# Automatising one－loop computations in the SM with RECOLA 

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#### Abstract

Automatisation of the computation of amplitudes for elementary processes in the Standard Model can be attained at one－loop level with the computer code RECOLA．Moving from Feynman dia－ grams to the language of off－shell currents，the code is based on a generalisation of the recursive Dyson－Schwinger equations at the one－loop level．Such relations have the advantage of improv－ ing the efficiency of the computation of the matrix elements，avoiding re－computation of sub－ amplitudes．The drawback is the lack of factorisation of the colour structures，which is a useful property of the diagrammatic approach．In this contribution we pay particular attention to this aspect showing how to optimise the treatment of colour in our approach and make RECOLA a competitive tool for the computation of both EW and QCD corrections．


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## 1. Introduction

After the discovery of the Higgs boson, an important task of the LHC is a precise study of the Standard Model and a comparison of theoretical predictions with experimental measurements for many processes. For reliable results, theoretical predictions beyond Born approximation are required, and large NLO contributions have to be precisely known. At hadron colliders QCD corrections are dominant, but also electroweak (EW) ones can have an important impact, e.g. in differential distributions at high energies. The computation of elementary processes in QCD at NLO is essentially under control and many codes are available [1, 2, 3, 4, 5, 6, 7]. For EW corrections, the situation is more involved, and a complete automatisation has not been achieved yet. Efficient automatic tools for the calculation of elementary particle reactions are important, because many steps are needed to get suitable theoretical predictions for physical hadronic processes (such as multichannel Monte Carlos, a proper treatment of real emission, convolution with parton distribution functions, parton showering, etc.) and they all need the partonic matrix elements as an essential ingredient. Here we present the code Recola which computes NLO amplitudes in the full SM [8]. The automatisation is achieved choosing an alternative approach to Feynman diagrams, based on recursion relations for off-shell currents.

## 2. Tree-level recursion relations

Starting from the Dyson-Schwinger equations [9], we build recursion relations for off-shell currents defined as follows. Writing a process with $L$ external legs in the form $P_{1}+\cdots+P_{L-1}+$ $P_{L} \rightarrow 0$, we select particle $P_{L}$ to be the last and denote the others as primary. We define the offshell current $w\left(P, \mathscr{C}, B_{\left\{l_{1}, \ldots, l_{n}\right\}}\right)$ as the sub-amplitude made of $n$ primary on-shell particles (with labels $l_{1}, \ldots, l_{n}$ ) and the off-shell particle $P$ (carrying colour labelled by $\mathscr{C}$ ):

$$
\begin{equation*}
w\left(P, \mathscr{C}, B_{\left\{l_{1}, \ldots, l_{n}\right\}}\right)=n \vdots \cdot P . \tag{2.1}
\end{equation*}
$$

The current does not contain the wave function of the off-shell particle $P$, which, for $n>1$, is replaced by its propagator. If, according to the Feynman rules of the theory, the selected $n$ external particles cannot generate $P$, the corresponding current vanishes. The Lorentz structure of $w$ is of scalar, spinor or vector type according to particle $P$. For $n=1$ the off-shell currents of the external legs coincide with their wave functions. Using the binary notation of Ref. [10], the labels $\left\{l_{1}, \ldots, l_{n}\right\}$ are expressed through a single tag number $B_{\left\{l_{1}, \ldots, l_{n}\right\}}$.

