

## Parity doubling among baryons in a Holographic QCD model at finite density

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We summarize our recent work where we developed the holographic mean field approach in a bottom-up holographic QCD model including baryons. We first fix the infrared cut-off  $z_m$  through the mesonic sector to analyse the amount of the ground state nucleon mass coming from the chiral symmetry breaking by varying the infrared boundary condition for the 5D baryon field. For the cold dense baryonic matter, by introducing the mean field for the baryon fields, we calculated the equation of state between the baryon number density and its corresponding chemical potential. Then, by using a Walecka type model, we get the density dependence of the effective nucleon mass. The result shows that the quark condensate and the effective mass decreases at high density region, and that the more amount of the proton mass comes from the chiral symmetry breaking, the faster the quark condensate and the effective nucleon mass decrease with increasing density.

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## 1. Introduction

There are some important features in low-energy QCD region, and one of them is the spontaneous chiral symmetry breaking ( $\chi$ SB). This is thought to be the origin of several kind of hadron masses such as the mass of the lightest nucleon. However, there is a possibility that only some kind of lightest nucleon mass is generated by the spontaneous  $\chi$ SB and the rest is the chiral invariant mass. This structure is represented by a so called parity doublet models. It is an interesting question to ask how much nucleon mass is generated by the spontaneous  $\chi$ SB, or to investigate the origin of nucleon mass. Since at high density region a partial restoration of chiral symmetry will occur, probing dense baryonic matter would give some clues to understand the origin of mass. In Ref. [1], we studied the effect of the chemical potential and the baryon number density on the equation of state by considering a parity doublet structure. In this write-up, we summarize main points of the work.

## 2. Parity doubling structure of the model

In this section we briefly review the holographic QCD model including baryons at zero density, which we used in Ref. [1].

### 2.1 model

To the present analysis, we consider the contribution from the scalar meson field  $X$  and two baryon fields  $N_1$  and  $N_2$ , and with the 5-dimensional gauge fields  $R_A$  and  $L_A$ . Which the bulk action given as [2]

$$S = S_{N_1} + S_{N_2} + S_{\text{int}} + S_X, \quad (2.1)$$

where

$$S_{N_1} = \int d^5x \sqrt{g} \left\{ \frac{i}{2} \bar{N}_1 e_A^M \Gamma^A \nabla_M N_1 - \frac{i}{2} \left( \nabla_M^\dagger \bar{N}_1 \right) e_A^M \Gamma^A N_1 - M_5 \bar{N}_1 N_1 \right\}, \quad (2.2)$$

$$S_{N_2} = \int d^5x \sqrt{g} \left\{ \frac{i}{2} \bar{N}_2 e_A^M \Gamma^A \nabla_M N_2 - \frac{i}{2} \left( \nabla_M^\dagger \bar{N}_2 \right) e_A^M \Gamma^A N_2 + M_5 \bar{N}_2 N_2 \right\}, \quad (2.3)$$

$$S_{\text{int}} = - \int d^5x \sqrt{g} G \left\{ \bar{N}_2 X N_1 + \bar{N}_1 X^\dagger N_2 \right\}, \quad (2.4)$$

$$S_X = \int d^5x \sqrt{g} \text{Tr} \left\{ |DX|^2 - m_5^2 |X|^2 - \frac{1}{4g_5^2} (F_L^2 + F_R^2) \right\}, \quad (2.5)$$

