

KN scattering with hadrons exchange potential in 3D technique

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We have formulated the KN scattering in a three-dimensional (3D) technique. The Lippmann-Schwinger equation is evaluated in 3D basis states without partial-wave (PW) expansion. The result is a set of two coupled integral equations for T-matrix elements in two variables, namely the momentum magnitude and the scattering angle. For the KN interaction we employ an already derived hadrons exchange model with the second order contributions only. The parameters of the model will be later determined by fitting to the experimental data for the cross section and some spin observables.

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1. Introduction

The partial-wave (PW) expansion has been a successful technique in solving few-particle systems, see for instance Ref. [1]. But as the energy increases PW calculations become more tedious both algebraically and numerically, since many more higher angular momentum states have to be considered. The three-dimensional (3D) technique appears to be a good alternative to the PW one in solving the Lippmann-Schwinger (LS) equation, see for example Ref. [2, 3]. In 3D technique the linear momentum state as part of the basis state is not expanded in the angular momentum states, thus, all partial waves are taken.

In Ref. [4] we compare PW calculations to 3D calculations for the cross section of KN scattering based on a simple Yukawa-type interaction with a spin-orbit force. We find that for higher energies of a few hundreds MeV, the PW calculations converge slowly. This suggests us more to use the 3D technique especially for higher energies. In this work we step further in our investigation on KN scattering using the 3D technique by taking a more realistic KN interaction derived in Ref. [5]. Our future plan is to refit the parameters of the potential by comparing our 3D calculations to experimental data for the cross section and some spin observables.

We describe the KN interaction that we take in Section 2. In Section 3 we show the 3D formulation. We summarize in Section 4.

2. Hadron Exchange Interaction

Here we describe briefly the KN interaction model developed by Büttgen et al. [5]. We show only those part of the model that we take in our present work. We take only up to the second order in accordance with our attempt to create a kind of one-hadron-exchange potential for KN system. The KN interaction model in Ref. [5] considers mesons and baryons as the degrees of freedom of the strong interaction dynamics. In this approach, the interactions are represented by the meson-meson-meson and baryon-baryon-meson vertices. For the baryon-baryon-scalar meson (bb_s), baryon-baryon-pseudoscalar meson (bb_p), baryon-baryon-vector meson (bb_v), pseudoscalar-pseudoscalar-scalar meson (pps), and pseudoscalar-pseudoscalar-vector meson (ppv) vertices, respectively, the interaction Lagrangians take the following forms

$$\mathcal{L}_{bb_s} = g_{bb_s} \bar{\Psi}_b \Psi_b \Phi_s \quad (2.1)$$

$$\mathcal{L}_{bb_p} = g_{bb_p} \bar{\Psi}_b i \gamma_5 \Psi_b \Phi_p \quad (2.2)$$

$$\mathcal{L}_{bb_v} = g_{bb_v} \bar{\Psi}_b \gamma_\mu \Psi_b \Phi_v^\mu + \frac{f_{bb_v}}{4m_N} \bar{\Psi}_b \sigma_{\mu\nu} \Psi_b (\partial^\mu \Phi_v^\nu - \partial^\nu \Phi_v^\mu) \quad (2.3)$$

$$\mathcal{L}_{pps} = g_{pps} m_p \Phi_p \Phi_p \Phi_s \quad (2.4)$$

$$\mathcal{L}_{ppv} = g_{ppv} \Phi_p (\partial_\mu \Phi_p) \Phi_v^\mu. \quad (2.5)$$

Here Ψ_b is the baryon field operator and Φ_s , Φ_p , and Φ_v^μ are the field operators for scalar, pseudoscalar, and vector mesons, respectively. Note that in the expressions above only the space-spin part is given. The isospin factor will be inserted in the equation describing the potential (see explanations below). As explained in Ref. [5] only the σ -meson is considered for the scalar meson, K -meson for the pseudoscalar meson, ρ - and ω -meson for the vector mesons, and the nucleon N , Λ -, and Σ -hyperon for the baryons. The second-order time-ordered diagrams are shown in Fig. 1.

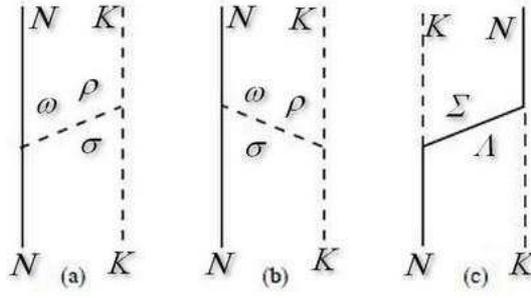


Figure 1: Time ordered diagrams for hadrons exchange interaction that is used in this work.

