PROCEEDINGS OF SCIENCE

Walking signals in eight-flavor QCD on the lattice

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We investigate walking signals of $N_f = 8$ QCD through the meson spectrum, using the HISQ action. Our data for $N_f = 8$ QCD are consistent with chiral perturbation theory (ChPT) in their chiral extrapolations, hence with the theory exhibiting spontaneous chiral symmetry breaking. Remarkably, while the $N_f = 8$ data near the chiral limit are well described by the ChPT, those for the relatively large fermion bare mass m_f away from the chiral limit actually exhibit a finite-size hyperscaling relation, suggesting a large anomalous dimension $\gamma_m \sim 1$. This implies that there exists a remnant of the infrared conformality, and suggests that a typical "one-family" technicolor model, as modeled by $N_f = 8$ QCD, can be a walking technicolor theory.

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1. Introduction

One of the candidates for the theory beyond the SM is Walking Technicolor (WTC) [1]. It suggests an approximate scale invariance with a large anomalous dimension $\gamma_m \simeq 1$ due to the "walking" coupling (Fig. 1), which is based on the scale-invariant gauge dynamics [2]. The walking behavior can in fact be realized in "large N_f QCD". $N_f = 8$ is particularly interesting from the model-building point of view: it is none other than so-called "one-family model" (Farhi-Susskind model [3]). Thus if the $N_f = 8$ turns out to be a walking theory, it would be a great message for the phenomenology, which is to be tested by the on-going experiments at LHC. (See Ref. [4] and references therein for the review of the recent lattice studies.)

As in Fig. 1, the fermion bare mass m_f obviously distorts the ideal behavior of the breaking of the scale symmetry. (This is in contrast to the conformal case, e.g. our $N_f = 12$ result [5].) Then, disregarding the effects of the lattice parameters L and a for the moment[1], we may imagine possible effects of m_f on the walking coupling of our target of study. Indeed, in our numerical exploration with $N_f = 8$ [7], while the data near the chiral limit are well described by the ChPT, those for the relatively large m_f away from the chiral limit actually exhibit a finite-size hyperscaling (FSHS) relation, suggesting a large anomalous dimension $\gamma_m \sim 1$. This is the first time that such a relation is observed in a theory with spontaneous chiral symmetry breaking[2] (SχSB).

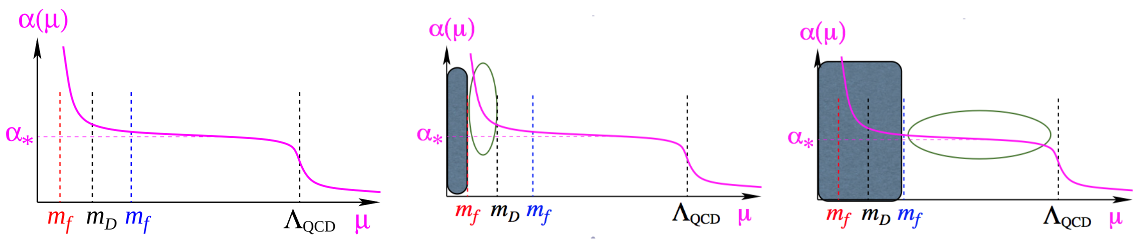

Figure 1: Schematic two-loop/ladder picture of the gauge coupling of the massless large-N_f QCD as a walking gauge theory in the SχSB phase near the conformal window (left). m_D is the dynamical mass of the fermion generated by the SχSB. Case 1: $m_f \ll m_D$ (red) well described by ChPT (center), and Case 2: $m_f \gg m_D$ (blue) well described by the hyperscaling (right). The SχSB order parameter on the lattice is not m_D but would be the chiral limit of F_π, $F = F_\pi(m_f = 0)$, which would be expected roughly as $m_D = \mathscr{O}(F)$.

2. Lattice simulation and results

In our simulation, we use the tree-level Symanzik/HISQ action [7] (without tadpole improvement and the mass correction in the Naik term) for the improvement of the flavor symmetry and the behavior towards the continuum limit. We carry out the simulation by using the standard Hybrid Monte-Carlo (HMC) algorithm and measure the mass of the pion M_π, ρ-meson M_ρ, the decay constant of the pion F_π and the chiral condensate $\langle\bar{\psi}\psi\rangle$ as the basic observables, for various quark

[1] In our simulation we use the parameter region where the effect of the system size is subdominant compared to the mass effect. This strategy is different from the one which is advocated by the authors of Ref. [6].