The off-shell leg of the currents allows to construct the currents recursively according to the following recursion relation:


Each term of the sums is obtained by multiplying the generating currents with the interaction vertex, marked by the small box, and the propagator of $P$, marked by the thick line. The first step of the recursion procedure is the construction of all 2-leg currents ( $n=2$ ), for whatever particle $P$ :


They can then be used to build the 3-leg currents:


The 4-leg currents are then given by


The procedure ends when all possible currents for $n \leq L-1$ have been computed. For $n=L-1$, just the current with $P$ coinciding with the antiparticle $\bar{P}_{L}$ of the last particle has to be considered. The amplitude $\mathscr{A}$ of the elementary process is then obtained by multiplying this unique last current with the inverse of the propagator of $\bar{P}_{L}$ and with its wave function:

$$
\begin{equation*}
\mathscr{A}=L-1 \overbrace{}^{\bar{P}_{L}} \times\left(\text { propagator of } \bar{P}_{L}\right)^{-1} \times \bullet \bar{P}_{L} . \tag{2.6}
\end{equation*}
$$

In this recursive approach each sub-graph (which would contribute to different Feynman diagrams) is computed automatically only once. Moreover the contribution to a current coming from different sub-graphs are summed at each step of the recursive procedure and the number of generated objects which are passed to the next step is minimised.

## 3. One-loop recursion relations

One-loop recursion relations are based on the decomposition of one-loop amplitudes in terms of the tensor integrals (TIs) $T_{(t)}^{\mu_{1} \cdots \mu_{r_{t}}}$, containing the dependence on the loop momentum $q$, and the tensor coefficients (TCs) $c_{\mu_{1} \cdots \mu_{r_{t}}}^{(t)}$ :

$$
\begin{equation*}
T_{(t)}^{\mu_{1} \cdots \mu_{r_{t}}}=\frac{(2 \pi \mu)^{4-D}}{\mathrm{i} \pi^{2}} \int \mathrm{~d}^{D} q \frac{q^{\mu_{1}} \cdots q^{\mu_{r_{t}}}}{D_{0}^{(t)} \cdots D_{k_{t}}^{(t)}}, \quad D_{k_{t}}^{(t)}=\left(q+p_{k_{t}}^{(t)}\right)^{2}-\left(m_{k_{t}}^{(t)}\right)^{2} \tag{3.1}
\end{equation*}
$$

Relying on external libraries for the computation of the TIs, the key point is to find an efficient procedure to determine numerically the TCs. In this framework the indices $\mu_{1}, \ldots, \mu_{r_{t}}$ are strictly 4-dimensional and the correct amplitude is obtained adding a rational part of type R 2 , which can be built by means of special Feynman rules [11] similar to those for the counterterms [12].

Starting from the work of van Hameren [13], who proposed a way to generalise the DysonSchwinger recursion relations to one-loop gluon amplitudes in QCD, we construct a recursive procedure to compute the TCs in the full SM. We exploit the correspondence between one-loop diagrams with $L$ external legs and tree diagrams with $L+2$ external legs, obtained by cutting one of the loop lines. We thus associate to a given loop process $P_{1}+\cdots+P_{L} \rightarrow 0$ a family of tree processes with two more legs $P_{1}+\cdots+P_{L}+P_{0}+\bar{P}_{0} \rightarrow 0$, for any particle $P_{0}$ of the SM. Since tree diagrams carry different symmetries with respect to loop diagrams, topological selection rules have to be applied in the generation of the tree diagrams associated to a given one-loop diagram. Essentially we have to discard redundant contributions originating from different choices of the cut line and of the direction of the "loop flow". Considering, for example, a three-point function at one loop, we find, without selection rules, six corresponding tree level diagrams:


Appropriate selection rules, expressed in the language of off-shell currents, are the following:

1) The current containing the first external line enters the loop flow first.
2) The 3 currents containing the external legs with the 3 smallest binary numbers enter the loop flow in fixed order (for example given by the ascending order of the 3 binaries).
The first rule practically fixes the cutting point, discarding in our example the last 4 diagrams:


The second rule fixes the loop flow, selecting the first tree diagram in the example:


The formal generation of the one-loop amplitude is reduced to the generation of a set of treelevel processes with two extra legs, for which the following recursion relation holds:


