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Upper limit on the amplitude of gravitational waves around 0.1Hz from the Global Positioning System

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We show upper bound on the amplitude of gravitational waves around 0.1 Hz from the global posioning system (GPS). Since the atomic clocks which these satellites have are stabilized with the accuracy $\Delta\nu/\nu \simeq 10^{-15}$, the GPS can provide the precise navigations and enable one to calibrate the primary standard for frequency on the ground. Although the attitude of these satellites is approximetaly twenty thousand kilometers, one can stabilize one's oscillator with the accuracy $\Delta\nu/\nu \simeq 10^{-12}$ with phase-locked loop circuits by receiving the radio wave emitted by GPS satellites for one or one hundred seconds. To take advantage of the Doppler tracking method, we have found that this it has already placed a meaningful constraint on the strain amplitude of continuous component of GWs as $h_c < 4.8 \times 10^{-12}$ at frequency range $10^{-2} \lesssim f \lesssim 10^{0}$ Hz. This work is documented in arXiv 1402.4521[1].

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1. Introduction

The existence of gravitational waves is predicted in general relativity and a lot of modified gravity theories. Gravitational waves (GWs) are expected to be emitted from the end of the life of massive stars, merger of neutron stars and black holes and the beginning of the Universe. While the beginning of the Universe, the existence of cosmological inflation is predicted in order to solve the horizon problem, flatness problem and the monopole problem (*see*. [2, 3, 4]). Especially many theories of the cosmological inflation predict that the spectrum of GWs originated from it is almost scale-invariant. In order to prove the existence of the cosmological inflation, we have to observe GWs with a wide frequency range and high sensitivity. Moreover, because GWs propagate freely since their generation, the thermal history of the Universe and the properties of inflaton field are imprinted on the power spectrum of gravitational wave background (GWB) ([5, 6]). In addition, the intermediate massive black hole (IMBH) binaries with mass $M = 10^5 M_\odot$ are expected emits GWs at the frequency range $10^{-4} \lesssim f \lesssim 10^0$ [Hz] when they collide. By detecting GWs from them, the number density of IMBH binaries can be revealed, and hence, the mysteries how super massive black holes (SMBHs), which exists at the center of almost all galaxies, form in the history of the Universe are expected to be solved.

When the strain amplitude of GWs h_c is large, radio waves which propagate over long distance are distorted by gravitational waves. Some previous studies of setting constraint on the strain amplitude of GWB use this property of GWs with ULYSSES and Cassini [7, 8]. However these planetary explorers fly so far from the Earth that the electrical noise precludes to set constraints on h_c at the frequency range $f \gtrsim 10^{-2}$ [Hz]. At this frequency range, torsion bar detector such as TOBA set a constraint on the strain amplitude as $h_\mathrm{c} < 2 \times 10^{-9}$ [9]. However, at this frequency range, the seismic oscillation of the ground interrupts the detection or setting on the constraint on h_c [1].

In this work, we set on the constraint on the strain amplitude of GWs with the global positioning system (GPS). The GPS is composed of thirty one satellites and provides detailed navigations and cruises. In order to maintain the accuracy of them, all GPS satellites have their own atomic clocks with 10^{-15} accuracy and emit precious, stable and high intensity radio waves synchronized with their atomic clocks for accurate navigations. It enables one to calibrate one's oscillator with 10^{-12} accuracy by receiving these radio waves for one or one hundred seconds. These oscillator is called GPS disciplined oscillators (GPSDOs) and use phase-locked loop circuits. This short time stability of the frequency of GPSDOs enables one to set a constraint on h_c at this frequency range. One of advantages of this method is that one can receive the radio waves from GPS satellites everytime and everywhere on the ground.

Throughout this proceedings, we adopt TT gauge to describe GWs. f and ν mean the frequency of GWs and electro-magnetic waves, respectively. A dot represents a partial derivative with respect to the physical time, i.e., $\dot{x} \equiv \dfrac{\partial x}{\partial t}$.

