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NNLO antenna subtraction for heavy quark pair production

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We report on the un-integrated and integrated antenna subtraction terms for the treatment of infrared (IR) divergences in processes of the form $S \to Q\overline{Q} + X$ at next-to-next-to leading order (NNLO) QCD, where S denotes an uncolored initial state and Q a massive quark. The integrated antenna functions are computed analytically in terms of cyclotomic harmonic polylogarithms. As a first application we calculate $R = \sigma(e^+e^- \to \gamma^* \to Q\overline{Q} + X)/\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)$ to order α_s^2 and compare with existing results.

Loops and Legs in Quantum Field Theory - LL 2014,
27 April - 2 May 2014
Weimar, Germany

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1. Introduction

The exploration of heavy quark production, in particular $t\bar{t}$ and single-top production, is a central issue at today's (and future) high energy colliders. At the LHC present experimental analyses of $t\bar{t}$ production reach a level of accuracy of a few percent, and this precision will increase in the future. This requires on the theoretical side precise predictions, in particular within the SM, which in view of the smallness of the gauge couplings at high energies means predictions at higher orders in perturbation theory, especially with respect to the coupling α_s of quantum chromodynamics (QCD), both for cross sections and differential distributions. Notable progress in this context has recently been made with the computation of the total hadronic $t\bar{t}$ cross section to order α_s^4 [1, 2].

As a contribution towards a fully differential NNLO treatment of $t\bar{t}$ production at hadron colliders, we report, within the antenna subtraction framework, on the construction of double-real and real-virtual subtraction terms for processes involving the production of a pair of massive quarks by an uncolored initial state S at next-to-next-to leading order (NNLO) QCD:

$$S \to Q\bar{Q} + X\,, \tag{1.1}$$

where S denotes, for example, an e^+e^- pair or an uncolored boson. Employing the subtraction method, the contribution of order α_s^2 to the cross section or to a differential distribution of an arbitrary IR safe observable associated with reaction (1.1) is given schematically by

$$\begin{aligned}\sigma_{\text{NNLO}} \,=\, &\int_{\Phi_4}\left(d\sigma^{RR}_{\text{NNLO}} - d\sigma^{S}_{\text{NNLO}}\right) + \int_{\Phi_3}\left(d\sigma^{RV}_{\text{NNLO}} - d\sigma^{T}_{\text{NNLO}}\right)\\ &+ \int_{\Phi_2} d\sigma^{VV}_{\text{NNLO}} + \int_{\Phi_4} d\sigma^{S}_{\text{NNLO}} + \int_{\Phi_3} d\sigma^{T}_{\text{NNLO}}\,.\end{aligned} \tag{1.2}$$

The exclusive double-virtual cross section $d\sigma^{VV}_{\text{NNLO}}$ involves the (renormalized) amplitudes for $S \to Q\bar{Q}$ at tree-level, one-loop and two-loop level. The real-virtual correction $d\sigma^{RV}_{\text{NNLO}}$ requires the tree-level and (renormalized) one-loop matrix elements for $S \to Q\bar{Q}g$. Finally the computation of the double-real radiation contribution $d\sigma^{RR}_{\text{NNLO}}$ demands the tree-level amplitudes $S \to Q\bar{Q}Q\bar{Q}$, $Q\bar{Q}gg$, and $Q\bar{Q}q\bar{q}$, where q denotes a massless quark. In general these individual contributions give rise to infrared (IR) singularities. While infrared singularities from virtual corrections are obtained immediately after integration over the loop-momenta, the infrared singularities due to soft and/or collinear real emission only become explicit after integrating the matrix elements over the corresponding phase space regions.

Therefore, one introduces subtraction terms, denoted by $d\sigma^{S}_{\text{NNLO}}$ and $d\sigma^{T}_{\text{NNLO}}$ in (1.2), which approximate, respectively, the double-real and the real-virtual contributions in all their singular limits and hence regulate their divergences. So, by construction, the integrals over $d\sigma^{RR}_{\text{NNLO}} - d\sigma^{S}_{\text{NNLO}}$ and over $d\sigma^{RV}_{\text{NNLO}} - d\sigma^{T}_{\text{NNLO}}$ are finite and can be evaluated numerically in four dimensions. Furthermore, in order to make the cancellation of IR singularities explicit in eq. (1.2), the integrals of these subtraction terms must be computed over the phase-space regions where IR singularities arise.

