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$\mathcal{O}(\alpha_s\alpha)$ corrections to Drell–Yan processes in the resonance region

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Drell–Yan-like W-boson and Z-boson production in the resonance region allows for some high-precision measurements that are crucial to carry experimental tests of the Standard Model to the extremes, such as the determinations of the W-boson mass and the effective weak mixing angle. We describe how the Standard Model prediction can be successfully performed in terms of a consistent expansion about the resonance pole, which classifies the corrections in terms of factorizable and non-factorizable contributions. The former can be attributed to the W/Z production and decay subprocesses individually, while the latter link production and decay by soft-photon exchange. At next-to-leading order we compare the full electroweak corrections with the pole-expanded approximations, confirming the validity of the approximation. At $\mathcal{O}(\alpha_s\alpha)$, we describe the concept of the expansion and report on results on the non-factorizable contributions, which turn out to be phenomenologically negligible. Moreover, we present first (preliminary) results on the dominant factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections, which originate from the interplay of initial-state QCD and final-state electroweak corrections. The naive factorization of NLO QCD and NLO EW corrections approximates the $\mathcal{O}(\alpha_s\alpha)$ corrections to the W-boson transverse-mass distribution, but, e.g., not the distribution in the lepton transverse momentum.

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1. Introduction

Drell–Yan-like W- or Z-boson production is among the most important standard candle processes at the LHC. Apart from delivering important information on parton distributions and allowing for the search for new gauge bosons in the high-mass range, these processes allow for high-precision measurements in the resonance regions. The weak mixing angle might be measured with LEP precision, and the W-boson mass M_W with an accuracy exceeding 10 MeV.

In the past two decades, great effort was made in the theory community to deliver precise predictions matching the required accuracy (for a list of references, see Ref. [1]). QCD corrections are known up to next-to-next-to-leading order, electroweak (EW) corrections up to next-to-leading order (NLO). Both on the QCD and on the EW side, there are further refinements such as leading higher-order effects, resummations, matched parton showers. In view of fixed-order calculations, the largest missing piece seems to be the mixed QCD–EW corrections of $\mathscr{O}(\alpha_s\alpha)$. Knowing the contribution of this order will also answer the question how to properly combine QCD and EW corrections in predictions. In Ref. [2] this issue is quantitatively discussed with special emphasis on observables that are relevant for the M_W determination, revealing percent corrections of $\mathscr{O}(\alpha_s\alpha)$ that should be calculated. First steps towards this direction have been taken by calculating two-loop contributions [3], the full $\mathscr{O}(\alpha_s\alpha)$ correction to the W/Z-decay widths [4], and the full $\mathscr{O}(\alpha)$ EW corrections to W/Z+jet production including the W/Z decays [5].

In this short article, we briefly report on our effort [1] to calculate the $\mathscr{O}(\alpha_s\alpha)$ corrections to Drell–Yan processes in the resonance region via the so-called *pole approximation*, which is based on a systematic expansion about the resonance pole. Specifically, we sketch the salient features of the approach, discuss its success at NLO, and describe results on the so-called non-factorizable contributions at $\mathscr{O}(\alpha_s\alpha)$, which comprise the most delicate contribution to the PA. Moreover, we present first (preliminary) results on the dominant factorizable corrections, originating from the interplay of initial-state QCD and final-state electroweak corrections.

2. Pole approximation for NLO corrections

The general idea [6] of a pole approximation (PA) for any Feynman diagram with a single resonance is the systematic isolation of all parts that are enhanced by a resonance factor $1/(p^2 - M_V^2 + \mathrm{i}M_V\Gamma_V)$, where p, M_V, and Γ_V are the momentum, mass, and width of the resonating particle V, respectively. In Drell–Yan production, V stands for a W or a Z boson. For W production different variants of PAs have been suggested and discussed at NLO already in Refs. [7, 8]. For the virtual corrections we follow the PA approach of Ref. [8]. Note, however, that we apply the PA to the real corrections as well, in contrast to Ref. [8] where they were based on full matrix elements.

