

## Nucleon decay via dimension 6 operators in anomalous $U(1)_A$ SUSY GUT models and $E_6 \times SU(2)_F$ SUSY GUT models

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We study nucleon decay via dimension 6 operators for various decay modes in the anomalous  $U(1)_A$  SUSY GUT scenarios, in which the unification scale  $\Lambda_u$  becomes smaller than the usual SUSY GUT scale  $\Lambda_G = 2 \times 10^{16}$  GeV in general. Since the predicted lifetime  $\tau(p \rightarrow \pi^0 + e^c)$  falls around the experimental lower bound, the discovery of the nucleon decay in future can be expected. We show that the two ratios  $R_1 = \frac{\Gamma_{n \rightarrow \pi^0 + \nu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$  and  $R_2 = \frac{\Gamma_{p \rightarrow K^0 + \mu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$  are important in identifying grand unification group,  $SU(5)$ ,  $SO(10)$ , and  $E_6$ . In these calculation we consider uncertainties of the unitary matrices diagonalizing Yukawa matrices.

In addition, we calculate the nucleon decay in anomalous  $U(1)_A E_6 \times SU(2)_F$  SUSY GUT model. In this model,  $SU(2)_F$  symmetry restricts Yukawa structure at GUT scale. As a result the diagonalizing matrices are restricted. Our calculation shows that in  $E_6$  models which have flavor symmetry  $SU(2)_F$  smaller  $R_1$  and  $R_2$  are preferable than these in  $E_6$  models which do not have flavor symmetry  $SU(2)_F$ .

This talk is based on Ref. [1, 2]

*KMI International Symposium 2013 on "Quest for the Origin of Particles and the Universe",  
11-13 December, 2013  
Nagoya University, Japan*

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## 1. Introduction

Supersymmetric grand unified theory (SUSY GUT) is one of the most attractive candidates for the theory beyond the standard model (SM). SUSY GUT realizes two unifications and each unification are supported by experiments. First, in SUSY GUT the SM gauge groups are unified into a grand unification group. In this work we treat  $SU(5)$ ,  $SO(10)$ , and  $E_6$  groups as the grand unification group. This unification is supported by the fact that three running gauge couplings meet at a scale  $\Lambda_{SUSYGUT} \sim 2 \times 10^{16}$  GeV in minimal supersymmetric SM (MSSM). Second the SUSY GUT realizes the unification of quarks and leptons. In  $SU(5)$  GUT model, one generation of SM quarks and leptons are unified into  $\bar{\mathbf{5}}$  and  $\mathbf{10}$  of  $SU(5)$ . If we assume that the  $\mathbf{10}$  quark and lepton induces stronger hierarchies for Yukawa couplings than the  $\bar{\mathbf{5}}$  quark and lepton, we can understand the various hierarchies of masses and mixings qualitatively. In minimal  $SO(10)$  GUT model in which one generation of quarks and leptons can be unified into  $\mathbf{16}$  multiplet and the MSSM Higgs are included in  $\mathbf{10}$  multiplet, it is hard to realize the above assumption. In this work to realize the above assumption we introduce new  $\mathbf{10}$  of  $SO(10)$  as a quark and lepton field [3] and we think the case in which  $\bar{\mathbf{5}}$  of  $SU(5)$  in  $\mathbf{10}$  of  $SO(10)$  become main mode of the second generation of  $\bar{\mathbf{5}}$ . In  $E_6$  GUT model, fundamental representation  $\mathbf{27}$  includes both  $\mathbf{16}$  and  $\mathbf{10}$  [4], and therefore, we can realize the above assumption naturally in this  $E_6$  model. Main mode of second generation  $\bar{\mathbf{5}}$  comes from  $\mathbf{10}_1$  of  $SO(10)$  which belongs to  $\mathbf{27}_1$  of  $E_6$ .

Nucleon decay is one of the most favorable candidates to identify GUT models. In SUSY GUT models, the nucleon decay is induced by dimension 6 and dimension 5 operators. Exchanges of X-type gauge boson, which are fundamental representation under  $SU(3)_C$  and  $SU(2)_L$  induce the nucleon decay via dimension 6 operators. If X-type gauge boson mass is around SUSY GUT scale, the nucleon lifetime becomes about  $10^{36}$  years. It is 100 times as large as current experimental lower bound  $\tau(p \rightarrow \pi^0 + e^c) \geq 1.3 \times 10^{34}$  [5]. Exchanges of triplet (colored) Higgs field induce the nucleon decay via dimension 5 operators. It is assumed that the triplet Higgs mass is around SUSY GUT scale, the nucleon lifetime becomes smaller than experimental lower bound. As a result in SUSY GUT models usually the nucleon decay via dimension 5 operators is significant and dangerous. To satisfy experimental lower bound, the triplet Higgs mass must be much larger than the SUSY GUT scale, while the GUT partner of triplet Higgs, must have mass of  $O(100)$  GeV. How to realize this mass splitting naturally? This problem is called the doublet-triplet (DT) splitting problem.