Among other methods, antenna subtraction provides a completely general framework to construct subtraction terms as products of various antenna functions and reduced matrix elements with remapped momenta. The antenna subtraction framework was initially formulated for massless final

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state partons [3, 4, 5], but has been extended to the case of hadronic collisions, both at NLO [6] and NNLO [7, 8, 9, 10]. In its massless from, the antenna method has lead to the successful description of the infrared structure of three-jet events in e^+e^- annihilation at NNLO [11, 12, 13, 14, 15]. Recently, it has successfully been applied to di-jet production at hadron colliders [16, 17, 18, 19, 20]. Extending the antenna method to NNLO QCD reactions with massive final state quarks is an ongoing effort [21, 22, 23, 24, 25, 26]. Intermediate results for $t\bar{t}$ production at hadron colliders have recently become available [22, 26, 27, 28].

2. Massive double-real antenna functions

The subtraction of all double unresolved limits in $d\sigma^{RR}_{\mathrm{NNLO}}$ requires new four-parton tree-level antenna functions: The antennae $A^0_4(Q,g,g,\bar{Q})$ and $\tilde{A}^0_4(Q,g,g,\bar{Q})$ govern the ordered and photon-like emission of two gluons between a massive quark-antiquark pair, whereas $B^0_4(Q,q,\bar{q},\bar{Q})$ is employed to subtract singular limits due to the emission of a massless quark-antiquark pair. They can be derived by appropriately normalizing the color-ordered squared tree-level matrix elements of $\gamma^* \to Q\bar{Q}gg$ and $\gamma^* \to Q\bar{Q}q\bar{q}$ and yield the correct unresolved factor in each limit [5, 25, 23].

The integrated subtraction term $\int_{\Phi_4} d\sigma^S_{\mathrm{NNLO}}$ involves the corresponding integrated antenna functions, which are schematically defined as follows:

$$\mathscr{X}^0_{ijkl} = (C(\varepsilon))^{-2} \int d\Phi_{X_{ijkl}}\, X^0_4(i,j,k,l)\,, \tag{2.1}$$

where $C(\varepsilon) = (4\pi)^{\varepsilon} e^{-\varepsilon\gamma_E}/(8\pi^2)$. Since the antenna phase space $d\Phi_{X_{ijkl}}$ is proportional to the normal four-particle phase space, the calculation of $\mathscr{X}^0_{ijkl}$ amounts to the integration of squared matrix element of $1 \to 4$ processes over the respective inclusive phase spaces. The integration has to be performed in $d = 4-2\varepsilon$ dimensions. In order to calculate this class of integrals we first write them in terms of unitarity cuts of massive three-loop propagator-type integrals [29, 30]. This step makes them accessible to the powerful techniques that have been developed for multi-loop computations, in particular, integration-by-parts reduction [31, 32, 33, 34, 35] and the method of differential equations [36, 37, 38].

As a result of the IBP reduction, we can express the integrated antenna functions $\mathscr{A}^0_{4,Qgg\bar{Q}}$, $\tilde{\mathscr{A}}^0_{4,Qgg\bar{Q}}$, and $\mathscr{B}^0_{4,Qq\bar{q}\bar{Q}}$ in terms of 15 master integrals shown in Fig.1. Analytic results of these integrals have been presented in Refs. [25, 23]. In case of topology (a), closed-form expressions for arbitrary d in terms of hypergeometric functions ${}_3F_2$ have been derived by employing phase space factorization along with standard identities and integral representations of hypergeometric functions. Their expansion near $d = 4$ was computed with the help of the computer program `HypExp` [39].

For the remaining master integrals we have derived a coupled system of first order differential equations in the variables q^2 and y [36, 37, 38]. This system has been solved in a bottom up approach order by order in ε by the aid of standard techniques. In order to fix the constants of integration, we have either imposed the vanishing of phase space at threshold, which is located at $y \to 1$, or matched the expressions to the known results in the massless limit $y \to 0$ [40].