The main difference with respect to the relation at tree level of the previous section is the dependence on the integration momentum $q$, generated by the Feynman rules for the vertex and the propagator. In the 't Hooft-Feynman gauge in the SM this $q$-dependence takes the standard form

$$
\begin{equation*}
(\text { vertex }) \times(\text { propagator })=\frac{a_{\mu} q^{\mu}+b}{(q+p)^{2}-m^{2}} \tag{3.4}
\end{equation*}
$$

where $a^{\mu}$ and $b$ have additional Dirac/Lorentz structures depending on the interacting particles, omitted here for simplicity. Repeated application of the recursion relation fixes the $q$-dependence of the loop currents. After $k$ steps one gets:

$$
\begin{equation*}
\operatorname{loop} \text { current }(q)=\sum_{r=0}^{k} a_{\mu_{1} \cdots \mu_{r}}^{(k, r)} \frac{q^{\mu_{1}} \cdots q^{\mu_{r}}}{\prod_{h=1}^{k}\left[\left(q+p_{h}\right)^{2}-m_{h}^{2}\right]} \tag{3.5}
\end{equation*}
$$

where $r$ is the tensor rank of each term of the linear combination. At the end of the recursive procedure the coefficients $a_{\mu_{1} \cdots \mu_{r}}^{(k, r)}$ are the contribution for that cut particle to the TCs $c_{\mu_{1} \cdots \mu_{r_{t}}}^{(t)}$. On the other hand, the $q$-dependent part of the loop currents of the last step is the integrand of the corresponding TI. In order to compute at each step of the recursion the coefficients $a_{\mu_{1} \cdots \mu_{r}}^{(k, r)}$ of the loop current and keep track of the corresponding loop propagators, we define the $q$-independent loop off-shell current

$$
\begin{equation*}
w_{i_{k}}\left(P, \mathscr{C}, B,\left\{p_{1}, \ldots, p_{k}\right\},\left\{m_{1}, \ldots, m_{k}\right\}\right)= \tag{3.6}
\end{equation*}
$$

where $\left\{p_{1}, \ldots, p_{k}\right\}$ and $\left\{m_{1}, \ldots, m_{k}\right\}$ are the sequences of momenta and masses of the propagators which link each current with its $q$-dependent part, allowing at the end of the procedure to associate each TC with the corresponding TI. Profiting of the symmetry properties of the tensor $q^{\mu_{1}} \cdots q^{\mu_{r}}$, the tensorial index $i_{k}$ has been introduced for the loop current $w_{i_{k}}$, which is in a one-to-one correspondence with the totally symmetric part of the coefficients $a_{\mu_{1} \cdots \mu_{r}}^{(k, r)}$ according to:

$$
\begin{equation*}
i_{k}=0 \rightarrow w_{i_{k}}=a^{(k, 0)}, \quad i_{k}=1, \ldots, 4 \rightarrow w_{i_{k}}=a_{\mu_{1}}^{(k, 1)}, \quad i_{k}=5, \ldots, 14 \rightarrow w_{i_{k}}=a_{\left\{\mu_{1} \mu_{2}\right\}}^{(k, 2)} \tag{3.7}
\end{equation*}
$$

## 4. Treatment of colour

An efficient treatment of colour can be achieved using the colour-flow representation introduced in Ref. [14], where the conventional 8 gluon fields $A_{\mu}^{a}$ are replaced by a $3 \times 3$ matrix

$$
\begin{equation*}
\left(\mathscr{A}_{\mu}\right)_{j}^{i}=\frac{1}{\sqrt{2}} A_{\mu}^{a}\left(\lambda^{a}\right)_{j}^{i} \quad \text { with the trace condition } \quad \sum_{i}\left(\mathscr{A}_{\mu}\right)_{i}^{i}=0 . \tag{4.1}
\end{equation*}
$$

