PROCEEDINGS OF SCIENCE

Towards Higgs production at N^{3}LO

Claude Duhr[*][†]
Institute for Particle Physics Phenomenology
Durham University South Road, DH1 3LE Durham, UK
E-mail: claude.duhr@durham.ac.uk

We report on the recent computation of the inclusive Higgs boson production cross section in gluon fusion to next-to-next-to-next-to-leading order in perturbative QCD in the soft-virtual approximation. We discuss the validity of the soft-virtual approximation at that order, and we give some outlook on the computation of the subleading terms in the threshold expansion.

Loops and Legs in Quantum Field Theory - LL 2014,
27 April - 2 May 2014
Weimar, Germany

[*]Speaker.

[†]The author would like to thank the organizers for their invitation to the conference and the participants for the many stimulating discussions.

PoS(LL2014)028

1. Introduction

The discovery of the Higgs boson by the ATLAS and CMS collaborations at the Large Hadron Collider (LHC) at CERN [1] has inaugurated a new era in high-energy particle physics. While so far all experimental measurements of the properties of the Higgs boson are consistent with the Standard Model (SM) expectations, new physics could still be hiding in small deviations arising from the presence of new virtual particles contributing to the production of a Higgs boson. Establishing the nature of the Higgs boson and measuring its couplings to SM particles thus requires a high level of precision, both at the experimental and the theoretical sides.

The inclusive Higgs boson production cross section at the LHC takes the form

$$\sigma(m_H^2,s) = \sum_{i,j}\int dx_1\,dx_2\,f_i(x_1)\,f_j(x_2)\,\hat{\sigma}_{ij}(m_H^2,x_1\,x_2\,s)\,, \tag{1.1}$$

where s and m_H^2 denote the hadronic centre-of-mass energy and the mass of the Higgs boson, $f_i(x_k)$ denote the parton density functions, and the sum runs over all parton species. The quantity $\hat{\sigma}_{ij}$ is the partonic cross section for two partons i and j to produce a Higgs boson. In the framework of the SM, the dominant partonic production mode of a Higgs boson at the LHC is the so-called gluon-fusion channel, where two gluons scatter to produce a Higgs boson via a top quark loop. If the mass of the Higgs boson is below the top-pair threshold, $m_H^2 < 4m_t^2$, the gluon-fusion process can be described by an effective Lagrangian where the top quark has been integrated out and the gluons couple directly to the Higgs boson via an operator of dimension five,

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{5,\text{SM}} - \frac{1}{4v}C(\mu^2)\,H\,G_{\mu\nu}^a\,G_a^{\mu\nu}\,, \tag{1.2}$$

where $\mathscr{L}_{5,\text{SM}}$ denotes the SM Lagrangian with $N_f = 5$ light quark flavours, and $v \simeq 246\,\text{GeV}$ is the vacuum expectation value of the Higgs field. The Wilson coefficient $C(\mu^2)$ describing the effective coupling is known as a perturbative expansion up to three loops in the $\overline{\text{MS}}$-renormalised strong coupling constant $\alpha_s(\mu^2)$ evaluated at the scale μ^2 [2]. The inclusive gluon-fusion cross section in the effective theory has been computed at next-to-leading order (NLO) [3] and next-to-next-to-leading order (NNLO) [4, 5, 6]. The remaining theoretical uncertainty on the cross section through NNLO due to variations of the renormalisation and factorisation scales is estimated to be of the order of 10%. Various corrections to the effective theory are known, including two-loop electroweak and mixed QED/QCD corrections, as well as top-mass corrections to the effective theory as an expansion in the inverse top-quark mass. In order to reduce the uncertainty on the cross section, the next important contributions are the next-to-next-to-next-to-leading order (N^{3}LO) corrections in the effective theory described by eq. (1.2). In this contribution we report on the progress in this direction, and we review the recent computation of the gluon-fusion cross section at N^{3}LO in the soft-virtual approximation. This result constitutes a first important step towards the full computation of a hadron collider observable at N^{3}LO in perturbative QCD.