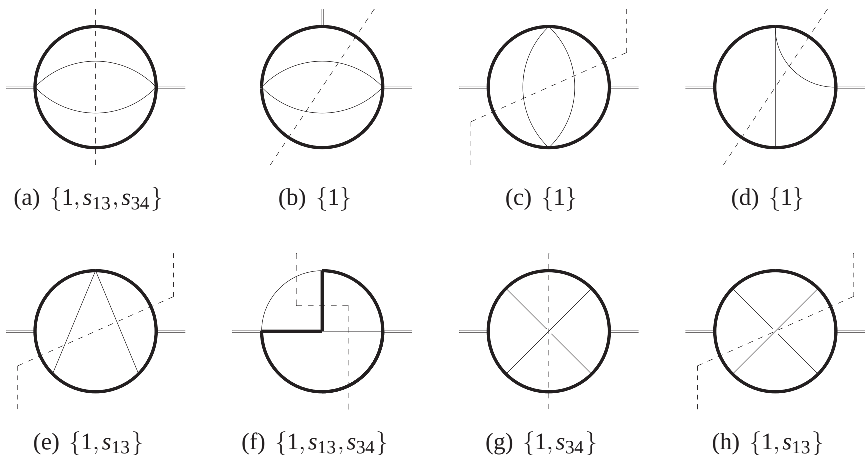


Figure 1: Definition of the double-real master integrals. In the diagrammatic representations bold (thin) lines refer to massive (massless) scalar propagators. The invariants in the curly brackets below the cut-diagrams denote irreducible numerators of the integrand. The double line represents the external momentum q, with $q^2 = s$. The dashed lines indicate the particles which are on-shell.

By this means, we obtain analytical results for all master integrals of Fig. 1 to all relevant orders in ε in terms of harmonic polylogarithms (HPL) [41] of argument $y = \frac{1-\beta}{1+\beta}$, where $\beta = \sqrt{1 - 4m_Q^2/q^2}$.

3. Massive real-virtual antenna functions

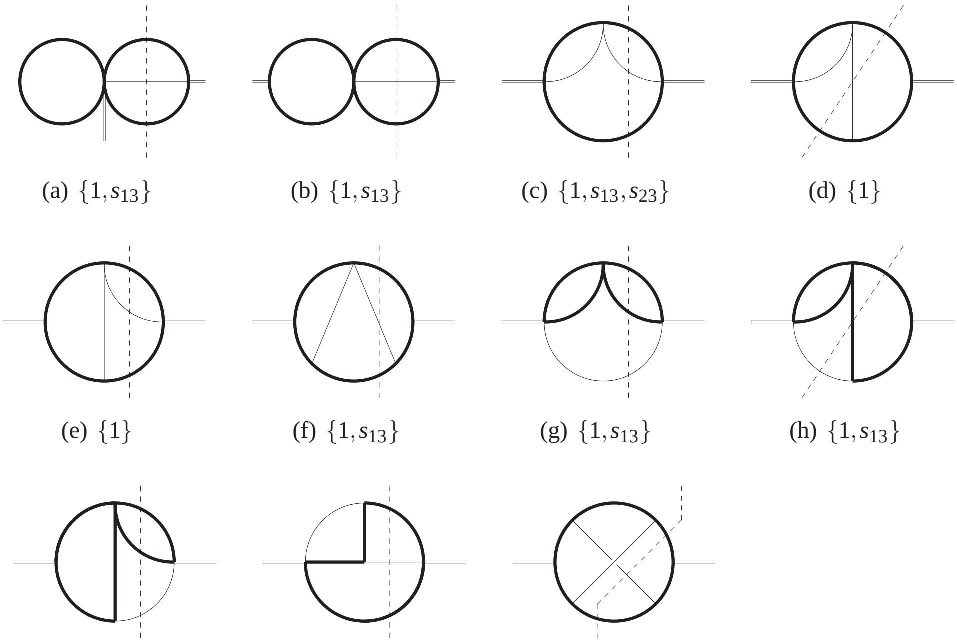
In order to render the real-virtual cross section for processes of the type (1.1) finite, we have to introduce the massive one-loop antenna functions $A^1_{3,Qg\bar{Q}}$ and $\tilde{A}^1_{3,Qg\bar{Q}}$, which can be determined from the interference of the Born amplitude and the leading and subleading color one-loop corrections to $\gamma^* \to Q\bar{Q}g$ [5, 42]. These antennae are also requied in their integrated form:

$$\mathscr{X}^1_{ijk} = (C(\varepsilon))^{-1} \int d\Phi_{X_{ijk}} X^1_3(i,j,k)\,, \tag{3.1}$$

with $d\Phi_{X_{ijk}}$ being proportional to the ordinary three-parton phase space. IBP reduction reveals that the analytic calculation of the integrated antenna functions $\mathscr{A}^1_{3,Qg\bar{Q}}$ and $\tilde{\mathscr{A}}^1_{3,Qg\bar{Q}}$ amounts to evaluating 22 master integrals, which are depicted in Fig. 2.

In the topologies (a) and (b) of Fig. 2, the dependence of the integrands on the loop momentum and the phase space momenta factorizes such that the results can be written in terms of ordinary products of known three-particle phase space integrals (cf. Ref. [21]) and scalar one-loop integrals. For the other master integrals we have derived differential equations in the variables q^2 and y in the same algorithmic fashion as in the case of the four-parton tree-level antenna functions (cf. Sec. 2). We have solved these differential equations in terms of cyclotomic harmonic polylogarithms [43, 44, 45, 46].