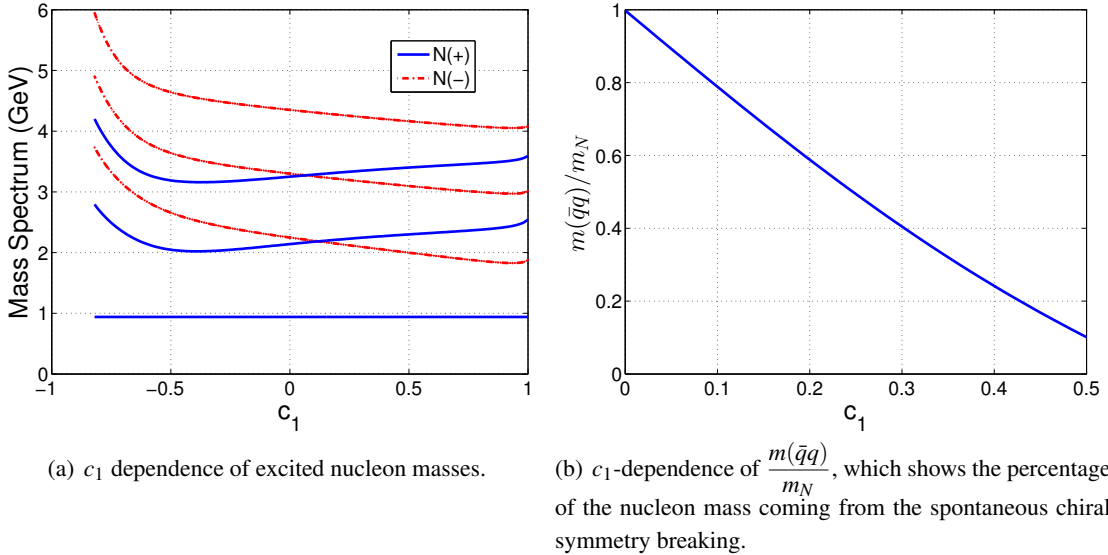
with  $M_5 = 5/2$  and  $m_5^2 = -3$  being the bulk masses corresponding to baryons and mesons,  $G$  the scalar-baryon coupling constant,  $g_5$  the gauge coupling constant. The vielbein  $e_M^A$  satisfies  $g_{MN} = e_M^A e_N^B \eta_{AB} = \frac{1}{z^2} \text{diag}(+ - - -)$ , where  $M$  labels the general space-time coordinate and  $A$  labels the local Lorentz space-time, with  $A, M \in (0, 1, 2, 3, z)$ . We take the vielbein as  $e_M^A = \frac{1}{z} \eta_M^A = \frac{1}{z} \text{diag}(+ - - -)$ , which fixes the gauge for the Lorentz transformation. The 5 dimensional Dirac matrices  $\Gamma^A$  are defined as  $\Gamma^\mu = \gamma^\mu$  and  $\Gamma^z = -i\gamma^5$  which satisfy the anti-commutation relation  $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$ . The covariant derivatives are defined as  $\nabla_M N_1 = (\partial_M + \frac{i}{4} \omega_M^{AB} \Gamma_{AB} - i(A_L^a)_{M^a}) N_1$ ,  $\nabla_M N_2 = (\partial_M + \frac{i}{4} \omega_M^{AB} \Gamma_{AB} - i(A_R^a)_{M^a}) N_2$ ,  $D_M X = \partial_M X - iA_{LM} X + iX A_{RM}$ , where  $\Gamma^{AB} = [\Gamma^A, \Gamma^B]/(2i)$ . The spin connection  $\omega_M^{AB}$  is given by  $\omega_M^{AB} = \frac{1}{z} (\eta_Z^A \eta_M^B - \eta_M^A \eta_Z^B) \eta^{ZZ}$ .

The two baryon fields  $N_1$  and  $N_2$  reproduce the chirality of baryon which transforms as the  $(2, 1)$  and  $(1, 2)$  representations of  $SU(2)_L \times SU(2)_R$  respectively. The interaction term mixes two towers of massive modes from each of  $N_1$  and  $N_2$ , and breaks their degeneracy into the parity doublet pattern.

## 2.2 parity doubling structure

The scalar field  $X$  has a solution which is obtained as [3, 4]  $X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\sigma z^3$ , where  $M$  corresponds to current quark mass and  $\sigma$  corresponds to quark condensate  $\langle \bar{q}q \rangle$ .