2. The gluon-fusion cross section at N^{3}LO in the soft-virtual approximation

The full computation of the gluon-fusion cross section at N^{3}LO requires the computation of thousands of Feynman diagrams. Fortunately, the steep fall of the parton density functions with the energy suggests that the cross section can be well approximated by an expansion close to threshold. This method was already employed at NNLO [6] and was shown to produce a reliable and fast-converging expansion for the cross section [7]. The expansion is controlled by the single parameter $z = m_H^2/\hat{s}$, where $\hat{s} = x_1 x_2 s$ denotes the partonic centre-of-mass energy. We can then approximate the cross section by its threshold expansion,

$$\hat{\sigma}_{ij}(s,z) = \delta_{ig}\,\delta_{jg}\,\hat{\sigma}_{gg}^{SV} + \sum_{k=0}^{\infty}(1-z)^k\,\hat{\sigma}_{ij}^{(k)}\,. \tag{2.1}$$

The leading term in the threshold expansion, the so-called *soft-virtual* term $\hat{\sigma}_{gg}^{SV}$, is only non-vanishing for the gluon initial state, and describes the production of a Higgs boson at threshold in association with soft final-state partons. If the soft-virtual term is expanded into a perturbative series in the strong coupling constant $\alpha_s = \alpha_s(\mu^2)$,

$$\hat{\sigma}_{gg}^{SV} = \frac{\pi C(\mu^2)^2}{v^2 V}\sum_{k=0}^{\infty}\left(\frac{\alpha_s}{\pi}\right)^{\ell}\hat{\eta}^{(\ell)}\,, \tag{2.2}$$

with $V = N^2-1$ the number of adjoint colours of $SU(N)$, then the perturbative coefficients can be schematically written in the form

$$\hat{\eta}^{(\ell)} = \omega^{(\ell)}\,\delta(1-z) + \sum_{k=0}^{2\ell-1}\rho_k^{(\ell)}\left[\frac{\log^k(1-z)}{1-z}\right]_+\,, \tag{2.3}$$

where the coefficients $\omega^{(\ell)}$ and $\rho_k^{(\ell)}$ are linear combinations of (multiple) zeta values whose coefficients are polynomials in the colour factors $C_A = N$ and $C_F = V/(2N)$ and the number of light flavours N_f. We emphasize that the entirety of the ℓ-loop corrections to the production cross section is contained in the coefficient $\omega^{(\ell)}$, while the coefficients $\rho_k^{(\ell)}$ only receive contributions from the emission of soft final-state partons. The soft-virtual term is known through NNLO [4, 5, 6, 7] in perturbative QCD, while at N^{3}LO only the coefficients $\rho_k^{(\ell)}$ have been computed [8]. In the rest of this section we review the recent computation of the full soft-virtual term $\hat{\eta}^{(\ell)}$ at N^{3}LO in perturbative QCD [9].

The partonic cross section for the production of a Higgs boson in association with a certain number of jets can schematically be written as

$$\hat{\sigma}_{ij}(i,j\to H+n\text{ partons}) \sim \int d\Phi_{n+1}\,|\mathscr{M}(i,j\to H+n\text{ partons})|^2\,, \tag{2.4}$$

where $|\mathscr{M}(i,j\to H+n\text{ partons})|^2$ denotes the matrix element (summed and averaged oder spins and colours) for two partons i and j to produce a Higgs boson in associations with n additional partons, and $d\Phi_{n+1}$ denotes the usual phase space measure in $D = 4-2\varepsilon$ dimensions,

$$d\Phi_{n+1} = (2\pi)^D\,\delta^{(D)}\left(p_i+p_j-p_H-\sum_{k=1}^{n}p_k\right)\frac{d^Dp_H}{(2\pi)^{D-1}}\,\delta_+(p_H^2-m_H^2)\prod_{k=1}^{n}\frac{d^Dp_k}{(2\pi)^{D-1}}\,\delta_+(p_k^2)\,, \tag{2.5}$$

with $\delta_+(p^2-m^2) = \delta(p^2-m^2)\,\theta(p^0)$. Note that, due to the presence of soft and collinear singularities, the phase space integration (2.4) is in general divergent and gives rise to poles in dimensional regularisation. These singularities cancel order by order in perturbation theory when summing up all the contributions from different final-sate multiplicities and the convolutions of lower-order cross sections with the splitting functions up to the required order.