[2] Ref. [8] found that $N_f = 8$ QCD with domain-wall fermions has SχSB and conformal properties depending on the fermion mass region.

masses and on various lattices, $L^3 \times T$ with fixed aspect ratio $T/L = 4/3$ for $L = 12$, 18, 24, 30 and 36 at $\beta = 3.8$.

We performed the analysis based on ChPT and FSHS. If the simulation region of m_f is in the SχSB region, physical quantities in the spectroscopy, M_H for $H = \pi, \rho, \cdots$ and F_π, are described by the ChPT. In that case, the masses and F_π depend on m_f up to chiral log as $M_\pi^2 = C_1^\pi m_f + C_2^\pi m_f^2 + \cdots$, $F_\pi = F + C_1^F m_f + C_2^F m_f^2 + \cdots$, where F is the value in the chiral limit. On the other hand, if the theory is in the conformal window, M_H and F_π in the infinite volume limit are described by the HS relation $M_H \propto m_f^{1/(1+\gamma_*)}$, and on the finite volume are described by the FSHS $\xi_H = LM_H = \mathscr{F}_H(Lm_f^{1/(1+\gamma_*)})$ for $H = \pi$, ρ or $F(M_H = F_\pi)$. The function, $\mathscr{F}_H$, is a some function (unknown *a priori*) of the scaling variable $X = Lm_f^{1/(1+\gamma_*)}$ in which γ_* denotes the mass anomalous dimension γ_m at the infrared fixed point and its value is universal for all channels.

3. Chiral perturbation theory (ChPT) analysis of F_π, M_ρ and $\langle\bar\psi\psi\rangle$ in small m_f

Based on the scenario explained in Fig. 1, and the blowup behaviors of both F_π/M_π (see Fig. 2) and M_ρ/M_π towards the chiral limit [7, 8], we first attempt the ChPT analysis in the small m_f.

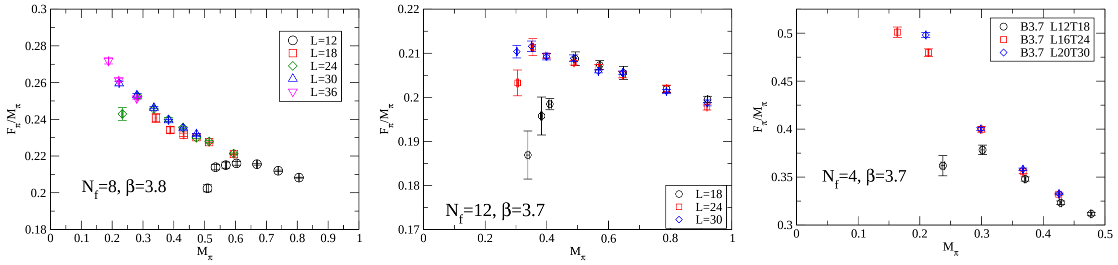

Figure 2: F_π/M_π as a function of M_π for $N_f = 8$ (left), $N_f = 12$ (center), and $N_f = 4$ (right).

Table 1 shows the results of the quadratic fit of F_π. (The fitted data are the result on the largest volume at each m_f region.) Particularly for the small region, $0.015 \le m_f \le 0.04$, the polynomial fit gives a good χ^2/dof (= 0.46). When we include the data at $m_f = 0.05$, χ^2/dof jumps up; this jump might be caused by the instability due to small dof. This suggests that there is a bound beyond which the ChPT does not describe the data well, and that bound is around $m_f \lesssim 0.05$. With this consideration and the good chiral behavior observed for other quantities (for instance M_ρ and $\langle\bar\psi\psi\rangle$) for small m_f region, we choose $m_f = 0.015 - 0.04$ for the fitting range of all quantities (roughly corresponding to Case 1 in Fig. 1). The above analysis suggests that our result in $N_f = 8$ is consistent with the SχSB phase with $F = 0.0310(13)$ up to chiral log corrections. Figure 3 shows the ChPT fit[3] in the range $0.015 \le m_f \le 0.04$ and the HS fit in the range $0.05 \le m_f$. Since the behavior in the small m_f region is well-described by the ChPT, therefore $N_f = 8$ QCD is consistent with SχSB.

We next analyze whether the chiral condensate shows SχSB behavior. We perform a direct measurement $\langle\bar\psi\psi\rangle = \mathrm{Tr}[D_{HISQ}^{-1}(x,x)]/4$, and the quantity $\Sigma \equiv \frac{F_\pi^2 M_\pi^2}{4m_f}$ through the GMOR relation.

