

In-medium η' mass and $\eta'N$ interaction in vacuum in the linear sigma model^{*}

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We investigate the $\eta'N$ two-body interaction in the context of the η' meson mass modification in the nuclear medium. It has been argued in several articles that the masses of η' and the other pseudoscalar mesons (π , K, η) should degenerate in the chiral-symmetric phase. It is expected that the reduction of the mass difference between η and η' would take place in the nuclear matter if one assumes that the decrease of the quark condensate at the normal nuclear density occurs with partial restoration of chiral symmetry. At low density, the in-medium self-energy giving the mass modification by the medium effect can be obtained by the $\eta'N$ two-body T matrix. Thus, we also estimate the $\eta'N$ interaction strength in vacuum with the linear sigma model which involves the effect of partial restoration of chiral symmetry. In the view of the linear sigma model, we find that the $\eta'N$ interaction is attractive and generated through the sigma meson exchange.We expect that the interaction is enough strong and for the existence of a bound state of the $\eta'N$ system.

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1. Introduction

The mass spectrum of hadrons reflects the symmetry of its fundamental theory, Quantum Chromodynamics (QCD). If one considers simply, nine Nambu–Goldstone (NG) bosons would appear according to the NG theorem when one assumes the axial part of $U(3)_L \times U(3)_R$ is spontaneously broken down to $U(3)_V$. Nevertheless, when one sees the pseudoscalar meson masses, one cannot find the singlet pseudoscalar meson around the pion mass [1]. Taking account of the $U_A(1)$ anomaly, we can regard the $U_A(1)$ symmetry as broken explicitly and the η' mass can be expained with the $U_A(1)$ anomaly [2].

Other than the $U_A(1)$ anomaly, chiral symmetry breaking also plays an important role for the generation of the η' mass. As discussed in Refs. [3], the pseudoscalar singlet meson and octet meson degenerate when chiral symmetry is restored, e.g. at high temperature or high density, in the chiral limit even if the $U_A(1)$ symmetry is broken explicitly. Taking account of the degeneracy of the pseudoscalar flavor-singlet and octet meson in the chiral restored phase and the partial restoration of chiral symmetry, we expect the reduction of the η' mass in the nuclear matter. Concerning the chiral restoration in the nuclear matter, the 35% reduction of the quark condensate is suggested from the analysis of the experimental data [4]. Recent experiments have suggested that the η' -nucleus optical potential could be attractive with certain strength and could hava a smaller imaginary part [5]. We can interpret the η' mass reduction in the nuclear matter as the attractive potential of η' in the nuclear matter. The existence of η' -mesic nuclei is suggested theoretically [6] and the experimental attempt to observe the η' and nucleon is attractive or repulsive despite the existence of some experimental data [8]. Such a poor knowledge of the $\eta'N$ interaction makes it difficult to analyze the η' properties in the nuclear matter.

In the below, we study the $\eta' N$ two-body interaction with the linear sigma model as a chiral effective model. In the construction of the model, we assume the 35% reduction of the quark condensate. In addition to the $\eta' N$ interaction, we calculate the in-medium η' mass, which is expected to reduce in the nuclear matter. The detail of this work is shown in Ref. [9].

2. Method

For the calculation, we use the SU(3) linear sigma model as a chiral effective model [10, 11]. The linear sigma model can describe both the chiral restored phase and the spontaneously broken phase. To describe the $\eta'N$ interaction, we introduce the nucleon degree of freedom explicitly based on the chiral symmetry. The Lagrangian is given as

$$\mathscr{L} = \frac{1}{2} \operatorname{tr} \partial_{\mu} M \partial^{\mu} M^{\dagger} - \frac{\mu^{2}}{2} \operatorname{tr} M M^{\dagger} - \frac{\lambda}{4} \operatorname{tr} (M M^{\dagger})^{2} - \frac{\lambda'}{4} \left(\operatorname{tr} M M^{\dagger} \right)^{2} + A \operatorname{tr} \left(\chi M^{\dagger} + M \chi^{\dagger} \right) + \sqrt{3} B \left(\det M + \det M^{\dagger} \right) + \bar{N} i \partial N - g \bar{N} \left(\frac{\sigma_{0}}{\sqrt{3}} + \frac{\sigma_{8}}{\sqrt{6}} + i \gamma_{5} \frac{\vec{\tau} \cdot \vec{\pi}}{\sqrt{2}} + i \gamma_{5} \frac{\eta_{0}}{\sqrt{3}} + i \gamma_{5} \frac{\eta_{8}}{\sqrt{6}} \right) N.$$

$$(2.1)$$

Here, the meson field, the nucleon field, and the quark mass are given, respectively, by

$$M = \sum_{a=0}^{8} \frac{\sigma_a \lambda_a}{\sqrt{2}} + i \sum_{a=0}^{8} \frac{\pi_a \lambda_a}{\sqrt{2}}, \ N = (p, n)^{t}, \ \chi = \text{diag}(m_q, m_q, m_s).$$
(2.2)



Figure 1: In-medium quark condensate. The **Figure 2:** In-medium meson mass. The solid, dashed and solid lines represent the in-medium dotted and dashed lines represent the η' , η and u, d and s quark condensates, respectively. π masses, respectively.

The Lagrangian is constructed to possess the same global symmetry as QCD. The term proportional to *A* expresses the effect of the current quark mass. Here, χ corresponds to the current quark mass and we assume the isospin symmetry, $m_q = m_u = m_d$, and introduce the SU(3) flavor symmetry breaking with $m_q \neq m_s$. The term proportional to *B* represents the effect of the U_A(1) anomaly and this term is not invariant under the U_A(1) transformation.