Inclusive phase-space integrals like eq. (2.4) can be efficiently computed using the *reverse-unitarity* technique [4, 10]. This technique, which was already successfully applied at NNLO, establishes a duality between inclusive phase-space integrals and cut loop integrals via Cutkosky's rule,

$$\text{Disc}\frac{1}{p^2-m^2+i\varepsilon} = 2\pi i\,\delta_+(p^2-m^2)\,. \tag{2.6}$$

The duality makes it possible to apply algebraic techniques developed for loop integrations directly to phase space integrals. In particular, it makes it possible to apply integration-by-parts (IBP) techniques to reduce all the phase space integrals to a small set of master integrals [11], and to set up a set of ordinary differential equations for the master integrals [12].

So far the discussion was generic, and the reverse-unitarity technique can immediately be applied to phase-space integrals in full kinematics. If we concentrate on the soft-virtual term at N^{3}LO, we can combine the reverse-unitarity technique with the method of *expansion by regions*. Indeed, the soft-virtual term at N^{3}LO receives contributions from interference diagrams with up to three additional partons, where each parton can be either real or virtual. Moreover, in the soft limit we can expand the integrand in the soft-parton momenta, where the momentum of each virtual gluon can be either hard, i.e., $\mathscr{O}(1)$, or soft, i.e., $\mathscr{O}(1-z)$, and each real final-state parton is soft. The coefficients appearing in the soft expansion still admit a diagrammatic interpretation, and they are thus still amenable to algebraic IBP reductions, giving rise to a small set of *soft master integrals*. We note, however, that these soft master integrals only admit trivial differential equation, because they are homogeneous in the expansion parameter $(1-z)$. As such, these integrals need to be computed by other means.

We have recently completed the computation of all soft master integrals that contribute to the soft virtual term at N^{3}LO. In the following we give a short summary of the different contributions:

1. **Three-loop virtual corrections**. The three-loop virtual corrections are given by the QCD form factor up to three loops, which was computed in ref. [13].

2. **Two-loop soft corrections to H + jet.** This contribution corresponds to interference diagrams involving two virtual gluons and one real final-state gluon. Note that there are two distinct cases to consider, corresponding to the square of the one-loop amplitude and the interference of the tree-level and two-loop amplitudes for $g\,g \to H\,g$. In the limit where the final-state gluon is soft, the interference terms exhibit a universal factorisation

$$\langle \mathscr{M}_3^{(k)}|\mathscr{M}_3^{(l)}\rangle \simeq -g_S^2\sum_{m=0}^{k}\sum_{n=0}^{l}(g_S\,\mu^{2\varepsilon})^{m+n}\langle \mathscr{M}_2^{(k-m)}|J_\mu^{(m)a}\,J_a^{(n)\mu}|\mathscr{M}_2^{(l-n)}\rangle\,, \tag{2.7}$$

where g_S is the strong coupling constant, $\mathscr{M}_N^{(k)}$ denotes the k-loop amplitude for $H+N$ gluons and $J_\mu^{(m)a}$ is the m-loop QCD soft current, known through two-loop order [14]. Note that each term (m,n) in the sum (2.7) corresponds $m+n$ soft and $l+k-n-m$ hard virtual gluons. The remaining phase space integration is trivial to perform, which completes the computation of this contribution.

3. **One-loop soft corrections to H + 2 jets.** There are two different contributions to consider, depending on whether the virtual gluon is hard or soft. The hard region is trivial to compute, and the soft region can easily be computed by expanding the integrand in the soft limit and reducing all the integrals to soft phase-space integrals that can be computed analytically [9, 15].

4. **Soft triple-real corrections to H + 3 jets.** In this case, all the gluons are real, and we can expand the tree-level matrix element for H + 3 jets in the soft final-state momenta. The remaining integrals can be reduced to eight soft master integrals that can all be evaluated analytically [16]. We observe that all the coefficients in front of the zeta values are integers in all cases, but we are currently lacking an explanation of this phenomenon.