Schematically each transition amplitude has the form

$$\mathscr{M} = \frac{W(p^2)}{p^2 - M_V^2 + \Sigma(p^2)} + N(p^2), \tag{2.1}$$

with functions W and N describing resonant and non-resonant parts, respectively, and Σ denoting the self-energy of V. The resonance of $\mathscr{M}$ is isolated in a gauge-invariant way as follows,

$$\mathscr{M} = \frac{W(\mu_V^2)}{p^2 - \mu_V^2}\,\frac{1}{1+\Sigma'(\mu_V^2)} + \left[\frac{W(p^2)}{p^2 - M_V^2 + \Sigma(p^2)} - \frac{W(\mu_V^2)}{p^2 - \mu_V^2}\,\frac{1}{1+\Sigma'(\mu_V^2)}\right] + N(p^2), \tag{2.2}$$

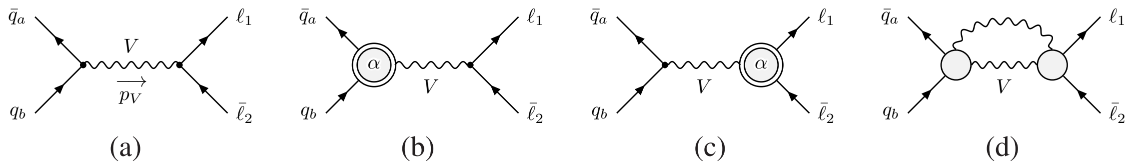

Figure 1: Generic diagrams for the lowest-order amplitude (a), for the EW virtual NLO factorizable corrections to production (b) and decay (c), as well as for virtual non-factorizable corrections (d), where the empty blobs stand for all relevant tree structures and the ones with "α" inside for one-loop corrections of $\mathscr{O}(\alpha)$.

where $\mu_V^2 = M_V^2 - \mathrm{i}M_V\Gamma_V$ is the gauge-invariant location of the propagator pole in the complex p^2 plane. Equation (2.2) can serve as a basis for the gauge-invariant introduction of the finite decay width in the resonance propagator, thereby defining the so-called *pole scheme*. In this scheme the term in square brackets is perturbatively expanded in the coupling α including terms up to $\mathscr{O}(\alpha)$, while the full p^2 dependence is kept. An application of this scheme to Z-boson production is, e.g., described in Ref. [9] in detail.

The PA for the amplitude results from the r.h.s. of (2.2) upon neglecting the last, non-resonant term and asymptotically expanding the term in square brackets in p^2 about the point $p^2 = \mu_V^2$, where only the leading, resonant term of the expansion is kept. The first term on the r.h.s. of (2.2) defines the so-called *factorizable* corrections in which on-shell production and decay amplitudes for V are linked by the off-shell propagator; these contributions are illustrated by diagrams (b) and (c) of Fig. 1. The term on the r.h.s. of (2.2) in square brackets contains the so-called *non-factorizable* corrections which receives resonant contributions from all diagrams where the limit $p^2 \to \mu_V^2$ in $W(p^2)$ or $\Sigma'(p^2)$ would lead to (infrared) singularities. At NLO, this happens if a soft photon of energy $E_\gamma \lesssim \Gamma_V$ is exchanged between the production process, the decay part, and the intermediate V bosons; a generic loop diagram is shown in Fig. 1(d). Although in principle the real-emission corrections can be based on full amplitudes without further approximations, the consistent application of the PA to both virtual and real corrections is necessary to make a separate discussion of factorizable and non-factorizable contributions possible.

Conceptually the evaluation of the non-factorizable corrections is the most delicate among all PA contributions. They possess rather interesting features. When the invariant mass of the resonance is integrated over, their contribution vanishes [10], i.e. they only tend to distort the resonance without changing the normalization of the cross section. Since they only involve soft photons, they take the form of a global correction factor to the lowest-order matrix element squared, with a non-trivial dependence on the virtuality $(p^2 - \mu_V^2)$ which gave the corrections their name. The virtual correction factor can be explicitly calculated with quite compact results, even for double resonances for which their calculation is described in detail in the literature [11]. The real corrections are better evaluated numerically to keep some flexibility in the treatment of photons in the event selection, using extended eikonal currents [11] that take into account the resonance distortion by soft photons of energy $E_\gamma \lesssim \Gamma_V$. Collinear singularities do not occur at all, since all relevant diagrams are of interference type.

In spite of the simple general idea of the PA, its consistent implementation in higher-order calculations involves subtle details. For instance, care has to be taken that subamplitudes appearing before or after the V resonance are based on subamplitudes with on-shell V bosons, otherwise