The necessary boundary conditions have been obtained from threshold expansions of the respective integrals which have been calculated by other means. Therefore, the integration constants



(a) $\{1, s_{13}\}$ (b) $\{1, s_{13}\}$ (c) $\{1, s_{13}, s_{23}\}$ (d) $\{1\}$

(e) $\{1\}$ (f) $\{1, s_{13}\}$ (g) $\{1, s_{13}\}$ (h) $\{1, s_{13}\}$

(i) $\{1, s_{13}\}$ (j) $\{1, s_{13}, s_{23}\}$ (k) $\{1, s_{13}\}$

Figure 2: Definition of the topologies for the combined phase space and loop integrations. In the diagrammatic representations bold (thin) lines refer to massive (massless) scalar propagators. The double line represents the external momentum q, with $q^2 = s$. The cut propagators are the ones intersected by the dashed line.

are given by values of cyclotomoic HPLs at argument $y = 1$. These quantities can either be computed numerically based on the defining integral representations of the cyclotomic HPLs, or one can exploit the various functional relations among the (cyclotomic) HPLs (shuffle relation etc.) in order to analytically reduce these objects to a smaller set of commonly known transcendental numbers. The latter approach relies on intensive usage of computer algebra and has been implemented by J. Ablinger and J. Blümlein. More details will be given in a future publication [47].

4. Cross section for $e^+e^- \to \gamma^* \to Q\bar{Q}X$ at order α_s^2

As a first application and check we compute the inclusive heavy quark-antiquark production cross section in e^+e^--annihilation via a virtual photon to order α_s^2 and to lowest order in $\alpha = e^2/(4\pi)$. The ratio R is defined by

$$R = \frac{\sigma(e^+e^- \to \gamma^* \to Q\bar{Q} + X)}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} = e_Q^2 \Big[N_c R^{(0)} + \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)\left(N_c^2 - 1\right) R^{(1)}$$
$$+ \left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2 \left(N_c^2 - 1\right)\left(N_c R^{(2)}_{\mathrm{LC}} - \frac{1}{N_c} R^{(2)}_{\mathrm{SC}} + 2T_R n_f R^{(2)}_f + 2T_R R^{(2)}_F\right) + \mathscr{O}(\alpha_s^3)\Big]. \quad (4.1)$$

In the following, we consider one heavy quark, carrying the electric charge e_Q (in units of the positron charge e) and n_f massless quark flavors. To order α_s, the ratio (4.1) has been known for a

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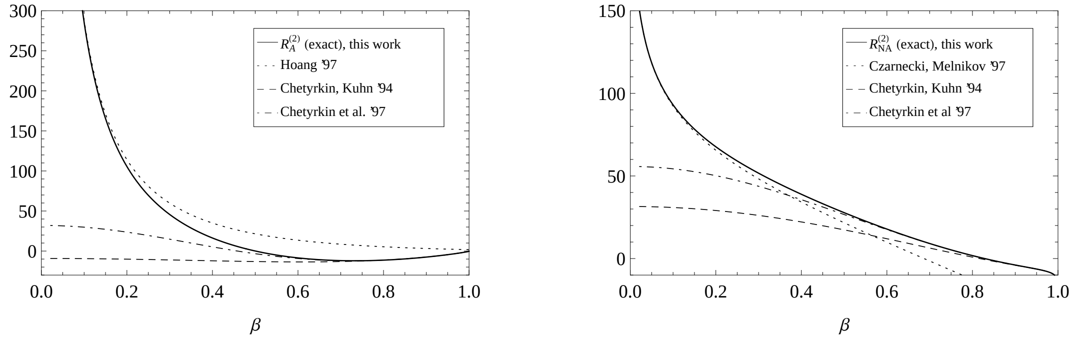

Figure 3: Exact results for $R_{\rm A}^{(2)} = 3R_{\rm SC}^{(2)}$ and $R_{\rm NA}^{(2)} = 3(R_{\rm LC}^{(2)} - R_{\rm SC}^{(2)})/2$ plotted against $\beta = \sqrt{1-4m_Q^2/s}$ (solid line). The renormalization scale is chosen to be $\mu = m_Q$. For comparison, the expansions in the threshold region (dotted curves) [55, 56] and in the asymptotic region (dashed and dash-dotted curves) [59, 60] are included as well.