The diagrams in Fig. 1 lead to the following hadron exchange interactions [5]. The σ -meson exchange (diagrams (a)+(b)) gives

$$\langle \vec{p}'\lambda' | V_\sigma | \vec{p}\lambda \rangle = \frac{g_{KK\sigma}g_{NN\sigma}}{16\pi^3} \frac{F_{KK\sigma}F_{NN\sigma}}{\sqrt{\omega_K\omega'_K}} \frac{\bar{u}(\vec{p}'\lambda')u(\vec{p}\lambda)}{q_\sigma^2 - m_\sigma^2}. \quad (2.6)$$

The vector meson ($v = \rho, \omega$) exchange (diagrams (a)+(b)) gives

$$\begin{aligned} \langle \vec{p}'\lambda' | V_v | \vec{p}\lambda \rangle = & -\frac{g_{KKv}F_{KKv}F_{NNv}}{32\pi^3} \frac{1}{\sqrt{\omega_K\omega'_K}} \frac{1}{q_v^2 - m_v^2} \left((g_{NNv} + f_{NNv})(p'_K + p_K)^\mu \bar{u}(\vec{p}'\lambda')\gamma_\mu u(\vec{p}\lambda) \right. \\ & \left. - \frac{f_{NNv}}{2m_N}(p_N + p'_N)^\mu (p_K + p'_K)_\mu \bar{u}(\vec{p}'\lambda')u(\vec{p}\lambda) \right). \end{aligned} \quad (2.7)$$

Since the ρ -meson has isospin 1, the corresponding contribution should be multiplied by an isospin factor $\tau_1 \cdot \tau_2$. Note that both kaon and nucleon have isospin $\frac{1}{2}$. The hyperon ($Y = \Lambda, \Sigma$) exchange (diagram (c)) gives

$$\langle \vec{p}'\lambda' | V_Y | \vec{p}\lambda \rangle = \frac{g_{KYN}^2}{32\pi^3} \frac{F_{KYN}^2}{\sqrt{\omega_K\omega'_K}} \frac{\bar{u}(\vec{p}'\lambda')(\gamma_\mu p_Y^\mu - m_Y)u(\vec{p}\lambda)}{E_Y(z - \omega'_K - \omega_K + E_Y + i\varepsilon)} \quad (2.8)$$

with $z = E_N + \omega_K$. The isospin factor for Λ -exchange is $\frac{1}{2}(1 + \tau_1 \cdot \tau_2)$ and for Σ -exchange is $\frac{1}{2}(3 - \tau_1 \cdot \tau_2)$. In these formulations, λ is the quantum number of the spin projection in some preferred direction and \vec{p} and \vec{p}' are the relative momentum of kaon-nucleon system in the initial and final states, respectively. In order to consider the hadron structure, Ref. [5] applies the form factor

$$F_h = \left(\frac{\Lambda_h^2 - m_h^2}{\Lambda_h^2 + \vec{q}_h^2} \right)^{n_h} \quad (2.9)$$

for meson-meson-meson vertices with $n_h = 1$ ($h = \sigma, \omega, \rho$) and $\vec{q}_h = \vec{p}' - \vec{p}$ is the transferred momentum. For hyperon containing vertices, Ref. [5] applies the form factor

$$F_Y = \left(\frac{\Lambda_Y^4 + m_Y^4}{\Lambda_Y^4 + q_Y^4} \right) \quad (2.10)$$

with q_Y is the transferred 4-momentum. As mentioned in Ref. [5], the reason for taking this form factors is to consider the off-shell behaviour of a hyperon exchanged.

3. Three-dimensional (3D) Formulation

The basis state in the 3D technique is defined as [4]

$$|\vec{p}\lambda\rangle \equiv |\vec{p}\rangle|\hat{z}\lambda\rangle \quad (3.1)$$

which is a direct product of the linear momentum state $|\vec{p}\rangle$ dan the spin state $|\hat{z}\lambda\rangle$ with the spin being quantized along the z -axis and $\lambda = \pm\frac{1}{2}$. In this basis, the LS equation is obtained as

$$T_{\lambda'\lambda}(\vec{p}', \vec{p}) = V_{\lambda'\lambda}(\vec{p}', \vec{p}) + \sum_{\lambda''} \int d\vec{p}'' V_{\lambda'\lambda''}(\vec{p}', \vec{p}'') G_0^+(E_{p''}, E_p) T_{\lambda''\lambda}(\vec{p}'', \vec{p}), \quad (3.2)$$

where

$$T_{\lambda'\lambda}(\vec{p}', \vec{p}) \equiv \langle \vec{p}'\lambda' | T | \vec{p}\lambda \rangle \quad (3.3)$$

$$V_{\lambda'\lambda}(\vec{p}', \vec{p}) \equiv \langle \vec{p}'\lambda' | V | \vec{p}\lambda \rangle = \sum_h \langle \vec{p}'\lambda' | V_h | \vec{p}\lambda \rangle \quad (3.4)$$

with $h = \sigma, \rho, \omega, \Lambda, \Sigma$. The free propagator is given as

$$G_0^+(E_{p''}, E_p) = \lim_{\varepsilon \rightarrow 0} \frac{1}{E_p + i\varepsilon - E_{p''}} \quad (3.5)$$

where $E_p = p^2/2\mu$ is the initial kinetic energy of the system, which is also the energy at which the scattering occurs.