In anomalous  $U(1)_A$  SUSY GUT model [3], the DT splitting is realized under natural assumptions. One of the most interesting predictions is the nucleon decay via dimension 6 operators dominates over the nucleon decay via dimension 5 operators. Therefore, we focus on the nucleon decay via dimension 6 operators in this work.

## 2. Nucleon decay via dimension 6 operators

In this section, we explain how to obtain dimension 6 operators which induce nucleon decay. First, I explain the quark and lepton sector. In  $E_6$  GUT models, three  $\mathbf{27}$  multiplets include all quarks and leptons. The fundamental  $\mathbf{27}$  is divided into several multiplets of  $SO(10)$  as

$$\mathbf{27} \rightarrow \mathbf{16} + \mathbf{10} + \mathbf{1}. \quad (2.1)$$

The spinor  $\mathbf{16}$  and the vector  $\mathbf{10}$  contain the SM multiplets as

$$\mathbf{16} \rightarrow \underbrace{q_L(\mathbf{3}, \mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1}, \mathbf{1})_1}_{\mathbf{10}} + \underbrace{d_R^c(\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}} + \underbrace{v_R^c(\mathbf{1}, \mathbf{1})_0}_{\mathbf{1}}, \quad (2.2)$$

$$\mathbf{10} \rightarrow \underbrace{D_R^c(\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} + L_L(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}} + \underbrace{\bar{D}_R^c(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}} + \bar{L}_L(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}}_{\mathbf{5}}, \quad (2.3)$$

where the numbers denote the representations under the SM gauge group  $SU(3)_C \times SU(2)_L \times U(1)_Y$ . In this paper SM multiplets which are represented as capital letter denote SM multiplets from the  $\mathbf{10}$  of  $SO(10)$  to distinguish them from the SM multiplets from the  $\mathbf{16}$  of  $SO(10)$ . Next we explain the gauge particle sector, especially the X-type gauge boson. In  $SU(5)$  GUT models, adjoint  $\mathbf{24}$  includes X-type gauge boson  $X(\bar{\mathbf{3}}, \mathbf{2})_{\frac{5}{6}}$ . In  $SO(10)$  GUT models,  $X'(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$  also induces nucleon decay. And in  $E_6$  GUT models, one more X-type gauge boson  $X''(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$  is added. Last we explain the Higgs sector. GUT Higgs VEVs induce GUT particle masses, especially X-type gauge boson masses. And these masses break the grand unification group to the SM gauge group. In  $SU(5)$  GUT model, we introduce adjoint Higgs  $A$  which obtains VEV in the SM gauge group singlet direction. In  $SO(10)$  GUT model, we add spinor Higgs  $C$  and  $\bar{C}$  which obtain VEVs  $\langle C \rangle = \langle \bar{C} \rangle = v_c$  in  $SU(5)$  singlet direction. In  $E_6$  GUT model, we add fundamental and anti fundamental Higgs  $\Phi$  and  $\bar{\Phi}$  which obtain VEVs  $\langle \Phi \rangle = \langle \bar{\Phi} \rangle = v_\phi$  in  $SO(10)$  singlet direction. In  $SO(10)$  and  $E_6$  GUT model, the adjoint Higgs has the Dimopoulos-Wilczek type VEV,

$$\langle \mathbf{45}_A \rangle = i\sigma_2 \times \begin{pmatrix} x & & & \\ & x & & \\ & & x & \\ & & & 0 \\ & & & & 0 \end{pmatrix}, \quad (2.4)$$

to realize the DT splitting [6]. Here  $\mathbf{45}_A$  is the  $\mathbf{45}$  component field of the  $E_6$  adjoint Higgs  $A$  in  $SO(10)$  decomposition, and  $\sigma_i$  ( $i = 1, 2, 3$ ) is the Pauli matrix.

We calculate the dimension-6 operators which induce the nucleon decay (here  $i, j$  is flavor index) as

$$\begin{aligned} \mathcal{L}_{eff} = & \frac{g_{GUT}^2}{M_X^2} \{ (\bar{e}_{Ri}^c u_{Rj}) (\bar{u}_{Lj}^c d_{Li}) + (\bar{e}_{Ri}^c u_{Rj}) (\bar{u}_{Li}^c d_{Lj}) + (\bar{e}_{Li}^c u_{Lj}) (\bar{u}_{Rj}^c d_{Ri}) + (\bar{E}_{Li}^c u_{Lj}) (\bar{u}_{Rj}^c D_{Ri}) \\ & - (\bar{v}_{Li}^c d_{Lj}) (\bar{u}_{Rj}^c d_{Ri}) - (\bar{N}_{Li}^c d_{Lj}) (\bar{u}_{Rj}^c D_{Ri}) \} \\ & + \frac{g_{GUT}^2}{M_{X'}^2} \{ (\bar{e}_{Li}^c u_{Lj}) (\bar{u}_{Ri}^c d_{Rj}) - (\bar{v}_{Li}^c d_{Lj}) (\bar{u}_{Ri}^c d_{Rj}) \} \\ & + \frac{g_{GUT}^2}{M_{X''}^2} \{ (\bar{E}_{Li}^c u_{Lj}) (\bar{u}_{Ri}^c D_{Rj}) - (\bar{N}_{Li}^c d_{Lj}) (\bar{u}_{Ri}^c D_{Rj}) \}. \end{aligned} \quad (2.5)$$