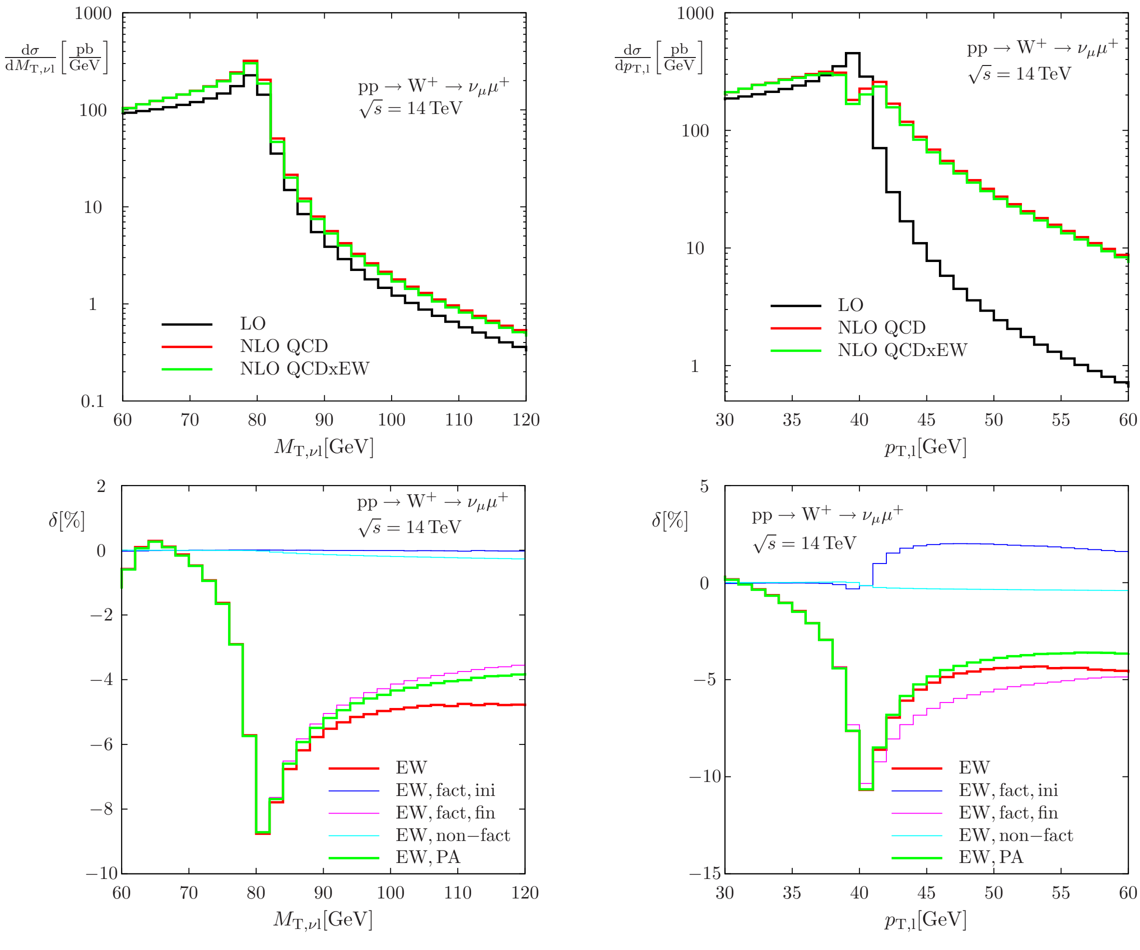

Figure 2: Distributions in the transverse mass (left) and transverse lepton momentum (right) for W^+ production at the LHC, with the upper plots showing the absolute distributions and the lower plots the relative NLO EW corrections in PA broken up into its factorizable and non-factorizable parts (taken from Ref. [1]).

gauge invariance cannot be guaranteed. Setting $p^2 = M_V^2$, instead of the problematic complex value $p^2 = \mu_V^2$, in the $\mathcal{O}(\alpha)$ corrections is certainly allowed in $\mathcal{O}(\alpha)$ approximation. However, the procedure is not unique, because the phase space is parametrized by more than one variable. The *on-shell projection* $p^2 \to M_V^2$ has to be defined carefully. Different variants may lead to results that differ within the intrinsic uncertainty of the PA, which is of $\mathcal{O}(\alpha/\pi \times \Gamma_V/M_V)$ in the resonance region when applied to $\mathcal{O}(\alpha)$ corrections. However, care has to be taken that virtual and real corrections still match properly in the (soft and collinear) infrared limits in order to guarantee the cancellation of the corresponding singularities.

