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Pion condensation phase at finite isospin chemical potential in a Holographic QCD model

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We report main results of our recent work for the study of the pion condensation for non-zero isospin chemical potential within a holographic QCD model. We confirmed that the second order phase transition with the mean field exponent occurs when the isospin chemical potential μ_I exceed the pion mass, which is consistent with the result obtained by the chiral effective Lagrangian at $O(p^2)$. Our result shows that the chiral condensate defined by $\tilde{\sigma} \equiv \sqrt{\langle\sigma\rangle^2 + \langle\pi^a\rangle^2}$ is almost constant in the small μ_I region, while it grows with μ_I in the large μ_I region showing an enhancement of the chiral symmetry breaking.

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1. Introduction

In this contribution, we report main results of the analysis done in Ref. [1], where we studied the pion condensation phase at finite chemical potential using a holographic QCD model [2].

Studying QCD at finite isospin chemical potential will give a clue to understand the symmetry energy which is important to describe the equation of state inside neutron stars. In addition, it may give some informations on the chiral symmetry structure of QCD. When we turn on the isospin chemical potential μ_I at zero baryon number density, the pion condensation is expected to occur at a critical point. Although there are so many works on the pion condensation at finite isospin chemical potential, not many works are for studying the strength of the chiral symmetry breaking.

In Ref. [1], we studied the pion condensation phase in a holographic QCD model [2] by introducing the mean fields for π, σ and the time component of ρ meson. Our results show that the phase transition is of the second order consistently with the one obtained in the $O(p^2)$ chiral Lagrangian [3]. It is remarkable that the chiral condensate defined by $\tilde{\sigma} \equiv \sqrt{\langle\sigma\rangle^2 + \langle\pi^a\rangle^2}$ is almost constant in the small μ_I region, while it grows with μ_I in the large μ_I region. This implies that the chiral symmetry breaking is enhanced by the existence of the isospin chemical potential.

2. Model

In Ref. [1], we used the following action of the hard-wall holographic QCD model given in Ref. [2]:

$$S_5 = \int d^4x \int_{\varepsilon}^{z_m} dz \sqrt{g} \mathrm{Tr}\left[|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right] + \mathscr{L}_5^{BD} , \tag{2.1}$$

where the metric is $ds^2 = a^2(z)\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$ with $a(z) = 1/z$ and the fifth direction z has the UV and the IR cutoffs, ε and z_m. This model consists of a scalar field X and gauge fields corresponding to the chiral symmetry $\mathrm{U(2)_L \times U(2)_R}$. The action contains the IR boundary term $\mathscr{L}_5^{BD}$ given by [4]

$$\mathscr{L}_5^{BD} = -\sqrt{g}\mathrm{Tr}\left\{\lambda z_m |X|^4 - m^2 z_m |X|^2\right\} \delta(z - z_m) , \tag{2.2}$$

where z_m in the coefficients of the $|X|^4$ term and the $|X|^2$ term are introduced in such a way that λ and m^2 carry no dimension. In the following analysis we adopt the $L_5 = R_5 = 0$ gauge, and use the IR boundary condition $F^L_{5\mu}|_{z_m} = F^R_{5\mu}|_{z_m} = 0$.

In the vacuum the chiral symmetry is spontaneously broken down to $\mathrm{U}(2)_V$ by the vacuum expectation value of X. This is determined by solving equation of motion and the solution of the X includes two parameters m_q and σ, where m_q corresponds to the current quark mass and σ to the quark condensate [2]. They are related with λ and m^2 by the IR boundary condition of the X. A parameter g_5^2 is determined by matching with QCD as $g_5^2 = \frac{12\pi^2}{N_c}$. The pion is described as a linear combination of the lowest eigenstate of π^a and the longitudinal mode of A^a_μ, and the ρ meson is the lowest eigenstate of V_μ. The values of the m_q and z_m together with that of the relation between these parameters λ and m^2 are fixed by fitting them to the pion mass $m_\pi = 139.6$ MeV, the ρ meson mass $m_\rho = 775.8$ MeV and the pion decay constant $f_\pi = 92.4$ MeV: $m_q = 2.29\,\mathrm{MeV}$, $z_m = 1/(323\,\mathrm{MeV})$. We use the a_0 meson mass $m_{a_0} = 980$ MeV as a reference value, which fixes $m^2 = 5.39$ and $\lambda = 4.4$, and see the dependence of our results on the scalar meson mass.

3. Pion condensation phase

In this section we study the pion condensation for finite isospin chemical potential μ_I in the holographic QCD model introduced in section 2.

The isospin chemical potential μ_I is introduced as a UV-boundary value of the time component of the gauge field of $\mathrm{SU(2)_V}$ symmetry as $V_0^3(z)|_\varepsilon = \mu_I$, where the superscript 3 indicates the third component of the isospin corresponding to the neutral ρ meson. Here we study the pion condensation phase for small μ_I, assuming that the rotational symmetry O(3) is not broken by e.g. the ρ meson condensation:$L_i = R_i = 0$. We also assume the time-independent condensate, then the vacuum structure is determined by studying the mean fields of five-dimensional fields which do not depend on the four-dimensional coordinate. Furthermore, we also take the mean fields for the neutral pion, the iso-triplet scalar meson (a_0 meson) and the iso-singlet pseudoscalar meson (η meson) to be zero in the pion condensation phase .