The Lagrangian contains 6 free parameters which cannot be fixed from the symmetry. We fix these parameters using the observed meson masses, the meson decay constants and the fact that the 35% reduction of the quark condensate at the normal nuclear density. For the calculation of the in-medium quantities, i.e. the meson masses, we introduce the effect of the symmetric nuclear matter with the mean field approximation of nucleon.

In the linear sigma model, the vacuum expectation value of the sigma field $\langle \sigma_0 \rangle$ is an order parameter of the chiral symmetry breaking. Now, we have non-zero $\langle \sigma_8 \rangle$ due to the explicit flavor symmetry breaking. We determine $\langle \sigma_0 \rangle$ and $\langle \sigma_8 \rangle$ to minimize the effective potential.

3. Results

3.1 In-medium meson mass

First, we show the in-medium quark condensate in Fig. 1. As mentioned above, we assume the 35% reduction of the quark condensate, so the value of the u, d quark condensate at the normal nuclear density is the input value. Here, we have assumed the isospin symmetry, so the u, d quark condensates coinside. Next, we discuss the in-medium meson mass. The in-medium self-energy of the mesons comes from the diagrams shown in Fig. 3. Diagram (*a*) in Fig. 3 comes from the nucleon mean field, while (*b*) and (*c*) come from the particle-hole excitation and the crossed channel of the particle-hole excitation, respectively. In the chiral limit, the η' mass can be written as

$$m_{n'}^2 = 6B \langle \sigma_0 \rangle. \tag{3.1}$$

B is the coefficient of the determinant term and represents the effect of the U_A(1) anomaly. From this expression, one can find the necessity of the U_A(1) anomaly and the chiral symmetry breaking for the generation of the η' mass, and the restoration of chiral symmetry, or the reduction of $\langle \sigma_0 \rangle$, leads to the reduction of the η' mass. The calculated in-medium meson masses are shown in Fig. 2.



Figure 3: The diagrams contributing to the inmedium meson self-energy. **Figure 4:** The diagrams which contribute to the $\eta'N$ interaction

From the calculation, the η' mass reduces about 80 MeV and the η mass enhances about 50 MeV at the normal nuclear density. The mass difference between the η and η' mass reduces about 130 MeV, going toward the degeneracy of η and η' , as we have expected.

3.2 $\eta' N$ **2-body interaction**

Here, we show the $\eta'N$ 2-body interaction evaluated with the same linear sigma model. In the linear sigma model, the diagrams shown in Fig. 4 contribute to the $\eta'N$ interaction. The diagram (*a*) in the Fig. 4 shows the contribution from the sigma meson exchange, while diagram (*b*) and (*c*) in Fig. 4 are the contributions from the Born term which contain the nucleon intermediate state. From these diagrams, we have obtained the low-energy $\eta'N$ 2-body interaction $V_{\eta'N}$ in the chiral limit as

$$V_{\eta'N} = -\frac{6gB}{\sqrt{3}m_{\sigma_0}^2}.$$
(3.2)

Substituting the value of the fixed parameters, we find that the $\eta'N$ interaction is comparably strong to the $\bar{K}N$ system. In the $\bar{K}N$ system, a bound state, $\Lambda(1405)$, exists due to the strong $\bar{K}N$ attraction. With the analogy to $\Lambda(1405)$, we expect the existence of the $\eta'N$ bound state. For the investigation of the possibility of the bound state, we analyzed the T-matrix of the $\eta'N$ system because the bound state appears as the pole of the T-matrix. We obtain the T-matrix with solving the single-channel Lippman-Schwinger equation,

$$T = V + VGT. \tag{3.3}$$

Here, G is the loop integral of η' and nucleon,

$$G(W) = 2m_N \int \frac{d^4q}{(2\pi)^4} \frac{1}{(P-q)^2 - m_N^2 + i\varepsilon} \frac{1}{q^2 - m_N^2 + i\varepsilon},$$
(3.4)

 $P = (W, \mathbf{0})$ is the 4-momentum of the $\eta' N$ in the center of mass system. As the interaction kernel V, we use the $\eta' N$ interaction obtained with linear sigma model shown in Eq. (3.2). Now, the interaction kernel is momentum-independent, so the equation can be solved with the algebraic way,

$$T(W) = \frac{1}{V^{-1} - G(W)}.$$
(3.5)

The obtained T matrix contains a divergence in the loop integral G. Here, we regulate the divergence with dimensional regularization and we fix the subtraction constant with the natural renormalization scheme, which excludes the other dynamics than η' and N [12]. We have found a $\eta'N$

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bound state as a pole of the obtained T matrix. The binding energy is 6.2 MeV, the scattering length is -2.7 fm and the effective range is 0.25 fm. The scattering length is the repulsive sign in our notation. Here, we note that the obtained $\eta'N$ scattering length is somewhat larger value compared to the value suggested in Ref. [8].

4. Conclusion

In this paper, we have calculated the in-medium meson mass and the $\eta'N$ 2-body interaction with the SU(3) linear sigma model. The medium effect is introduced as one nucleon loop for the calculation of the in-medium meson mass. We have obtained about 80 MeV reduction of the η' mass and 130 MeV decrease of the mass difference between η and η' . Concerning the $\eta'N$ twobody interaction, we have found the strong attraction of $\eta'N$ comparable to the $\bar{K}N$ system. The $\eta'N$ interaction obtained from the linear sigma model is provided from the sigma meson exchange. This is a different character from that of the ordinary NG boson, the Weinberg–Tomozawa interaction which is energy dependent. With the analogy of $\Lambda(1405)$, we have investigated the possibility of the $\eta'N$ bound state. As a result, we found a $\eta'N$ bound state with the binding energy 6.2 MeV and the scattering length -2.7 fm. The coupling between σ_0 and η' is necessary for the generation of the η' mass and the η' - σ_0 coupling leads to the attraction through the sigma meson exchange.

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