0$ gauge in the following analysis. Note that equation (2.3) for $N = w$ is $J^w(w) = 0$. J^i ($i = 1, 2, 3$) may be non-zero in general. We assume that the possible non-zero component is given by J^3 and we set $J^1 = J^2 = 0$ without loss of generality. The condition $J^w = J^1 = J^2 = 0$ is realized by taking $\Psi = (\Psi_1, 0, \Psi_3, 0)^T$ with real Ψ_1 and Ψ_3 . One can easily confirm that this Ψ with $A_1 = A_2 = 0$ solves the coupled equations (2.2) and (2.3).

Now the equations of motion (EOM) for Ψ_1 and Ψ_3 are

$$\begin{aligned} (A_0(w) - m_5(w)) \Psi_1(w) + (A_3(w) + \partial_w) \Psi_3(w) &= 0, \\ (A_0(w) + m_5(w)) \Psi_3(w) + (A_3(w) - \partial_w) \Psi_1(w) &= 0. \end{aligned} \quad (2.4)$$

As we will show in the next section, $A_\mu(w)$ transform as $A_0(w) \rightarrow A_0(-w)$ and $A_i(w) \rightarrow -A_i(-w)$, under the parity transformation. Using this we decompose the HMF into the parity-even component $A_\mu^{(e)}(w)$ and the parity-odd component $A_\mu^{(o)}(w)$ as

$$A_0^{(e)}(w) = \frac{1}{2} [A_0(w) + A_0(-w)], \quad A_0^{(o)}(w) = \frac{1}{2} [A_0(w) - A_0(-w)], \quad (2.5)$$

$$A_i^{(e)}(w) = \frac{1}{2} [A_i(w) - A_i(-w)], \quad A_i^{(o)}(w) = \frac{1}{2} [A_i(w) + A_i(-w)]. \quad (2.6)$$

In the present analysis, we assume that $A_\mu^{(o)}(w) = 0$, i.e., $A_0(w)$ is an even function and $A_i(w)$ is an odd function of w . Then, from (2.4), one can easily see that Ψ_1 (Ψ_3) is either an even (odd) function of w or an odd (even) function [3].

The physical solution is constrained by the boundary coconditions

$$\Psi(\pm w_{\max}) = 0, \quad A_0(\pm w_{\max}) = \mu_0, \quad A_3(\pm w_{\max}) = \pm \mu_3, \quad (2.7)$$

and by the regularity conditions $\partial_w A_0(0) = A_3(0) = 0$. Here μ_0 is the baryon chemical potential and μ_3 is the source of the axial baryon current. Note that $\Psi(\pm w_{\max}) = 0$ guarantees that the one-point function of the 4d fermionic operator associated with Ψ vanishes [6].

We identify the action with the grand potential as discussed in [7, 8, 9]: $S = -\int d^4x \Omega(\mu, \mu_3)$. Once the solution of EOM is substituted, S_Ψ in (2.1) vanishes, while S_A is non-zero in which the information of baryons are book-kept. Note that Ω diverges since the \mathcal{L}_A goes infinity at the UV boundaries. Thus we regularize the grand potential Ω as $\Omega_{\text{reg}} = \Omega - \Omega_0$, where Ω_0 is the grand potential at zero density limit: $\int d^4x \Omega_0 \equiv -S|_{n_B=0}$.

In the present analysis we assume that the ground state forms a baryonic matter, which is achieved by taking Ψ_1 to be an even function with no node. We solve the coupled equations of motion at given values of μ and μ_3 to calculate Ω_{reg} . In Fig. 1, we plot μ_3 dependence of Ω_{reg} at $\mu_0/m_B = 1.01$, where m_B is the mass of the ground state baryon. The standard thermodynamics tells that the slope of Ω_{reg} gives the expectation value of the axial baryon number current operator conjugate to the source μ_3 . Figure 1 shows that a non-zero axial current remains even at $\mu_3 = 0$, showing that the rotational invariance is broken. As far as we examined, we could not find any rotationally invariant solution at finite baryon density.

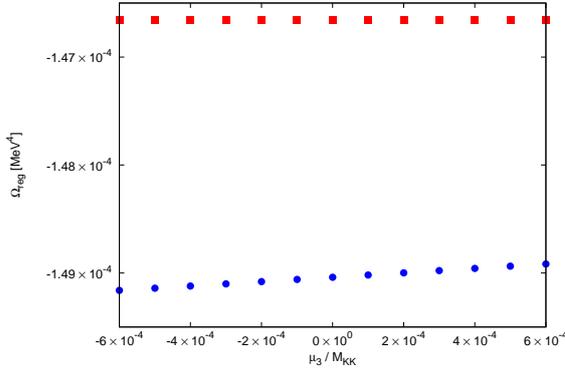


Figure 1: Ω_{reg} as a function of μ_3 at $\mu/m_B = 1.01$. The circular plots are shared by both the original model and the extended model whereas the box plots are only for the extended model.