Based on our results derived in Ref. [1], Figure 2 exemplarily shows the NLO QCD and EW corrections to the transverse-mass and transverse-lepton-momentum distributions for W^+ production at the LHC and, in particular, illustrates the structure and quality of the PA applied to the EW corrections. The distributions show the well-known Jacobian peaks at $M_{T,\nu l} \sim M_W$ and $p_{T,l} \sim M_W/2$, respectively, which play a central role in the measurement of the W-boson mass M_W at hadron colliders. The EW corrections significantly distort the distributions and shift the peak position. Note also the extremely large QCD corrections above the peak in the $p_{T,l}$ distribution, which are induced by the recoil of the W boson against the hard jet of the real QCD correction. The lower panels of Fig. 2 compare the full NLO EW corrections (without photon-induced processes from $q\gamma$ collisions) to the result of the PA, which is also broken up into factorizable corrections to the initial/final state and non-factorizable contributions. Near the Jacobian peaks, the PA turns out to be good within some 0.1%. Interestingly, the impact of the non-factorizable corrections is suppressed to the 0.1% level and, thus, phenomenologically negligible. A similar conclusion even holds for the factorizable initial-state corrections when one takes into account that the percentage correction to the $p_{T,l}$ distribution actually should be normalized to the full cross section including QCD corrections, which are overwhelming above the Jacobian peak. Thus, the relevant part of the NLO EW corrections near the peaks entirely results from the factorizable final-state corrections, where the bulk originates from collinear final-state radiation from the decay leptons.

More details and results on the PA at NLO are discussed in Ref. [1], in particular for Z production, for which the PA works similarly well.

3. Pole approximation at $\mathscr{O}(\alpha_s\alpha)$

Though technically more complicated, the concept of the PA, described at NLO in the previous section, can be carried over to higher orders in a straightforward way. The corresponding virtual, real, and mixed virtual–real contributions involve various different interference diagrams. Exemplarily we depict the generic graphs for the non-factorizable $\mathscr{O}(\alpha_s\alpha)$ corrections in Fig. 3 and for the factorizable contributions combining initial-state QCD and final-state EW corrections in Fig. 4.

As at NLO, the non-factorizable corrections originate from the virtual exchange or real emission of soft photons with energies $E_\gamma \lesssim \Gamma_V$, however, without any restriction on the kinematics of virtual gluons or real jet radiation. In Ref. [1] we have discussed in detail the factorization properties of the virtual and real photonic parts of the non-factorizable $\mathscr{O}(\alpha_s\alpha)$ corrections, which result from the soft nature of the effect. Using gauge-invariance arguments borrowed from the classic YFS paper [12], we show that this factorization of the photonic factors even hold to any order in the strong coupling α_s. We have verified this statement diagrammatically and, for the purely virtual corrections, also with effective-field-theory techniques [13]. Both the virtual and real photonic corrections can be written as correction factors to squared matrix elements containing gluon loops or external gluons, i.e. the necessary building blocks are obtained from tree-level and one-loop calculations. Our numerical study [1] shows that the non-factorizable corrections of $\mathscr{O}(\alpha_s\alpha)$ below the 0.1% level and, thus, phenomenologically negligible, both for W-boson and Z-boson production.

Of course, one could have speculated on this suppression, since the impact of non-factorizable photonic corrections is already at the level of some 0.1% at NLO. However, the $\mathscr{O}(\alpha_s\alpha)$ corrections mix EW and QCD effects, so that small photonic corrections might have been enhanced by the strong jet recoil effect observed in the $p_{T,l}$ distribution. This enhancement is seen in the virtual and real corrections separately, but not in their sum. Furthermore, the existence of gluon-induced (qg) channels implies a new feature in the non-factorizable corrections. In the $q\bar{q}$ channels, and thus in the full NLO part of the non-factorizable corrections, the soft-photon exchange proceeds between initial- and final-state particles, whereas in the qg channels at $\mathscr{O}(\alpha_s\alpha)$ the photon is also exchanged

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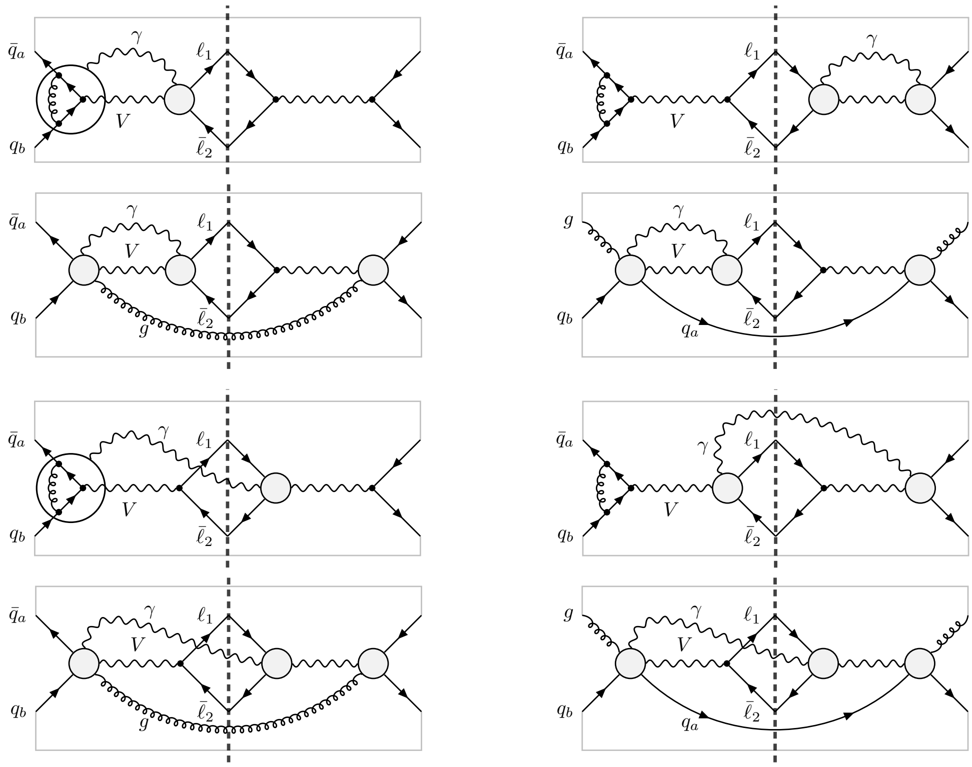