Gluons are labelled by a pair of indices $i, j=1,2,3$, while quarks and antiquarks maintain the usual colour index $i=1,2,3$. The Feynman rules in this representation are obtained by multiplying the standard Feynman rules by $\left(\lambda^{a}\right)_{j}^{i}$ for each gluon line. Their colour structure turns out to be simply a combination of Kronecker $\delta$ s. For example one gets

$$
\begin{align*}
& j \longrightarrow_{p} i=\delta_{j}^{i} \frac{\mathrm{i}(\not p+m)}{p^{2}-m^{2}}, \quad{\underset{j}{i_{1}}}_{j_{1}}^{\underset{p}{\mu}{ }_{p}^{v}}{ }_{i_{2}}^{j_{2}}=\delta_{j_{2}}^{i_{1}} \delta_{j_{1}}^{i_{2}} \frac{-\mathrm{i} g_{\mu \nu}}{p^{2}}, \\
& \underbrace{i_{1}}_{j_{2}}{\underset{\sim}{\infty}}_{\mu}^{j_{i_{3}}^{j_{3}}}=\left(\delta_{j_{3}}^{i_{1}} \delta_{j_{2}}^{i_{3}}-\frac{1}{N_{\mathrm{c}}} \delta_{j_{2}}^{i_{1}} \delta_{j_{3}}^{i_{3}}\right) \frac{\mathrm{i} g_{\mathrm{s}}}{\sqrt{2}} \gamma^{\mu} . \tag{4.2}
\end{align*}
$$

These colour structures determine the structure of the amplitude which for a process with $k$ external gluons and $m$ external quark-antiquark pairs takes the form

$$
\begin{equation*}
\mathscr{A}_{\substack{j_{1} \cdots j_{n}}}^{i_{1} \cdots i_{n}}=\sum_{P\left(j_{1}, \ldots, j_{n}\right)} \delta_{j_{1}}^{i_{1}} \cdots \delta_{j_{n}}^{i_{n}} \mathscr{A}_{P} \quad n=k+m \tag{4.3}
\end{equation*}
$$

where the sum has been performed on the permutations $P\left(j_{1}, \ldots, j_{n}\right)$ of the lower indices (equivalently one could have chosen the upper ones). The computation of the sum over colours of the squared amplitude can follow two strategies:

- The $N_{\mathrm{c}}^{2 n}$ colour-dressed amplitudes $\mathscr{A}_{j_{1}, \ldots, j_{n}}^{i_{1} \ldots, i_{n}}$ are computed (i.e. for all possible colour indices $\left.i_{1}, \ldots, i_{n}, j_{1}, \ldots, j_{n}\right)$ and the summed squared amplitude is given by

$$
\begin{equation*}
\mathscr{A}^{2}=\sum_{i_{1} \ldots i_{n}, j_{1}, \ldots, j_{n}}\left(\mathscr{A}_{j_{1} \cdots j_{n}}^{i_{1} \cdots i_{n}}\right)^{*} \mathscr{A}_{\substack{i_{1} \cdots j_{n}}}^{i_{1} \cdots i_{n}} . \tag{4.4}
\end{equation*}
$$

- Alternatively the decomposition of Eq. (4.3) can be used and the summed squared amplitude is obtained by summing, over the permutations $P$, the $n$ ! "structure-dressed" amplitudes $\mathscr{A}_{P}$ :

$$
\begin{equation*}
\mathscr{A}^{2}=\sum_{P, P^{\prime}} \mathscr{A}_{P}^{*} C_{P P^{\prime}} \mathscr{A}_{P^{\prime}} \tag{4.5}
\end{equation*}
$$

The coefficients $C_{P P^{\prime}}$ can be trivially calculated and are given by positive powers of $N_{\mathrm{c}}$. This second strategy reduces the number of amplitudes to compute and is essentially the starting point of the known approach based on colour-ordered amplitudes.

This second strategy is implemented in REcola, by assigning a colour structure (labelled by the index $\mathscr{C}$ ) to each off-shell current. At each step of the recursion procedure the colour structure of the outgoing current is computed from the colour structures of the incoming ones. This allows to generate automatically all structure-dressed amplitudes $\mathscr{A}_{P}$ simultaneously in one recursive procedure, in such a way that structure-dressed currents contributing at intermediate steps to several $\mathscr{A}_{P}$ are computed just once.