[1]Dyson showed that GWB at this frequency resonates with the Earth and the strain amplitude of GWB can be constrained by measuring the distortion of the Earth surface in 1969 [10]. Recently, Coughlin & Harms had measured the seismic oscillations at twenty points on the Earth for one year and revised the constraint as $h_\mathrm{c} < 4.1 \times 10^{-14}$ at 0.1 Hz[11].

2. Method

We assume that GWBs are expected to be isotropic and stationary (see [12]) and its wavelength are longer than the typical distance between GPS satellites and GPSDOs (detectors) on the ground. In this case, the frequency of radio waves emitted from the satellites is modulated by GWs as

$$\frac{\Delta\nu}{\nu} = \frac{l}{2c}\dot{h}(t)\sin^2\theta \tag{2.1}$$

where $\dot{h}(t)$, θ, l and c are the time derivative of the amplitude of GWs at time t, the angle between the directions of propagation of radio waves and gravitational waves, the distance between the GPS satellite and the observer and the speed of light, respectively. In the case that $\theta = \pi/2$, the characteristic magnitude of $\dot{h}(t)\sin^2\theta$ can be written with h_c as $2\pi f h_\mathrm{c}$. Therefore the effect of GWs on propagating electromagnetic waves can be written as the following,

$$\frac{\Delta\nu}{\nu} = \frac{\pi l f}{c}h_\mathrm{c} \tag{2.2}$$

In reality, the frequency modulation sourced by GWs is buried within the noise. Therefore, if one can receive the radio waves which is emitted at a distance of l with a time variance of the frequency fluctuations σ [13], one can set the upper bound of h_c as

$$h_\mathrm{c} < \frac{c}{\pi l f}\sigma \tag{2.3}$$

For the GPS, the distance from GPS satellites from observers l is approximately 2×10^7 [m] and the standard variation of frequency from GPS satellites converges to $\sigma \simeq 1\times10^{-12}$ by receiving the signal from GPS satellites for the period from one second to one hundred seconds [14]. By substituting them into Eqs. (2.2) and (2.4), we obtain the constraint on the strain amplitude of h_c as

$$h_\mathrm{c} < 4.8\times10^{-12}\left(\frac{1\mathrm{Hz}}{f}\right). \tag{2.4}$$

The probed range of this method is determined by the time of integrating signal of the radio waves from GPS satellite t_i and the time of flight of the radio wave from the GPS satellite and observer $t_\mathrm{f} = l/c$. It can be written as

$$t_\mathrm{i}^{-1} < f < t_\mathrm{f}^{-1}\,. \tag{2.5}$$

In Fig. 1, we plot the constraint on the strain amplitude of the continuous component of GWs and compare the result from the torsion bar detector [9].

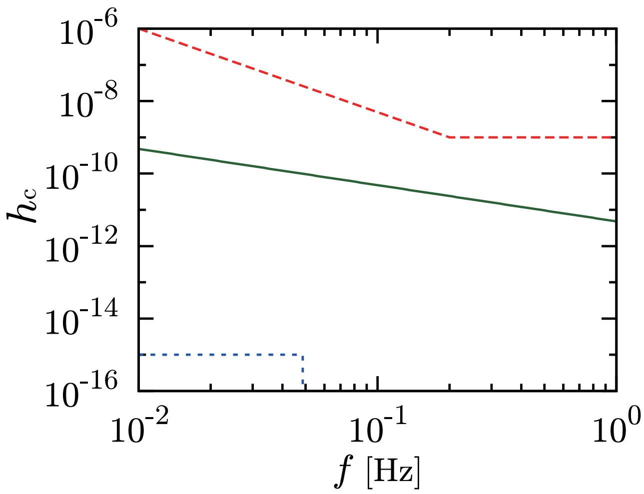

Figure 1: The upper limit on the strain amplitude of GWBs from GPS satellites (solid line). The dashed line represents the constraint from the torsion bar detector [9]. The dotted line represents the constraint from the Doppler tracking method[7].