Figure 3: Generic diagrams for the non-factorizable corrections of $\mathscr{O}(\alpha_s\alpha)$.

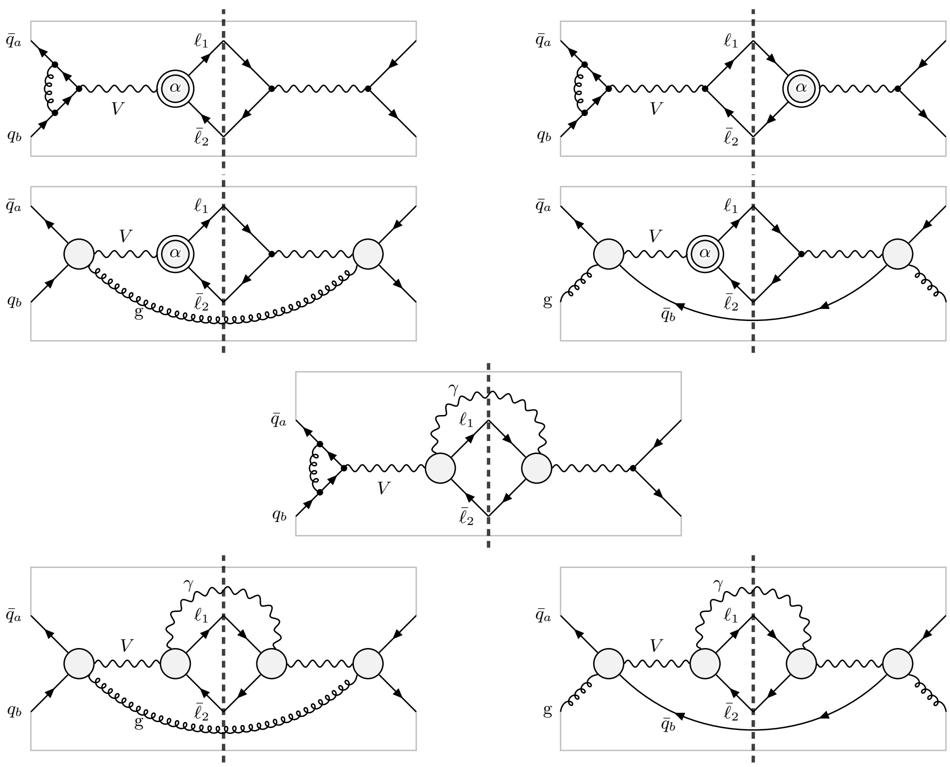

Figure 4: Generic diagrams for the $\mathscr{O}(\alpha_s\alpha)$ factorizable corrections of the "initial–final" type.