The computation can be further optimised for those currents that differ only by the colour structure. Just one of such currents is actually computed in RECOLA and the others are obtained by multiplying that one by a colour coefficient. To understand how it works, let us consider the process $\bar{u} u g g g \rightarrow 0$. Labelling the external particles with the binary numbers $1,2,4,8,16$, we concentrate on the currents generated combining gluons 4 and 8 and quark 2 :

$$
2 \rightarrow \beta=w\left(u, \delta_{\beta}^{i_{2}}, 2\right) \quad 4 \infty_{\beta}^{\alpha}=w\left(g, \delta_{\beta}^{i_{4}} \delta_{j_{4}}^{\alpha}, 4\right) \quad 8 \infty_{\beta}^{\alpha}=w\left(g, \delta_{\beta}^{i_{8}} \delta_{j_{8}}^{\alpha}, 8\right)
$$

Combining gluon 4 and 8 , two gluonic currents are generated:


Each of these two currents generates two quark currents after combining with quark 2 :


The two $u$-quark currents with the same colour structure $\delta_{j_{8}}^{i_{4}} \delta_{j_{4}}^{i_{8}} \delta_{\beta}^{i_{2}}$ are actually two contributions to the same current, which can be summed up. According to the Feynman rules the currents generated in these two steps are numerically related and one needs just to compute two of them, $w\left(g, \delta_{j_{4}}^{i_{8}} \delta_{\beta}^{i_{4}} \delta_{j_{8}}^{\alpha}, 12\right)$ and $w\left(u, \delta_{j_{8}}^{i_{2}} \delta_{j_{4}}^{i_{8}} \delta_{\beta}^{i_{4}}, 14\right)$ for instance, and the others can be extracted from them. In particular the two contributions with structure $\delta_{j_{8}}^{i_{4}} \delta_{j_{4}}^{i_{8}} \delta_{\beta}^{i_{2}}$ turn out to differ by a sign, summing up to zero; their computation can thus be completely avoided.

## 5. Features of RECOLA

The code Recola allows for the computation of tree and one-loop amplitudes in the full SM, including the rational parts of type R2 and all counterterms (on-shell in the EW sector and $\overline{\mathrm{MS}}$ for the strong coupling constant). A consistent treatment of unstable particles is provided based on the complex-mass scheme [15]. The code offers the possibility for every given process to select/unselect arbitrary powers in the strong coupling constant, as well as specific resonant contributions. Soft and collinear divergences can be treated either in dimensional or mass regularisation, while ultraviolet finiteness can be checked numerically. Dynamical running of $\alpha_{\mathrm{s}}$ is also implemented. In addition to an efficient treatment of colour, the sum over helicity configurations has been optimised: recalculation of identical currents appearing in different helicity configurations is avoided and helicity conservation for massless fermions is used. Moreover RECOLA calculates all colour- and spin-correlated amplitudes needed for the application of the dipole subtraction method [16]. For the computation of the tensor integrals RECOLA relies on the Collier library [17].

The code is written in Fortran90 and is structured in three parts:

- First the processes to be computed are defined by calling the subroutine
define_process_rcl:
call define_process_rcl(1,'u g -> u g e+ e-','NLO')
call define_process_rcl(2,'u g -> u g e+[+] e-[-]','NLO')
call define_process_rcl(3,'u g -> u g Z(e+ e-)','NLO')
The number in the call is the process label, while the last argument specifies the loop order (LO or NLO). The process is given by the user through a string, where incoming and outgoing particles are separated by the sybmbol ->; specific helicity contributions are selected by the symbols $[+],[-],[0]$, while the round brackets are used to select resonant contributions.
- Secondly the skeleton of all defined processes is generated:
call generate_processes_rcl
- Finally the amplitudes are computed calling the subroutine compute_process_rcl with the proper process labels, according to

```
call compute_process_rcl(1,p,A2lo(1),A2nlo(1))
call compute_process_rcl(2,p,A2lo(2),A2nlo(2))
call compute_process_rcl(3,p,A2lo(3),A2nlo(3))
```