These contributions, together with the knowledge of the three-loop Wilson coefficient [2], beta function [17] and splitting functions [18] (as well as the lower order cross sections to higher order in the dimensional regulator ε [19]), are sufficient to obtain the full result for the gluon-fusion cross section at N^{3}LO in the soft-virtual approximation. Adding all the contributions together, all the poles in dimensional regularisation cancel. The finite terms, however, do not cancel, and constitute the final result for $\hat{\eta}^{(3)}$ [9]. Note that we reproduce exactly the predictions for $\rho_k^{(3)}$ of ref. [8]. The coefficient $\omega^{(3)}$ is genuinely new, and includes in particular the full three-loop corrections to Higgs production in gluon fusion.

3. Towards phenomenology

Even though the computation of the soft-virtual approximation is complete, it would be premature to draw strong phenomenological conclusions. Indeed, whenever we truncate a series expansion, an ambiguity is introduced which can be quantified by multiplying the result by an arbitrary function $g(z)$ with $\lim_{z\to 1} g(z) = 1$. Indeed, it is easy to see that

$$\sum_{i,j}\int dx_1\,dx_2\,[f_i(x_1)\,f_j(x_2)\,z\,g(z)]\left[\frac{\hat{\sigma}_{ij}(m_H^2,x_1\,x_2\,s)}{z\,g(z)}\right]_{\text{threshold}}, \tag{3.1}$$

has the same formal accuracy in the soft-virtual approximation, as long as we make sure that $g(z)$ approaches 1 in the soft limit. Despite the fact that formal accuracy is the same in the soft limit, the numerical impact on the cross section can be quite sizeable, and a detailed analysis of the numerical impact of different choices for $g(z)$ was presented in ref. [20]. Note that this ambiguity was already known at NNLO, where it was shown that the soft-virtual approximation underestimates the full NNLO cross section by a large amount. At the same time, it was observed a posteriori that choosing $g(z) = z$ at NNLO leads to a very good approximation to the full NNLO result. In particular, this choice reproduced correctly the leading logarithmic behaviour of the first subleading term, $\hat{\sigma}_0$, in the soft expansion (2.1). Whether the same choice leads to a good approximation also at N^{3}LO remains, however, still an open question that can most likely only be resolved once more terms in the threshold expansion are known.

4. Conclusion and outlook

In this contribution we have reported on the first computation of the inclusive gluon-fusion cross section at N^3LO in the soft-virtual approximation. Although it is still premature to draw strong phenomenological conclusions at this point, our result constitutes an important step forward in performing perturbative computations for hadron colliders. In particular, we are confident that it should be possible to consistently improve our result in order to overcome the ambiguities inherent to the soft-virtual approximation. First, the techniques we have developed to perform the threshold expansion are by no means restricted to the soft-virtual term, but they can equally-well be applied to the subleading terms. This was already demonstrated in ref. [16], where also the first subleading term appearing in the soft expansion of the triple-real contribution was computed. For the mixed real-virtual contributions, a new effect appears beyond the soft-virtual approximations, because the virtual gluons may not only be hard or soft, but we also need to take into account contributions from collinear regions, where the virtual gluons are collinear to the external momenta. Finally, we note that the contribution coming from the square of the one-loop amplitude for H + jet has been computed exactly without any approximation [21]. Indeed, in this case the phase space integration reduces to a trivial integration over a two-body phase space. Moreover we note that the two-loop matrix element for H + jet is also available [22], and so similar techniques should also be applicable in this case. This is currently under investigation.

References

[1] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012); S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012).

[2] K. G. Chetyrkin, B. A. Kniehl and M. Steinhauser, Nucl. Phys. B **510**, 61 (1998); Y. Schroder and M. Steinhauser, JHEP **0601**, 051 (2006); K. G. Chetyrkin, J. H. Kuhn and C. Sturm, Nucl. Phys. B **744**, 121 (2006).

[3] D. Graudenz, M. Spira and P. M. Zerwas, Phys. Rev. Lett. **70**, 1372 (1993); S. Dawson, Nucl. Phys. B **359**, 283 (1991); A. Djouadi, M. Spira and P. M. Zerwas, Phys. Lett. B **264**, 440 (1991); M. Spira, A. Djouadi, D. Graudenz and P. M. Zerwas, Nucl. Phys. B **453**, 17 (1995) [hep-ph/9504378].

[4] C. Anastasiou and K. Melnikov, Nucl. Phys. B **646**, 220 (2002).

[5] V. Ravindran, J. Smith and W. L. van Neerven, Nucl. Phys. B **665**, 325 (2003).