The KN potential in momentum space has a general structure as

$$\begin{aligned} V_h(\vec{p}', \vec{p}) &= \langle \vec{p}' | V_h | \vec{p} \rangle \\ &= A_h(\vec{p}', \vec{p}) + B_h(\vec{p}', \vec{p}) \vec{\sigma} \cdot \vec{p}' \vec{\sigma} \cdot \vec{p}. \end{aligned} \quad (3.6)$$

with $A_h(\vec{p}', \vec{p})$ and $B_h(\vec{p}', \vec{p})$ being some spin-independent function and $\vec{\sigma}$ are the Pauli's matrices. We evaluate the matrix element of the spin operator in the last term of Eq.(3.6) and obtain

$$\begin{aligned} \langle \hat{z}\lambda' | \vec{\sigma} \cdot \vec{p}' \vec{\sigma} \cdot \vec{p} | \hat{z}\lambda \rangle &= \delta_{\lambda'\lambda} \left(\cos \theta' \cos \theta + e^{-2i\lambda(\phi' - \phi)} \sin \theta' \sin \theta \right) \\ &\quad + \delta_{\lambda', -\lambda} 2\lambda e^{2i\lambda\phi'} \left(\sin \theta' \cos \theta - e^{-2i\lambda(\phi' - \phi)} \cos \theta' \sin \theta \right). \end{aligned} \quad (3.7)$$

In the case $\vec{p} = p\hat{z}$, the azimuthal behaviour of $V_{\lambda'\lambda}(\vec{p}', \vec{p})$ and $T_{\lambda'\lambda}(\vec{p}', \vec{p})$ show up as

$$V_{\lambda'\lambda}(\vec{p}', p\hat{z}) = e^{-i(\lambda' - \lambda)\phi'} V_{\lambda'\lambda}(p', p, \theta') \quad (3.8)$$

$$T_{\lambda'\lambda}(\vec{p}', p\hat{z}) = e^{-i(\lambda' - \lambda)\phi'} T_{\lambda'\lambda}(p', p, \theta'). \quad (3.9)$$

This azimuthal behaviour leads to the final form of the integral equation for the T -matrix element

$$\begin{aligned} T_{\lambda'\lambda}(p', p, \theta') &= V_{\lambda'\lambda}(p', p, \theta') + 2\mu \lim_{\varepsilon \rightarrow \infty} \sum_{\lambda''} \int_0^\infty dp'' \frac{p''^2}{p^2 + i\varepsilon - p''^2} \\ &\quad \times \int_{-1}^1 d\cos \theta'' V_{\lambda'\lambda''}^\lambda(p', p'', \theta', \theta'') T_{\lambda''\lambda}(p'', p, \theta'') \end{aligned} \quad (3.10)$$

with

$$V_{\lambda'\lambda''}^{\lambda}(p', p'', \theta', \theta'') = \int_0^{2\pi} d\phi'' V_{\lambda'\lambda''}(\vec{p}', \vec{p}'') e^{i(\lambda'\phi' - \lambda''\phi'')} e^{-i\lambda(\phi' - \phi'')}. \quad (3.11)$$

The symmetry behaviour of $T_{\lambda'\lambda}(p', p, \theta')$ shows up as [4]

$$T_{\lambda'\lambda}(p', p, \theta') = (-)^{\lambda' - \lambda} T_{-\lambda', -\lambda}(p', p, \theta'), \quad (3.12)$$

thus, allowing us to solve Eq.(3.10) only for $T_{\frac{1}{2}\frac{1}{2}}(p', p, \theta')$ and $T_{-\frac{1}{2}\frac{1}{2}}(p', p, \theta')$.

Having the T -matrix element $T_{\frac{1}{2}\frac{1}{2}}(p', p, \theta')$ and $T_{-\frac{1}{2}\frac{1}{2}}(p', p, \theta')$, we can calculate the spin-averaged differential cross section as

$$I_0 \equiv \frac{d\sigma}{d\Omega} = (4\pi^2\mu)^2 \left(\left| T_{\frac{1}{2}\frac{1}{2}}(p, p, \theta') \right|^2 + \left| T_{-\frac{1}{2}\frac{1}{2}}(p, p, \theta') \right|^2 \right) \quad (3.13)$$

and the polarization, as one example of the spin observable, as

$$P_y = \frac{2}{I_0} (4\pi^2\mu)^2 \text{Im} \left(T_{\frac{1}{2}\frac{1}{2}}^*(p, p, \theta') T_{-\frac{1}{2}\frac{1}{2}}(p, p, \theta') \right). \quad (3.14)$$

4. Summary

The KN scattering has been formulated by using a 3D technique. The hadrons exchange model has been applied for the KN interaction by considering the second order diagrams only. The cross section and some spin observables will be utilized in determining the parameters of the model. All this is needed for our intention to make a one-hadron-exchange KN interaction in the next future.

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