long time [48, 49]. To order α_s^2, the leading-color correction $R_{\rm LC}^{(2)}$, the subleading-color correction $R_{\rm SC}^{(2)}$ and the massless flavor correction $R_f^{(2)}$ receive the following contributions ($R_F^{(2)}$ is not discussed here): The double-virtual correction from the process $\gamma^* \to Q\bar{Q}$ (i.e. 2-loop times Born and 1-loop squared) can be obtained in analytic form from the literature [50]. The real-virtual contribution associated with $\gamma^* \to Q\bar{Q}g$ (1-loop times Born) involves the master integrals of Fig. 2, whereas the contribution induced by the squared Born amplitudes $\gamma^* \to Q\bar{Q}gg$ and $\gamma^* \to Q\bar{Q}q\bar{q}$ can be expressed in terms of the integrals shown in Fig. 1. Furthermore, $R_{\rm SC}^{(2)}$ receives a contribution from the squared tree-level matrix element of $\gamma^* \to Q\bar{Q}Q\bar{Q}$ (in the following denoted by $R_E^{(2)}$), which is completely finite and will be discussed in more detail below.

The contributions from the various subprocesses exhibit explicit poles in ε of IR origin. Verifying the IR finiteness of $R_{\rm LC}^{(2)}$, $R_{\rm SC}^{(2)}$, and $R_f^{(2)}$ as anticipated according to the KLN theorem [52, 53] provides an important check of our calculations. Indeed, we find that in $R_{\rm LC}^{(2)}$, $R_{\rm SC}^{(2)}$, and $R_f^{(2)}$ all poles cancel analytically.

As discussed in Ref. [25], our result for $R_f^{(2)}$ is in full agreement with the one of Ref. [54], which was obtained in $d = 4$ by the aid of different techniques. For the leading and subleading color corrections, $R_{\rm LC}^{(2)}$ and $R_{\rm SC}^{(2)}$, approximate results in terms of truncated power series expansions have been computed, both, at pair production threshold $s \gtrsim 4m_Q^2$ (including terms of order β) [55, 56, 57] and in the high energy region $s \gg m_Q^2$ (through order m_Q^{12}/s^6) [58, 59, 60]. After expanding our expression for $R_{\rm LC}^{(2)}$ in the respective regions, we find full agreement with the existing results to all available orders. The same is true for $R_{\rm SC}^{(2)}$ in the threshold region. Note that due to the constraint $s \geq 16m_Q^2$, the term $R_E^{(2)}$ can be omitted for center-of-mass energies s close to the pair-production threshold at $s \gtrsim 4m_Q^2$. In the limit $m_Q^2/s \to 0$, the term $R_E^{(2)}$ becomes divergent. However, when we combine the known expressions of the logarithmically enhanced and finite terms of $R_E^{(2)}$ in this limit (cf. Ref. [61]) with the other contributions to $R_{\rm SC}^{(2)}$, we recover the massless result $R_{\rm SC}^{(2)}|_{m_Q=0} = -\frac{3}{32}$.

Finally, Fig. 3 shows our exact expressions for $R_{\rm A}^{(2)} = N_c R_{\rm SC}^{(2)}$ and $R_{\rm NA}^{(2)} = N_c(R_{\rm LC}^{(2)} - R_{\rm SC}^{(2)})/2$ plotted against the velocity β in the entire physical region $0 \leq \beta \leq 1$.

5. Summary and outlook

We addressed, within the antenna subtraction framework, the treatment of infrared singularities that arise in the computation of observables, in particular distributions, for processes at NNLO QCD, where a heavy quark-pair is produced by an uncolored initial state. We constructed the massive NNLO antenna functions that form part of the double-real and real-virtual antenna subtraction terms and outlined the analytic computation of their integrated counterparts in terms of (cyclotomic) HPLs. Our results include also analytical expressions for sets of master integrals, which we expect to be useful for other applications, too.

As a first application and check of our results we derived exact expressions for the order α_s^2 corrections to the total heavy quark antiquark production cross section in e^+e^--annihilation. We verified the analytic cancellation of all infrared poles in these contributions. Furthermore, the finite pieces are in full agreement with existing (approximate) results.

The (integrated) antenna functions discussed above provide the last missing building blocks for the numerical calculation of cross sections and differential distributions for heavy quark pair production by uncolored initial states at NNLO QCD within the antenna framework. Future applications include, for example, the forward-backward asymmetry for b- and t-quarks in e^+e^--annihilation.

Acknowledgments

Collaboration with W. Bernreuther and C. Bogner, as well as with J. Ablinger and J. Blümlein is hereby acknowledged. We thank Gabriel Abelof, Aude Gehrmann-De Ridder and Thomas Gehrmann for discussions. This work was supported in parts by Deutsche Forschungsgemeinschaft (DFG) Sonderforschungsbereich/Transregio 9 “Computergestützte Theoretische Teilchenphysik” and by the Cluster of Excellence Precision Physics, Fundamental Interactions and Structure of Matter (PRISMA-EXC 1098).