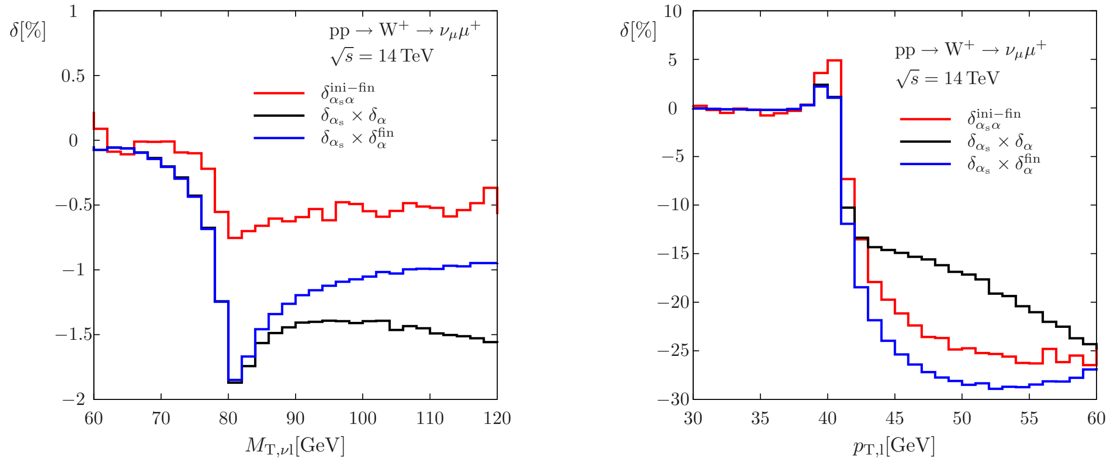

Figure 5: Relative factorizable corrections (in red) of $\mathcal{O}(\alpha_s\alpha)$ induced by initial-state QCD and final-state EW contributions to the distributions in $M_{T,\nu l}$ (left) and $p_{T,l}$ (right) for W^+ production at the LHC. The naive products of the NLO correction factors δ_{α_s} and δ_α are shown for comparison (see text).

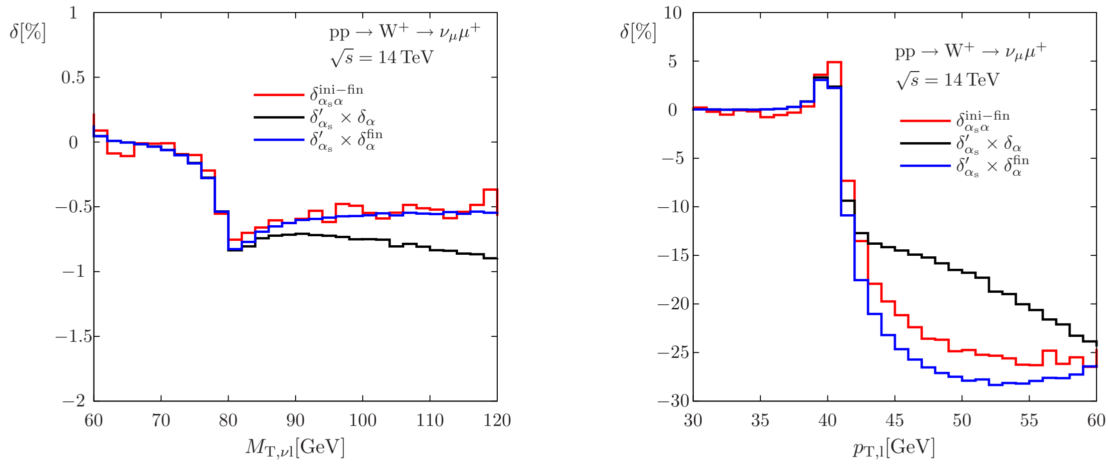

Figure 6: As in Fig. 5, but with the naive product of QCD and EW corrections based on δ'_{α_s} instead of δ_{α_s}.

between final-state particles. The known suppression mechanisms in non-factorizable corrections work somewhat differently in those cases [11].

Figures 5 and 6 show first preliminary results on the "initial–final" factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections $\delta^{\text{ini-fin}}_{\alpha_s\alpha}$ induced by initial-state QCD and final-state EW contributions. From the results of the PA for the NLO EW corrections, one has to expect that those contributions furnish the by far dominant part at $\mathcal{O}(\alpha_s\alpha)$, while the two other types of factorizable contributions of "initial–initial" and "final–final" type are much smaller. In detail, the figures compare the factorizable initial–final corrections (red curves) to different versions of naive products of the NLO QCD and NLO EW correction factors. To define the naive products, we write the NLO QCD cross section σ^{NLO_s} as

$$\sigma^{\text{NLO}_s} \equiv \sigma^{\text{LO}}(1+\delta_{\alpha_s}) = \sigma^0 + \sigma^{\text{LO}}\left(\frac{\sigma^{\text{LO}}-\sigma^0}{\sigma^{\text{LO}}} + \delta_{\alpha_s}\right) \equiv \sigma^0 + \sigma^{\text{LO}}\delta'_{\alpha_s}, \tag{3.1}$$

where σ^{LO} and σ^0 denote the LO cross section evaluated with LO and NLO PDFs, respectively. The standard QCD K factor is, thus, given by $K^{\text{NLO}_s} = 1+\delta_{\alpha_s}$, and δ'_{α_s} differs from δ_{α_s} by the