[3] The left panel of Fig. 3 also shows the $L = 12$ data of F_π, denoted as "region-A" in which F_π linearly goes to zero towards the chiral limit. We therefore do not include such data in the analysis.

fit range (m_f)	F	$\mathscr{X}(m_f^{\min}=0.015)$	$\mathscr{X}(m_f=m_{\max})$	χ^2/dof	dof
0.015–0.04	0.0310(13)	3.74	11.80	0.46	1
0.015–0.05	0.0278(8)	4.64	19.28	5.56	2
0.015–0.10	0.0311(3)	3.70	37.0	7.85	6

Table 1: Results of the chiral fit of F_π with $F_\pi = F + C_1^F m_f + C_2^F m_f^2$ and the the expansion parameter in ChPT, $\mathscr{X} = N_f \left(\frac{M_\pi}{4\pi F/\sqrt{2}}\right)^2$, for various fit ranges. This is in contrast to the $N_f = 12$ case [5] showing $\mathscr{X} \simeq 40$ at $m_f^{\min}$.

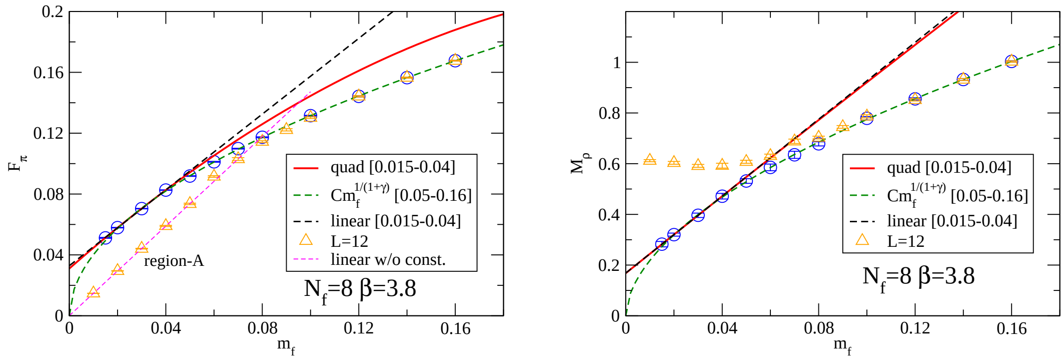

Figure 3: Fitting results of F_π (left) and M_ρ (right). Linear and quadratic fits in $0.015 \le m_f \le 0.04$, Power fit ($y = Cm_f^\alpha$) in $0.05 \le m_f \le 0.16$.

We also estimate the chiral limit, $F^2 \cdot \left(\frac{M_\pi^2}{4m_f}\right)\Big|_{m_f\to 0} = 0.00050(3)$, by multiplying F with the extrapolated value of M_π^2/m_f, as the alternative of the chiral limit. All the results in the chiral limit are non-zero and are consistent with one another.

From the analyses up to chiral log of all the observables, F_π, M_π, M_ρ and $\langle\overline{\psi}\psi\rangle$, the chiral behavior of $N_f = 8$ QCD is consistent with that of SχSB in $0.015 \le m_f \le 0.04$.

4. Study of remnants of conformality, Hyperscaling analysis in the intermediate m_f

We find the two regions of m_f having qualitatively different properties: $0.015 \le m_f \le 0.04$ and $0.05 \le m_f \le 0.16$ by the analysis of ChPT shown in the previous section. Furthermore, based on the scenario of WTC in Fig. 1, if this theory is near the conformal phase boundary, it is expected that some remnants of the conformality appear in physical quantities. It is remarkable that in Fig. 3 the fit results in the mass range, $m_f \gtrsim 0.05$, are consistent with the HS behavior. This suggests that, although $N_f = 8$ QCD is in the SχSB phase, there exists a remnant of the conformality in the m_f region away from the chiral limit. Therefore, we will carry out further in depth analysis, which employs the FSHS test with the mass correction term [9], to investigate whether the remnant of the conformality persists.

4.1 Finite size Hyper-Scaling (FSHS) fits with the correction term

Our data of $N_f = 8$ cannot satisfy the FSHS with universal γ in the whole range of m_f [7], because we showed that the theory is in the SχSB phase[4]. However, because of the power behavior in the middle range of the fermion mass as mentioned in Fig. 3, we carry out the FSHS test with the mass correction term [9] in our data to look for a remnant of the conformality.

Although it might be possible to obtain a common value of the γ, FSHS is only expected for larger mass region, where mass corrections may not be negligible [9], $\xi_H = C_0^H + C_1^H X + C_2^H L m_f^{\alpha}$ for $m_f \geq 0.05$ and $\xi_\pi \geq 8$. We perform a simultaneous fit with mass correction term [7] in this region[5] of m_f and ξ_π using M_π, F_π, and M_ρ with a common γ. The example with $\alpha = 1$ of this analysis is shown in Fig. 4, as a typical result of the simultaneous fit. Under the assumption that all the observables give a universal γ, we estimate $\gamma = 0.78$–0.93 with $\chi^2/\text{dof} = O(0.1)$. These estimated values of γ would be identified as the mass anomalous dimension in the walking regime.