The bulk fields  $N_1$  and  $N_2$  can be decomposed as  $N_1 = N_{1L} + N_{1R}$ ,  $N_2 = N_{2L} + N_{2R}$ , where  $N_{1L} = i\Gamma^z N_{1L}$ ,  $N_{1R} = -i\Gamma^z N_{1R}$ ,  $N_{2L} = i\Gamma^z N_{2L}$ ,  $N_{2R} = -i\Gamma^z N_{2R}$ . The baryon mass spectrum is determined by solving the equations of motion of  $N_1$  and  $N_2$ . Two IR boundary values of two baryon fields are set to be 1 and  $c_1$  with  $c_1$  being a free parameter to be determined later. While the UV boundary values are set to be zero for normalizable modes. We show the  $c_1$ -dependence of the masses of higher excited nucleons in Fig. 1(a). Here  $N(+)$  denotes the positive-parity state and  $N(-)$  the negative one. There are two parts in the Fig. 1(a): for  $c_1 > c_1^* \approx 0.12$ , the first excited state carries the negative parity and the second the positive parity, while for  $c_1 < c_1^*$ , the positive-parity excited nucleon be the first excited state which is consistent with the experimental data. For understanding the meaning of  $c_1$ , we investigate the effect of dynamical chiral symmetry breaking on the nucleon mass in the following way: for given value of  $c_1$  we take  $\sigma = 0$  and calculate the mass eigenvalue by solving the corresponding equation of motion. The lowest eigenvalue  $m_0^{(1)}$ , denoted as  $m_0$  in the following, is considered as the chiral invariant mass of nucleon. In Fig. 1(b), we plot the  $c_1$  dependence of the value of  $1 - m_0/m_N \equiv \frac{m(\bar{q}q)}{m_N}$  which shows the percentage of the nucleon mass coming from the spontaneous chiral symmetry breaking. From Fig. 1(b) we



**Figure 1:**  $c_1$  dependence of excited nucleon masses and  $\frac{m(\bar{q}q)}{m_N}$ .

conclude that, in the case of  $c_1 = 0$ , which is chosen in Ref. [2], all the nucleon mass comes from

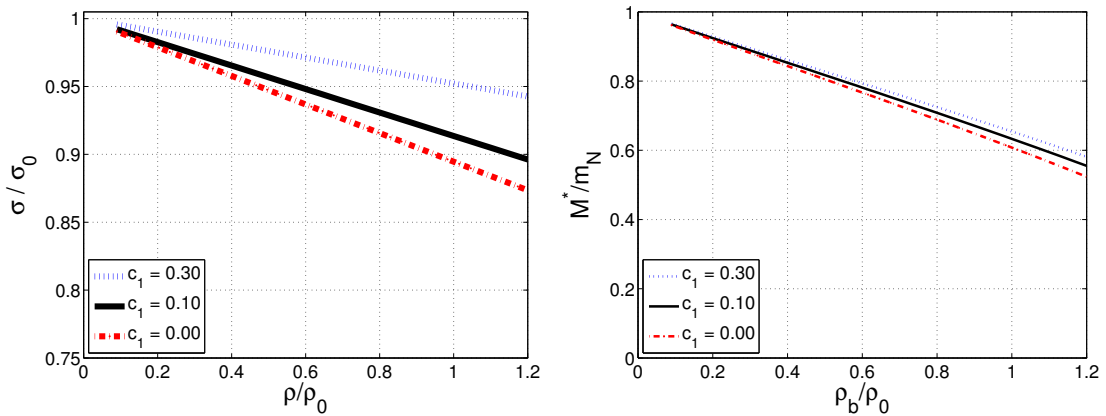
the spontaneous chiral symmetry breaking. On the other hand, when  $c_1 > 0.25$ , more than half of the nucleon mass is the chiral invariant mass.