We show the resultant relation between μ_I and the isospin density in Fig. 1 for $\lambda = 1$, 4.4 and 100 corresponding to $m_{a_0} = 610$MeV, 980MeV and 1210MeV. This shows that the phase transition

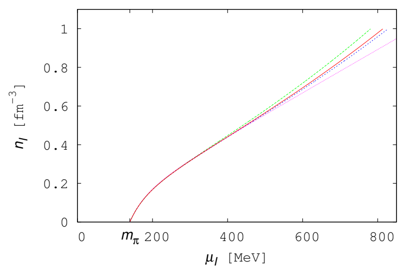

Figure 1: Relation between the isospin number density n_I and the isospin number chemical potential μ_I. The green, red and blue curves show our results for $\lambda = 1$, 4.4 and 100, respectively. The pink dashed-curve shows the result given by the chiral Lagrangian in Ref. [3]. Each choice of λ corresponds to $m_{a_0} = 610$MeV, 980MeV and 1210MeV, respectively.

is of the second order and the critical chemical potential is equal to the pion mass. Our result on the relation between isospin number density and isospin chemical potential for small μ_I agrees with the following one obtained by $\mathrm{O}(p^2)$ chiral Lagrangian [3]: $n_I = f_\pi^2 \mu_I \left(1 - \frac{m_\pi^4}{\mu_I^4}\right)$. For $\mu_I > 500$ MeV, there is a difference between our predictions and the one from $\mathrm{O}(p^2)$ chiral Lagrangian, which can be understood as the higher order contribution as we showed in Ref. [1].

We show the μ_I dependences of the "σ"-condensate denoted as $\langle\sigma\rangle$ and the π-condensate denoted as $\langle\pi^1\rangle$ in Fig. 2. This shows that the "σ"-condensate decreases rapidly after the phase transition where the π-condensate grows rapidly. The "σ"-condensate becomes very small for $\mu_I \gtrsim 400$ MeV, while the π-condensate keeps increasing. Using the form $\langle\pi^a\rangle \propto (\mu_I - \mu_I^c)^\nu$ near the phase transition point, we fit the critical exponent ν to obtain the mean field exponent $\nu = \frac{1}{2}$.

We also show the "chiral circle" in Fig. 3. It is remarkable that the value of the "chiral condensate" defined by $\langle\tilde{\sigma}\rangle = \sqrt{\langle\sigma\rangle^2 + \langle\pi^a\rangle^2}$ is constant for increasing isospin chemical potential μ_I for $\mu_I \lesssim 300$ MeV, and that it grows rapidly in the large μ_I region.

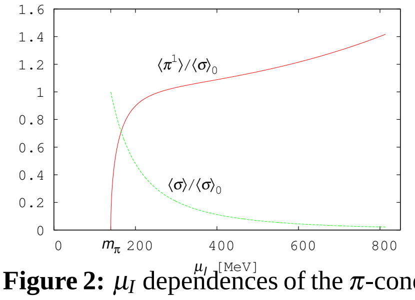

Figure 2: μ_I dependences of the π-condensate (red curve) and the "σ"-condensate (green curve).

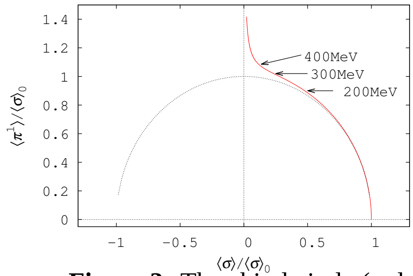

Figure 3: The chiral circle (red curve).

4. A summary and discussions

We studied the phase transition to the pion condensation phase for finite isospin chemical potential μ_I using the holographic QCD model given in Refs. [2], by introducing μ_I as a UV-boundary value of the time component of the gauge field of SU(2)$_V$ symmetry as $V_0^3(z)|_\varepsilon = \mu_I$. We assumed non-existence of vector meson condensates since we are interested in studying the small μ_I region. Furthermore, we assumed that the neutral pion does not condense. We solved the coupled equations of motion for the π- and "σ"-condensates together with V_0^3 to determine μ_I.

Our result shows that the phase transition is of the second order and the critical chemical potential is predicted to be equal to the pion mass. This is consistent with the result obtained by the chiral Lagrangian approach in Ref. [3].

We also studied the μ_I dependence of the π-condensate and "σ"-condensate. Our result shows that the value of the "chiral condensate" defined by $\langle\tilde{\sigma}\rangle = \sqrt{\langle\sigma\rangle^2 + \langle\pi^a\rangle^2}$ is constant for $\mu_I \lesssim$ 300 MeV, and that it grows rapidly in the large μ_I region. This indicates that the chiral symmetry restoration at finite baryon density and/or finite temperature will be delayed when non-zero isospin chemical potential is turned on.

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