LO prediction induced by the transition from LO to NLO PDFs. The EW correction factor is used in two different versions, first based on the full NLO correction (δ_α, black curves), and second based on the dominant EW final-state correction of the PA ($\delta_\alpha^{\text{fin}}$, blue curves). The relative EW correction is always derived from the ratio of the NLO EW contribution $\Delta\sigma^{\text{NLO}_{\text{ew}}}$ and the LO contribution σ^0 to the NLO cross section, $\delta_\alpha = \Delta\sigma^{\text{NLO}_{\text{ew}}}/\sigma^0$, so that the EW correction factors are practically independent of the PDF set. An ansatz for the cross section $\sigma^{\text{NNLO}_{\text{s}\otimes\text{ew}}}_{\text{naive fact}}$ based on the naive factorization of QCD and EW corrections then reads

$$\sigma^{\text{NNLO}_{\text{s}\otimes\text{ew}}}_{\text{naive fact}} = \sigma^{\text{NLO}_\text{s}}(1+\delta_\alpha) = \sigma^{\text{LO}}(1+\delta_{\alpha_\text{s}})(1+\delta_\alpha). \tag{3.2}$$

This ansatz should approximate the full NLO QCD+EW cross section improved by our calculated $\mathcal{O}(\alpha_s\alpha)$ correction $\Delta\sigma^{\text{NNLO}_{\text{s}\otimes\text{ew}}}_{\text{ini-fin}}$,

$$\sigma^{\text{NNLO}_{\text{s}\otimes\text{ew}}} = \sigma^{\text{NLO}_\text{s}} + \Delta\sigma^{\text{NLO}_{\text{ew}}} + \Delta\sigma^{\text{NNLO}_{\text{s}\otimes\text{ew}}}_{\text{ini-fin}}, \tag{3.3}$$

which is consistently evaluated with NLO PDFs.

Figures 5 and 6 compare the $\mathcal{O}(\alpha_s\alpha)$ correction $\delta^{\text{ini-fin}}_{\alpha_\text{s}\alpha} = \Delta\sigma^{\text{NNLO}_{\text{s}\otimes\text{ew}}}_{\text{ini-fin}}/\sigma^{\text{LO}}$ with the two versions $\delta_{\alpha_\text{s}}\delta_\alpha$ and $\delta'_{\alpha_\text{s}}\delta_\alpha$, respectively. The corrections to the W-boson transverse-mass distribution are approximated by the naive factorization $\delta'_{\alpha_\text{s}}\delta_\alpha$ quite well. The preference of the factorization variant $\delta'_{\alpha_\text{s}}\delta_\alpha$ over $\delta_{\alpha_\text{s}}\delta_\alpha$ can be interpreted upon inspecting the difference between $\sigma^{\text{NNLO}_{\text{s}\otimes\text{ew}}}$ and the factorized ansatz $\sigma^{\text{NNLO}_{\text{s}\otimes\text{ew}}}_{\text{naive fact}}$,

$$\frac{\sigma^{\text{NNLO}_{\text{s}\otimes\text{ew}}} - \sigma^{\text{NNLO}_{\text{s}\otimes\text{ew}}}_{\text{naive fact}}}{\sigma^{\text{LO}}} = \delta^{\text{ini-fin}}_{\alpha_\text{s}\alpha} - \delta'_{\alpha_\text{s}}\delta_\alpha, \tag{3.4}$$

i.e. the validity of the factorization approximation $\sigma^{\text{NNLO}_{\text{s}\otimes\text{ew}}}_{\text{naive fact}}$ is signalled by a small difference between $\delta^{\text{ini-fin}}_{\alpha_\text{s}\alpha}$ and $\delta'_{\alpha_\text{s}}\delta_\alpha$. Naive factorization works for the $M_{\text{T},\nu l}$ distribution, which is rather insensitive to any recoil effect of the W boson with respect to additional jet emission. The factorization ansatz fails, however, for the distribution in the lepton transverse momentum, which is sensitive to the interplay between QCD and photonic real-emission effects. It remains to be seen whether the $\mathcal{O}(\alpha_s\alpha)$ corrections to the $p_{\text{T,l}}$ distribution can be reproduced by attaching a photonic shower, which is based on the asymptotics of collinear photon emission, to a QCD-based prediction.

The results shown in Figs. 5 and 6 are obtained by photon recombination, which combines collinear photons and leptons to a quasi-particle as described in Refs. [8,9], while the NLO EW results shown in Fig. 2 involve "bare muons" without any photon recombination. Photon recombination restores the level of inclusiveness required by the KLN theorem to imply a cancellation of the collinear singularity, which enhances the effect of final-state radiation. Photon recombination typically reduces the size of photonic corrections by a factor of two.