References

[1] P. Bärnreuther, M. Czakon and A. Mitov, *Percent Level Precision Physics at the Tevatron: First Genuine NNLO QCD Corrections to* $q\bar{q} \to t\bar{t} + X$, Phys. Rev. Lett. **109**, 132001 (2012) [arXiv:1204.5201 [hep-ph]].

[2] M. Czakon, P. Fiedler and A. Mitov, *Total Top-Quark Pair-Production Cross Section at Hadron Colliders Through* $\mathcal{O}(\alpha_S^4)$, Phys. Rev. Lett. **110**, no. 25, 252004 (2013) [arXiv:1303.6254 [hep-ph]].

[3] D. A. Kosower, *Antenna factorization in strongly ordered limits,* Phys. Rev. D **71**, 045016 (2005) [hep-ph/0311272].

[4] D. A. Kosower, *Antenna factorization of gauge theory amplitudes,* Phys. Rev. D **57**, 5410 (1998) [hep-ph/9710213].

[5] A. Gehrmann-De Ridder, T. Gehrmann and E. W. N. Glover, *Antenna subtraction at NNLO,* JHEP **0509**, 056 (2005) [hep-ph/0505111].

[6] A. Daleo, T. Gehrmann and D. Maitre, *Antenna subtraction with hadronic initial states,* JHEP **0704**, 016 (2007) [hep-ph/0612257].

[7] A. Daleo, A. Gehrmann-De Ridder, T. Gehrmann and G. Luisoni, *Antenna subtraction at NNLO with hadronic initial states: initial-final configurations*, JHEP **1001**, 118 (2010) [arXiv:0912.0374 [hep-ph]].

[8] R. Boughezal, A. Gehrmann-De Ridder and M. Ritzmann, *Antenna subtraction at NNLO with hadronic initial states: double real radiation for initial-initial configurations with two quark flavours*, JHEP **1102**, 098 (2011) [arXiv:1011.6631 [hep-ph]].

[9] T. Gehrmann and P. F. Monni, *Antenna subtraction at NNLO with hadronic initial states: real-virtual initial-initial configurations*, JHEP **1112**, 049 (2011) [arXiv:1107.4037 [hep-ph]].

[10] A. Gehrmann-De Ridder, T. Gehrmann and M. Ritzmann, *Antenna subtraction at NNLO with hadronic initial states: double real initial-initial configurations*, JHEP **1210**, 047 (2012) [arXiv:1207.5779 [hep-ph]].

[11] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, *Infrared structure of $e^+e^- \to 3$ jets at NNLO*, JHEP **0711**, 058 (2007) [arXiv:0710.0346 [hep-ph]].

[12] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and G. Heinrich, *Jet rates in electron-positron annihilation at $\mathcal{O}(\alpha_s^3)$ in QCD*, Phys. Rev. Lett. **100**, 172001 (2008) [arXiv:0802.0813 [hep-ph]].

[13] S. Weinzierl, *NNLO corrections to 2-jet observables in electron-positron annihilation*, Phys. Rev. D **74**, 014020 (2006) [hep-ph/0606008].

[14] S. Weinzierl, *NNLO corrections to 3-jet observables in electron-positron annihilation*, Phys. Rev. Lett. **101**, 162001 (2008) [arXiv:0807.3241 [hep-ph]].

[15] S. Weinzierl, *The infrared structure of $e^+e^- \to 3$ jets at NNLO reloaded*, JHEP **0907**, 009 (2009) [arXiv:0904.1145 [hep-ph]].

[16] E. W. Nigel Glover and J. Pires, *Antenna subtraction for gluon scattering at NNLO*, JHEP **1006**, 096 (2010) [arXiv:1003.2824 [hep-ph]].

[17] A. Gehrmann-De Ridder, E. W. N. Glover and J. Pires, *Real-Virtual corrections for gluon scattering at NNLO*, JHEP **1202**, 141 (2012) [arXiv:1112.3613 [hep-ph]].

[18] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and J. Pires, *Double Virtual corrections for gluon scattering at NNLO*, JHEP **1302**, 026 (2013) [arXiv:1211.2710 [hep-ph]].

[19] A. Gehrmann-De Ridder, T. Gehrmann, E. W. N. Glover and J. Pires, *Second order QCD corrections to jet production at hadron colliders: the all-gluon contribution*, Phys. Rev. Lett. **110**, no. 16, 162003 (2013) [arXiv:1301.7310 [hep-ph]].