where the output values $\mathrm{A} 21 \circ(\mathrm{n})$ and $\mathrm{A} 2 \mathrm{nlo}(\mathrm{n})$ are the LO and NLO amplitudes for the process with label $n$. The first two steps have to be run just once, while this last subroutine needs to be called for each phase-space point $\mathrm{p}(1: \operatorname{legs}, 0: 3)$.

RECOLA requires negligible amount of memory for executables, object files and libraries, while the RAM needed does not exceed 2 Gbyte even for complicated processes. To give an idea of the efficiency of the code in the following tables we give the CPU time needed for the generation ( $t_{\mathrm{gen}}$ ) and the computation ( $t_{\mathrm{TIs}}$ for the tensor integrals with COLLIER, $t_{\mathrm{TCs}}$ for the tensor coefficients) for a single phase-space point ${ }^{1}$ of some processes of physical interest at LHC:

- QCD corrections:

| $u \bar{d} \rightarrow W^{+} g g$ | $t_{\mathrm{gen}}:$ | 2.4 s | $t_{\mathrm{TIs}}:$ | 4.0 ms | $t_{\mathrm{TCs}}:$ | 1.1 ms |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $u \bar{d} \rightarrow W^{+} g g g$ | $t_{\mathrm{gen}}:$ | 15 s | $t_{\mathrm{TIs}}:$ | 67 ms | $t_{\mathrm{TCs}}:$ | 45 ms |
| $u \bar{u} \rightarrow W^{+} W^{-} g g$ | $t_{\mathrm{gen}}:$ | 76 s | $t_{\mathrm{TIs}}:$ | 83 ms | $t_{\mathrm{TCs}}:$ | 16 ms |

- EW + QCD corrections:

| $u \bar{u} \rightarrow l^{+} l^{-} g g$ | $t_{\text {gen }}:$ | 3.2 s | $t_{\mathrm{TIs}}:$ | 27 ms | $t_{\mathrm{TCs}}:$ | 25 ms |
| :--- | ---: | ---: | :--- | ---: | :--- | ---: |
| $u \bar{u} \rightarrow l^{+} l^{-} u \bar{u}$ | $t_{\mathrm{gen}}:$ | 5 s | $t_{\mathrm{TIs}}:$ | 68 ms | $t_{\mathrm{TCs}}:$ | 35 ms |
| $u \bar{u} \rightarrow l^{+} l^{-} g g g$ | $t_{\text {gen }}:$ | 44 s | $t_{\mathrm{TIs}}:$ | 331 ms | $t_{\mathrm{TCs}}:$ | 684 ms |
| $u \bar{u} \rightarrow l^{+} l^{-} u \bar{u} g$ | $t_{\mathrm{gen}}:$ | 50 s | $t_{\mathrm{TIs}}:$ | 835 ms | $t_{\mathrm{TCs}}:$ | 632 ms |

In these sample processes colour and helicity are summed for all particles, except for the massive vector bosons which decay leptonically. The processes have been computed on a personal computer with processor $\operatorname{Intel}(\mathrm{R})$ Core(TM) i5-2450M CPU @ 2.50 GHz .

[^1]
## 6. Conclusion

We have presented the code RECOLA for the computation of EW and QCD processes with elementary particles in the SM at NLO, based on a one-loop generalisation of Dyson-Schwinger recursion relations. The code has been used for the computation of EW corrections to the process $p p \rightarrow l^{+} l^{-} j j[8,19]$.

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[^1]:    ${ }^{1}$ The numbers have been produced averaging the CPU time of 100 phase-space points generated by RAMBO [18].