3. An Extended Model

Note that the action (2.1) is not constructed in a parity invariant way [10, 11]. Let us extend our model to maintain the parity invariance. In order to construct a parity invariant theory, we need to introduce two fermionic fields Ψ^I and Ψ^{II} with opposite signature of the mass terms [10, 11]¹.

Let us coconsider action of the baryon sector given by $S_\Psi = S_\Psi^I + S_\Psi^{II}$, where S_Ψ^I is the same as (2.1) with Ψ replaced with Ψ^I . The second term is defined as

$$S_\Psi^{II} = \int dx^4 dw \left[\frac{i}{2} \overline{\Psi^{II}} \Gamma^M D_M \Psi^{II} - \frac{i}{2} (D_M \Psi^{II}) \Gamma^M \Psi^{II} + m_5(w) \overline{\Psi^{II}} \Psi^{II} \right]. \quad (3.1)$$

Note that Ψ^I and Ψ^{II} transform as $\Psi^I \rightarrow -\Gamma^0 \Gamma^w \Psi^{II}$ and $\Psi^{II} \rightarrow -\Gamma^0 \Gamma^w \Psi^I$ under the parity transformation, and one can confirm that the total action S_Ψ is now parity invariant.

In this extended model, the currents $J^N(w)$ in (2.3) are replaced with the contributions from Ψ^I and Ψ^{II} as follows:

$$J_N(w) = J_N^I(w) + J_N^{II}(w), \quad J_N^{I,II}(w) = \overline{\Psi^{I,II}}(w) \Gamma^N \Psi^{I,II}(w). \quad (3.2)$$

We impose the condition $J^w = J^1 = J^2 = 0$ by taking $\Psi^I = (\Psi_1^I, 0, \Psi_3^I, 0)^T$ and $\Psi^{II} = (\Psi_1^{II}, 0, \Psi_3^{II}, 0)^T$ with real $\Psi_1^{I,II}$ and $\Psi_3^{I,II}$. The equation of motion for $\Psi^I(w)$ is the same as (2.4) whereas the equation of motion for $\Psi^{II}(w)$ is given by

$$\begin{aligned} (A_0(w) + m_5(w)) \Psi_1^{II}(w) + (A_3(w) + \partial_w) \Psi_3^{II}(w) &= 0, \\ (A_0(w) - m_5(w)) \Psi_3^{II}(w) + (A_3(w) - \partial_w) \Psi_1^{II}(w) &= 0. \end{aligned} \quad (3.3)$$

We should impose the boundary condition $\Psi^{II}(\pm w_{\text{max}}) = 0$, on top of the boundary conditions (2.7) and the regularity conditions $\partial_w A_0(0) = A_3(0) = 0$ given in the previous section.

The numerical results from the extended model are shown in Fig. 1. The solutions given by the circular plots, which break the rotational invariance at $\mu_3 = 0$, remain solutions in the extended model, whereas new solutions given by boxes appear. We can read the expectation value of the axial current from the slope of the each plots, and we find that the new solutions give vanishing current at $\mu_3 = 0$, restoring the rotational invariance.

¹See also another model [12] based on a bottom-up approach [13].

However the grandpotential of the new solution is greater than the rotationally non-invariant solution. This suggests that the rotational invariance is still broken at zero temperature at small baryon density corresponding to $\mu/m_B = 1.01$.

4. Discussions

We found that the original model proposed by [3] does not have rotationally invariant solution at zero temperature at small baryon density. In the present work, we proposed a parity-invariant extension of the model of [3], and we found that the extended model has a rotationally invariant state. The two baryon fields with opposite parity in the extended model remind us of the parity doublet model [4]. A picture of parity doublet model is naturally requested in the holographic mean field approach.

As far as we have examined within the present work, the rotationally invariant state turned out not to be the ground state, suggesting a spontaneous breaking of the rotational invariance with finite expectation value of the axial current operator. The finite expectation value of axial current operator may be understood as a condensation of axial vector mesons.

However, we postpone a definite conclusion here. The main statement in the present work is to show the presence of a rotationally invariant state in the extended model at finite baryon densities. In order to reach the final conclusion, we think that we should relax the ansatz we have employed, and we need to numerically survey wider range of the possible solutions. These issues remain to be investigated in future work [6].

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