[20] J. Currie, A. Gehrmann-De Ridder, E. W. N. Glover and J. Pires, *NNLO QCD corrections to jet production at hadron colliders from gluon scattering*, JHEP **1401**, 110 (2014) [arXiv:1310.3993 [hep-ph]].

[21] A. Gehrmann-De Ridder and M. Ritzmann, *NLO Antenna Subtraction with Massive Fermions*, JHEP **0907**, 041 (2009) [arXiv:0904.3297 [hep-ph]].

[22] G. Abelof and A. Gehrmann-De Ridder, *Antenna subtraction for the production of heavy particles at hadron colliders*, JHEP **1104**, 063 (2011) [arXiv:1102.2443 [hep-ph]].

[23] W. Bernreuther, C. Bogner and O. Dekkers, *The real radiation antenna functions for $S \to Q\bar{Q}gg$ at NNLO QCD*, JHEP **1310**, 161 (2013) [arXiv:1309.6887 [hep-ph]].

[24] G. Abelof, O. Dekkers and A. Gehrmann-De Ridder, *Antenna subtraction with massive fermions at NNLO: Double real initial-final configurations*, JHEP **1212**, 107 (2012) [arXiv:1210.5059 [hep-ph]].

[25] W. Bernreuther, C. Bogner and O. Dekkers, *The real radiation antenna function for $S \to Q\bar{Q}q\bar{q}$ at NNLO QCD,* JHEP **1106**, 032 (2011) [arXiv:1105.0530 [hep-ph]].

[26] G. Abelof and A. Gehrmann-De Ridder, *Double real radiation corrections to $t\bar{t}$ production at the LHC: the all-fermion processes*, JHEP **1204**, 076 (2012) [arXiv:1112.4736 [hep-ph]].

[27] G. Abelof and A. Gehrmann-De Ridder, *Double real radiation corrections to $t\bar{t}$ production at the LHC: the $gg \to t\bar{t}q\bar{q}$ channel*, JHEP **1211**, 074 (2012) [arXiv:1207.6546 [hep-ph]].

[28] G. Abelof, A. Gehrmann-De Ridder, P. Maierhofer and S. Pozzorini, *NNLO QCD corrections to top-antitop production in the $q\bar{q}$ channel*, arXiv:1404.6493 [hep-ph].

[29] R. E. Cutkosky, *Singularities and discontinuities of Feynman amplitudes,* J. Math. Phys. **1**, 429 (1960).

[30] C. Anastasiou and K. Melnikov, *Higgs boson production at hadron colliders in NNLO QCD*, Nucl. Phys. B **646**, 220 (2002) [hep-ph/0207004].

[31] K. G. Chetyrkin and F. V. Tkachov, *Integration by Parts: The Algorithm to Calculate beta Functions in 4 Loops,* Nucl. Phys. B **192**, 159 (1981).

[32] S. Laporta, *High precision calculation of multiloop Feynman integrals by difference equations,* Int. J. Mod. Phys. A **15**, 5087 (2000) [hep-ph/0102033].

[33] A. V. Smirnov, *Algorithm FIRE – Feynman Integral REduction,* JHEP **0810**, 107 (2008) [arXiv:0807.3243 [hep-ph]].

[34] A. V. Smirnov and V. A. Smirnov, *FIRE4, LiteRed and accompanying tools to solve integration by parts relations,* Comput. Phys. Commun. **184**, 2820 (2013) [arXiv:1302.5885 [hep-ph]].

[35] C. Anastasiou and A. Lazopoulos, *Automatic integral reduction for higher order perturbative calculations,* JHEP **0407**, 046 (2004) [hep-ph/0404258].

[36] A. V. Kotikov, *Differential equations method: New technique for massive Feynman diagrams calculation*, Phys. Lett. B **254**, 158 (1991).

[37] E. Remiddi, *Differential equations for Feynman graph amplitudes,* Nuovo Cim. A **110**, 1435 (1997) [hep-th/9711188].

[38] T. Gehrmann and E. Remiddi, *Differential equations for two loop four point functions,* Nucl. Phys. B **580**, 485 (2000) [hep-ph/9912329].

[39] T. Huber and D. Maitre, *HypExp 2, Expanding Hypergeometric Functions about Half-Integer Parameters,* Comput. Phys. Commun. **178**, 755 (2008) [arXiv:0708.2443 [hep-ph]].

[40] A. Gehrmann-De Ridder, T. Gehrmann, G. Heinrich, *Four particle phase space integrals in massless QCD,* Nucl. Phys. **B682** (2004) 265-288. [hep-ph/0311276].