### 3. Equation of state in the holographic mean field approach to the model

In this section, we study the finite density system using the holographic mean field theory proposed in Ref. [5]. The main idea of holographic mean field theory is to introduce the mean fields for all the 5D fields including fermion fields. In the present analysis, we consider only the symmetric nuclear matter, in which the proton and the neutron have the same mean fields. Furthermore, we assume that only the  $U(1)_V$  gauge field within the vector and axial-vector gauge fields and the trace part of the scalar field have their own mean fields. The baryon number density is written in terms of the baryon fields as  $\rho_b = \int \frac{dz}{2z^4} (N_+^\dagger N_+ + N_-^\dagger N_-) = \int dz \rho(z)$ , which is controlled by the IR values of  $N_+$  and  $N_-$ . The baryon chemical potential  $\mu$  is introduced as the value of  $V_0$  at the UV boundary.

It is believed that chiral symmetry is partially restored at the higher density region. Defining the in-medium condensate through the holographic mean field  $X(z)$  as  $\sigma = \frac{2X(z)}{z^3} \Big|_{z=z_{UV}}$ , we study the density dependence of the chiral condensate for checking the partial chiral restoration. In Fig. 2(a) we plot the density dependence of the  $\sigma$  normalized by the vacuum value  $\sigma_0$ . The figure shows that, when the number density is increased, the quark condensate  $\sigma$  decreases. This can be understood as a sign of the partial chiral symmetry restoration. The amount of the chiral condensate at higher density is consistent with the one obtained in Ref. [6].

For studying the density dependence of nucleon mass, we define the effective nucleon mass using the Walecka type model (see e.g. Refs. [7]), in which the relation between the effective nucleon mass  $M^*$  and the chemical potential  $\mu$  is expressed as:  $\mu = \sum_{n=1}^{\infty} \frac{g_{\omega^{(n)}NN}^2}{m_{\omega^{(n)}}^2} \rho_b + \sqrt{k_F^2 + M^{*2}}$ , where  $\rho_b$  is the baryon number density,  $g_{\omega^{(n)}NN}$  is the coupling for  $n$ th eigenstate of the omega mesons,  $m_{\omega^{(n)}}$  is its mass,  $k_F$  is the Fermi momentum. In Fig. 2(b) we plot the density dependence of the effective mass  $M^*$ . From this figure we find that the effective mass decreases with increasing



(a) Density dependence of  $\sigma/\sigma_0$  for several choices of  $c_1$ . (b) Density dependence of the effective nucleon mass  $M^*$ .

**Figure 2:** Density dependence of  $\sigma/\sigma_0$  and the effective nucleon mass  $M^*$ .

density. The decreasing rate is larger than the one obtained in Ref. [6], which is the reflection of the iterative corrections included through the holographic mean field theory. It should be noted that for smaller value of  $c_1$ , the decreasing of  $M^*$  is more rapid. This could be understood as that, the more rapidly the effective mass  $M^*$  decreases with density due to the larger amount of the mass coming from the chiral symmetry breaking at vacuum.

#### 4. A summary and discussions

By applying the holographic mean field approach in a bottom-up holographic QCD model proposed in Ref. [2]. We found the IR boundary value ( $c_1$ ) for one of two baryon fields controls the percentage of the chiral invariant mass: for  $c_1 = 0$  all of the mass of the ground-state nucleon is generated by the spontaneous chiral symmetry breaking, while for  $c_1 > 0.25$ , more than half of the nucleon mass is actually the chiral invariant mass.

We also studied the density dependence of the chiral condensate using the holographic mean field approach proposed in Ref. [5]. Our result shows that while number density increases, the quark condensate  $\sigma$  decreases, which means that the chiral symmetry is partially restored at high density region. We next calculated the effective mass of nucleon and the baryon number density. We found the larger percentage of the mass coming from the spontaneous  $\chi$ SB, the more rapidly the effective nucleon mass decreasing in large baryon number density.

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