[6] R. V. Harlander and W. B. Kilgore, Phys. Rev. Lett. 88, 201801 (2002).

[7] S. Catani, D. de Florian and M. Grazzini, JHEP 0105, 025 (2001); R. V. Harlander and W. B. Kilgore, Phys. Rev. D 64, 013015 (2001).

[8] S. Moch and A. Vogt, Phys. Lett. B **631**, 48 (2005).

[9] C. Anastasiou, C. Duhr, F. Dulat, E. Furlan, T. Gehrmann, F. Herzog and B. Mistlberger, [arXiv:1403.4616 [hep-ph]].

[10] C. Anastasiou, L. J. Dixon, K. Melnikov and F. Petriello, Phys. Rev. Lett. **91** (2003) 182002 [hep-ph/0306192]; C. Anastasiou and K. Melnikov, Phys. Rev. D **67** (2003) 037501 [hep-ph/0208115]; C. Anastasiou, L. J. Dixon, K. Melnikov and F. Petriello, Phys. Rev. D **69** (2004) 094008 [hep-ph/0312266].

[11] F. V. Tkachov, Phys. Lett. B **100** (1981) 65; K. G. Chetyrkin and F. V. Tkachov, Nucl. Phys. B **192** (1981) 159.

[12] A. V. Kotikov, Phys. Lett. B **259**, 314 (1991); A. V. Kotikov, Phys. Lett. B **267**, 123 (1991); T. Gehrmann and E. Remiddi, Nucl. Phys. B **580** (2000) 485 [hep-ph/9912329].

[13] P.A. Baikov, K.G. Chetyrkin, A.V. Smirnov, V.A. Smirnov, M. Steinhauser, Phys. Rev. Lett. **102**, 212002 (2009); T. Gehrmann, E. W. N. Glover, T. Huber, N. Ikizlerli, C. Studerus, JHEP **1006**, 094 (2010).

[14] S. Catani, M. Grazzini, Nucl. Phys. B **591**, 435 (2000); S. D. Badger and E. W. N. Glover, JHEP **0407**, 040 (2004) [hep-ph/0405236]; C. Duhr and T. Gehrmann, Phys. Lett. B **727**, 452 (2013); Y. Li and H. X. Zhu, JHEP **1311**, 080 (2013).

[15] Y. Li, A. von Manteuffel, R. M. Schabinger and H. X. Zhu, arXiv:1404.5839 [hep-ph].

[16] C. Anastasiou, C. Duhr, F. Dulat and B. Mistlberger, JHEP **1307** (2013) 003 [arXiv:1302.4379 [hep-ph]].

[17] O. V. Tarasov, A. A. Vladimirov and A. Y. .Zharkov, Phys. Lett. B **93**, 429 (1980); S. A. Larin and J. A. M. Vermaseren, Phys. Lett. B **303**, 334 (1993); T. van Ritbergen, J. A. M. Vermaseren and S. A. Larin, Phys. Lett. B **400**, 379 (1997); M. Czakon, Nucl. Phys. B **710**, 485 (2005).

[18] S. Moch, J. A. M. Vermaseren and A. Vogt, Nucl. Phys. B **688**, 101 (2004); Nucl. Phys. B **691**, 129 (2004).

[19] C. Anastasiou, S. Bühler, C. Duhr and F. Herzog, JHEP **1211**, 062 (2012); M. Höschele, J. Hoff, A. Pak, M. Steinhauser, T. Ueda, Phys. Lett. B **721**, 244 (2013); S. Bühler and A. Lazopoulos, JHEP **1310**, 096 (2013).

[20] F. Herzog and B. Mistlberger, in: Proceedings of the *49^{th} Rencontres de Moriond*, [arXiv:1405.5685 [hep-ph]].

[21] C. Anastasiou, C. Duhr, F. Dulat, F. Herzog and B. Mistlberger, JHEP **1312**, 088 (2013); W. B. Kilgore, Phys. Rev. D **89** (2014) 073008 [arXiv:1312.1296 [hep-ph]].

[22] T. Gehrmann, M. Jaquier, E. W. N. Glover and A. Koukoutsakis, JHEP **1202** (2012) 056 [arXiv:1112.3554 [hep-ph]].

PoS(LL2014)028