[41] E. Remiddi and J. A. M. Vermaseren, *Harmonic polylogarithms,* Int. J. Mod. Phys. A **15**, 725 (2000) [hep-ph/9905237].

[42] O. Dekkers, W. Bernreuther, *The real-virtual antenna functions for $S \to Q\bar{Q}X$ at NNLO QCD*, To be published.

[43] J. Ablinger, *A Computer Algebra Toolbox for Harmonic Sums Related to Particle Physics,* arXiv:1011.1176 [math-ph].

[44] J. Ablinger, *Computer Algebra Algorithms for Special Functions in Particle Physics,* arXiv:1305.0687 [math-ph].

[45] J. Ablinger, J. Blümlein and C. Schneider, *Harmonic Sums and Polylogarithms Generated by Cyclotomic Polynomials,* J. Math. Phys. **52**, 102301 (2011) [arXiv:1105.6063 [math-ph]].

[46] J. Ablinger, J. Blümlein and C. Schneider, *Analytic and Algorithmic Aspects of Generalized Harmonic Sums and Polylogarithms,* J. Math. Phys. **54**, 082301 (2013) [arXiv:1302.0378 [math-ph]].

[47] O. Dekkers, W. Bernreuther, J. Ablinger and J. Blümlein, In preparation.

[48] A. O. G. Kallen and A. Sabry, *Fourth order vacuum polarization,* Kong. Dan. Vid. Sel. Mat. Fys. Med. **29**, no. 17, 1 (1955).

[49] J. S. Schwinger, *Particles, Sources And Fields. Volume II,* Reading 1973, 459p

[50] W. Bernreuther, R. Bonciani, T. Gehrmann, R. Heinesch, T. Leineweber, P. Mastrolia and E. Remiddi, *Two-loop QCD corrections to the heavy quark form-factors: The Vector contributions,* Nucl. Phys. B **706**, 245 (2005) [hep-ph/0406046].

[51] R. K. Ellis, D. A. Ross and A. E. Terrano, *The Perturbative Calculation of Jet Structure in e^+e^- Annihilation,* Nucl. Phys. B **178**, 421 (1981).

[52] T. Kinoshita, *Mass singularities of Feynman amplitudes,* J. Math. Phys. **3**, 650 (1962).

[53] T. D. Lee and M. Nauenberg, *Degenerate Systems and Mass Singularities,* Phys. Rev. **133**, B1549 (1964).

[54] A. H. Hoang, J. H. Kuhn and T. Teubner, *Radiation of light fermions in heavy fermion production,* Nucl. Phys. B **452**, 173 (1995) [hep-ph/9505262].

[55] A. H. Hoang, *Two loop corrections to the electromagnetic vertex for energies close to threshold,* Phys. Rev. D **56**, 7276 (1997) [hep-ph/9703404].

[56] A. Czarnecki and K. Melnikov, *Two loop QCD corrections to the heavy quark pair production cross-section in e^+e^- annihilation near the threshold,* Phys. Rev. Lett. **80**, 2531 (1998) [hep-ph/9712222].

[57] M. Beneke, A. Signer and V. A. Smirnov, *Two loop correction to the leptonic decay of quarkonium,* Phys. Rev. Lett. **80**, 2535 (1998) [hep-ph/9712302].

[58] S. G. Gorishnii, A. L. Kataev and S. A. Larin, *Three Loop Corrections of Order $\mathcal{O}(m^2)$ to the Correlator of Electromagnetic Quark Currents,* Nuovo Cim. A **92**, 119 (1986).

[59] K. G. Chetyrkin and J. H. Kuhn, *Quartic mass corrections to R_{had},* Nucl. Phys. B **432**, 337 (1994) [hep-ph/9406299].

[60] K. G. Chetyrkin, R. Harlander, J. H. Kuhn and M. Steinhauser, *Mass corrections to the vector current correlator,* Nucl. Phys. B **503**, 339 (1997) [hep-ph/9704222].

[61] S. Catani and M. H. Seymour, *Corrections of $\mathcal{O}(\alpha_S^2)$ to the forward backward asymmetry,* JHEP **9907**, 023 (1999) [hep-ph/9905424].

[62] K. G. Chetyrkin, J. H. Kuhn and M. Steinhauser, *Three loop polarization function and $\mathcal{O}(\alpha_s^2)$ corrections to the production of heavy quarks,* Nucl. Phys. B **482**, 213 (1996) [hep-ph/9606230].

[63] D. Maitre, *HPL, a mathematica implementation of the harmonic polylogarithms,* Comput. Phys. Commun. **174**, 222 (2006) [hep-ph/0507152].

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