The VEVs which we introduced above induce the X-type gauge boson masses as

$$M_X^2 = g_{GUT}^2 x^2, \quad M_{X'}^2 = g_{GUT}^2 (x^2 + v_c^2), \quad M_{X''}^2 = g_{GUT}^2 \left( \frac{1}{4} x^2 + v_\phi^2 \right). \quad (2.6)$$

In this work VEVs are  $x = 1 \times 10^{16}$  GeV,  $v_c = 5 \times 10^{14}$  GeV, and  $v_\phi = 5 \times 10^{15}$  GeV. This is one of typical VEVs in anomalous  $U(1)_A$  SUSY GUT models.  $x > v_c$  is always satisfied in the anomalous  $U(1)_A$  SUSY GUT to realize DT splitting. As a result, a relation between the X-type gauge boson masses  $M_{X'}^2 \sim M_X^2$  is satisfied. Therefore, the effect of  $X'$  becomes almost maximal in anomalous  $U(1)_A$  SUSY GUT.

Next, we explain the diagonalizing matrices which make Yukawa matrices diagonal and translate flavor eigenstate to mass eigenstate. The diagonalizing matrices are important for the calculation of nucleon decay. In this work, to realize realistic quark and lepton masses and mixings we assume that the  $\mathbf{10}$  of  $SU(5)$  quark and lepton induces stronger hierarchies for Yukawa couplings than the  $\bar{\mathbf{5}}$  quark and lepton. As a result, the diagonalizing matrix for  $\mathbf{10}$  of  $SU(5)$  quark and lepton is CKM-type matrix  $U_{CKM-type}$  and the diagonalizing matrix for  $\bar{\mathbf{5}}$  quark and lepton is MNS-type matrix  $U_{MNS-type}$ , which are obtained as

$$U_{CKM-type} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad U_{MNS-type} = \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}. \quad (2.7)$$

Each component has the  $O(1)$  coefficient  $C$ , and we take  $0.5 \leq |C| \leq 2$ .

In next section we show the result of my work. But to calculate nucleon decay width we have to consider other effects. In Ref. [1, 2], we show the detail to calculate nucleon decay width.

### 3. Result

In Ref. [1], we calculate lifetime of each decay mode in anomalous  $U(1)_A$  SUSY GUT model with  $SU(5)$ ,  $SO(10)$ , and  $E_6$  grand unification group. As a result of this calculation, the predicted lifetime of  $p \rightarrow \pi^0 + e^c$  is predicted as  $(2-8) \times 10^{34}$  years, which is not far from the experimental lower bound  $\tau(p \rightarrow \pi^0 + e^c) \geq 1.3 \times 10^{34}$  years, therefore we can expect discovery of the proton decay in future. To identify GUT model we use the  $p \rightarrow \pi^0 + e^c$ ,  $p \rightarrow K^0 + \mu^c$ , and  $n \rightarrow \pi^0 + \nu^c$  mode, because these decay mode have small dependence on the  $O(1)$  coefficients in the diagonalizing matrices.

We use  $R_1 = \frac{\Gamma_{n \rightarrow \pi^0 + \nu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$  and  $R_2 = \frac{\Gamma_{p \rightarrow K^0 + \mu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$ .  $R_1$  is very useful to identify grand unification group. From eq. (2.5) the number of operators which induce nucleon to anti electron decay mode is larger than that of operators which induce nucleon to anti neutrino in  $SU(5)$  GUT model. On the other hand, in the added operators of  $SO(10)$  and  $E_6$  GUT model these numbers are the same. Therefore  $R_1$  tend to be lager as grand unification group become large. As a result of the calculation, we show that three anomalous  $U(1)_A$  SUSY GUT models, with  $SU(5)$ ,  $SO(10)$  and  $E_6$  grand unification group can be identified by measuring the two ratios. Especially if  $R_1 > 1$ , the grand unification group is implied to be  $E_6$ .

In Ref. [2], we calculate  $R_1$  and  $R_2$  in anomalous  $U(1)_A E_6 \times SU(2)_F$  SUSY GUT model. In this model  $SU(2)_F$  symmetry restricts Yukawa matrix at GUT scale so the predictions are expected to be more restricted. In the  $E_6$  models without  $SU(2)_F$ , we have 27 real parameters in Yukawa matrix for up-type quark, down-type quark, and charged lepton. In this model, we have 9 real parameters (+ 2 CP phases) in Yukawa matrix at GUT scale. Therefore we can expect that there are

18 conditions for quark and lepton masses and mixing at GUT scale. But most of these conditions have scale dependence. In this work we use 9 conditions which do not have scale dependence to calculate  $R_1$  and  $R_2$ . As a result of calculation we show that the predicted region for  $R_1$  and  $R_2$  is more restricted than in the  $E_6$  model without  $SU(2)_F$  as expected.

## References

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