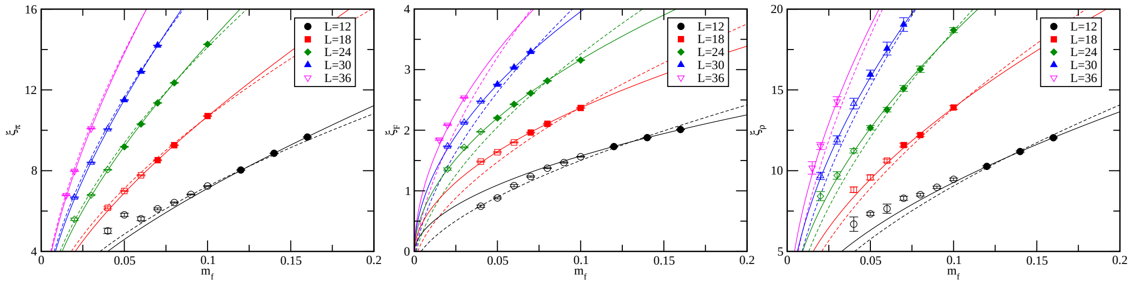

Figure 4: Simultaneous FSHS fit in ξ_π(left), ξ_F(center) and ξ_ρ(right) with $\alpha = 1$. The filled symbols are included in the fit, but the open symbols are omitted. The fitted region is $m_f \geq 0.05$ and $\xi_\pi \geq 8$. The solid curve is the fit result. For a comparison, the simultaneous fit result without correction terms is also plotted by the dashed curve, whose $\chi^2/\text{dof} = 83$.

5. Summary and Discussion

In search of a candidate for WTC theory, we have investigated the meson spectrum of $N_f = 8$ QCD using lattice simulations based on the HISQ action for $\beta = 6/g^2 = 3.8$ [7]. We have found that the data of F_π, M_π, M_ρ and $\langle\overline{\psi}\psi\rangle$ are well described by the ChPT in the small m_f region, $0.015 \leq m_f \leq 0.04$ (Case 1 in Fig. 1), suggesting that $F = 0.031(1)(^{+2}_{-10})$, $\langle\overline{\psi}\psi\rangle|_{m_f\to 0} = 0.00052(5)(^{+8}_{-29})$, and $\frac{M_\rho}{F/\sqrt{2}} = 7.7(1.5)(^{+3.8}_{-0.4})$ in the chiral limit extrapolation. In contrast, the data for the relatively large fermion bare mass $m_f \geq 0.05$, away from the chiral limit, actually exhibited a naive FSHS with the non-universal γ. This implies that there exists a remnant of the IR conformality where the SχSB effects are negligible (Case 2 in Fig. 1). Therefore, there could exist large mass corrections on the FSHS and we obtained $0.78 \lesssim \gamma \lesssim 0.93$ in simultaneous FSHS fits with mass correction for $m_f \geq 0.05$ and $\xi_\pi \geq 8$. Summarizing all our analyses we may infer that a typical technicolor

[4]The exponents for F_π, M_ρ and M_π obtained from naive FSHS relation are $\gamma = 0.928(8)$, $0.798(20)$ and $0.567(3)$ respectively, being non-universal value. This non-universal value of γ is also shown in Ref. [8].

[5]It is noted that a simultaneous fit including the lighter mass with $m_f \geq 0.015$ in $\xi_\pi \geq 6.8$ fails with a large $\chi^2/\text{dof} = 3.5$ even if the mass correction is included. This is because the chiral properties are dictated by SχSB and should not be consistent with HS near the chiral limit.

("one-family model") as modeled by $N_f = 8$ QCD can be a WTC theory having an approximate scale invariance with large anomalous dimension $\gamma_m \sim 1$.

Finally, we should comment on the possible light flavor-singlet scalar meson in $N_f = 8$ QCD. The WTC predicts a light composite Higgs-like scalar boson (techni-dilaton) as a pseudo Nambu-Goldstone boson of the approximate scale invariance. We studied [10] both the flavor-singlet scalar and scalar glueballs in $N_f = 12$ for a hint of a scalar bound state; we then studied the flavor-singlet scalar in $N_f = 8$ and found it as light as π. The result suggests that a light flavor-singlet scalar composite does exist [11] in WTC theories.

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