4. Conclusions

Here we have briefly summarized the main results of Ref. [1], where we have shown how the $\mathcal{O}(\alpha_s\alpha)$ corrections to Drell–Yan processes can be approximated by the leading term in an expansion about the resonance pole. The quality of such an approximation achieved at NLO strongly supports the expectation that this approach is sufficient for observables that are dominated by the

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resonance, which include, e.g., the ones relevant for precision determinations of the W-boson mass. The pole approximation classifies corrections into factorizable and non-factorizable contributions. Our results show that the latter, which link production and decay subprocesses via soft-photon exchange, are phenomenologically negligible. The phenomenologically relevant corrections, thus, are of factorizable nature and can be attributed to corrections to the initial or final states. Again, the pattern of contributions to the pole approximation at NLO gives a clear picture on the expected hierarchy in higher orders. At $\mathscr{O}(\alpha_s\alpha)$, the bulk of corrections will be contained in the combination of QCD corrections to the production and EW corrections to the decay processes, with a particular enhancement induced by final-state radiation off the charged leptons. Here we have presented first preliminary results on those contributions, revealing that the naive factorization of NLO QCD and NLO EW corrections approximates the $\mathscr{O}(\alpha_s\alpha)$ corrections to the W-boson transverse-mass distribution, but not the lepton transverse-momentum distribution, which is particularly sensitive to recoil effects of the W boson. The completion of our calculation of $\mathscr{O}(\alpha_s\alpha)$ corrections and the discussion of their phenomenological implications are in progress.

References

[1] S. Dittmaier, A. Huss and C. Schwinn, arXiv:1403.3216 [hep-ph], to appear in Nucl. Phys. B.

[2] G. Balossini *et al.*, JHEP **1001** (2010) 013 [arXiv:0907.0276 [hep-ph]].

[3] A. Kotikov, J. H. Kühn and O. Veretin, Nucl. Phys. B **788** (2008) 47 [hep-ph/0703013];
R. Bonciani, PoS EPS-HEP2011 (2011) 365;
W. B. Kilgore and C. Sturm, Phys. Rev. D **85** (2012) 033005 [arXiv:1107.4798 [hep-ph]].

[4] A. Czarnecki and J. H. Kühn, Phys. Rev. Lett. **77** (1996) 3955 [hep-ph/9608366];
D. Kara, Nucl. Phys. B **877** (2013) 683 [arXiv:1307.7190].

[5] A. Denner, S. Dittmaier, T. Kasprzik and A. Mück, JHEP **0908** (2009) 075 [arXiv:0906.1656 [hep-ph]]; JHEP **1106** (2011) 069 [arXiv:1103.0914 [hep-ph]] and Eur. Phys. J. C **73** (2013) 2297 [arXiv:1211.5078 [hep-ph]].

[6] R. G. Stuart, Phys. Lett. B **262**, 113 (1991);
A. Aeppli, F. Cuypers and G. J. van Oldenborgh, Phys. Lett. B **314** (1993) 413 [hep-ph/9303236];
A. Aeppli, G. J. van Oldenborgh and D. Wyler, Nucl. Phys. B **428** (1994) 126 [hep-ph/9312212];
H. G. J. Veltman, Z. Phys. C **62** (1994) 35.

[7] D. Wackeroth and W. Hollik, Phys. Rev. D **55** (1997) 6788 [hep-ph/9606398].

[8] S. Dittmaier and M. Krämer, Phys. Rev. D **65** (2002) 073007 [hep-ph/0109062].

[9] S. Dittmaier and M. Huber, JHEP **1001** (2010) 060 [arXiv:0911.2329 [hep-ph]].

[10] V. S. Fadin, V. A. Khoze and A. D. Martin, Phys. Rev. D **49** (1994) 2247 and Phys. Lett. B **320** (1994) 141 [hep-ph/9309234].

[11] K. Melnikov and O. I. Yakovlev, Nucl. Phys. B **471** (1996) 90 [hep-ph/9501358];
W. Beenakker, A. P. Chapovsky and F. A. Berends, Nucl. Phys. B **508** (1997) 17 [hep-ph/9707326];
A. Denner, S. Dittmaier and M. Roth, Nucl. Phys. B **519** (1998) 39 [hep-ph/9710521].

[12] D. R. Yennie, S. C. Frautschi and H. Suura, Annals Phys. **13** (1961) 379.

[13] M. Beneke, A. P. Chapovsky, A. Signer and G. Zanderighi, Phys. Rev. Lett. **93** (2004) 011602 [hep-ph/0312331] and Nucl. Phys. B **686** (2004) 205 [hep-ph/0401002].