The accuracy of frequency fluctuations of received radio waves from GPS satellites determined mostly on the time resolution of the received electric signal in the A/D converter of the GPS receiver σ_r. It is reported that $\sigma_r \gtrsim 2 \times 10^{-13}/t_i$, and the largest error comes from the digitization of received radio waves [15]. It is difficult to improve the GPS constraints on the strain amplitude until the resolution of the quantization in the A/D converter is much improved.

The intensity of GWBs can be characterized by the dimensionless cosmological density parameter $\Omega_{gw}(f)$. The parameter is defined as

$$\Omega_{gw}(f) \equiv \frac{10\pi^2}{3H_0^2}(fh_c)^2 , \tag{2.6}$$

where H_0 is the Hubble constant. The Planck collaborations reported it by measuring the temperature fluctuations of cosmic microwave background (CMB) as

$$H_0 = 67.11\text{km}/\text{sec}/\text{Mpc} = 2.208 \times 10^{-18}\text{sec}^{-1} . \tag{2.7}$$

By substituting Eqs. (2.4) and (2.7) into (2.6), we obtain

$$\Omega_{gw}(f) < 1.7 \times 10^{14} \text{ for } 10^{-2} \lesssim f \lesssim 10^{0} \text{ [Hz]} . \tag{2.8}$$

In Fig 2, we compare our constraint with the previous studies.

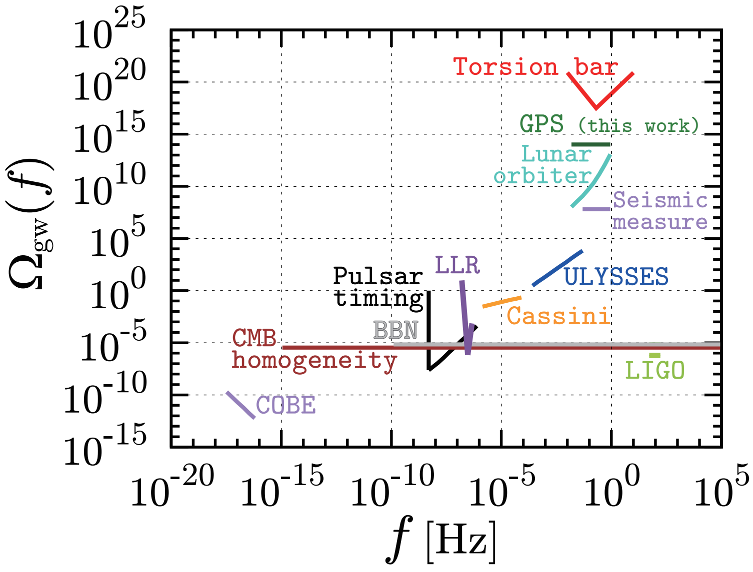

Figure 2: Summary of the constraints on GW background in terms of $\Omega_{gw}(f)$, which includes `COBE` [12], `CMB homogeneity` and `BBN` [16], `Pulsar timing` [17], `LLR` [18, 19], `Cassini` [8], `ULYSSES` [7], `Lunar orbiter` [20], `Seismic measure` [11] `Torsion bar` [9], `LIGO` [21, 22], and `GPS`.

3. Conclusion

We set a constraint on the strain amplitude of the continuous component of GWs as $h_c < 4.8 \times 10^{-12}(1\,\mathrm{Hz}/f)$ at the frequency range $10^{-2} \lesssim f \lesssim 1$ [Hz] with the radio waves emitted by GPS satellites in operation. The sensitivity to the GWs is limited to time resolutions of the digitization of the A/D converter on the GPS receiver and it is difficult to improve this constraint by this methods.

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