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Triune Pairing Revelation

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A remarkable quantitative consistency is shown between three manifestations of pairing: 1) the even-odd mass differences; 2) the rotational moments of inertia; 3) the low energy level densities. The gap parameter extracted from rotational states nicely agrees with that obtained from even-odd mass differences. The natural log of experimental nuclear level densities at low energy is linear with energy. This can be interpreted in terms of a nearly 1st order phase transition from a superfluid to an ideal gas of quasi particles. The transition temperature coincides with the BCS critical temperature and yields gap parameters in good agreement with the values extracted from even-odd mass differences from rotational states. This converging evidence greatly supports the application of the BCS theory to atomic nuclei.

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1. Introduction: The anomalous quasiparticle spectrum

It is well known that the excitation spectrum with a “macroscopic” gap near the ground state is responsible for superfluid and superconducting states in bosonic and fermionic systems, respectively.

In nuclei, the pairing Hamiltonian produces just this kind of spectrum, as shown in fig.(1) where the paired quasi particle spectrum is compared with the unpaired particle/hole spectrum.

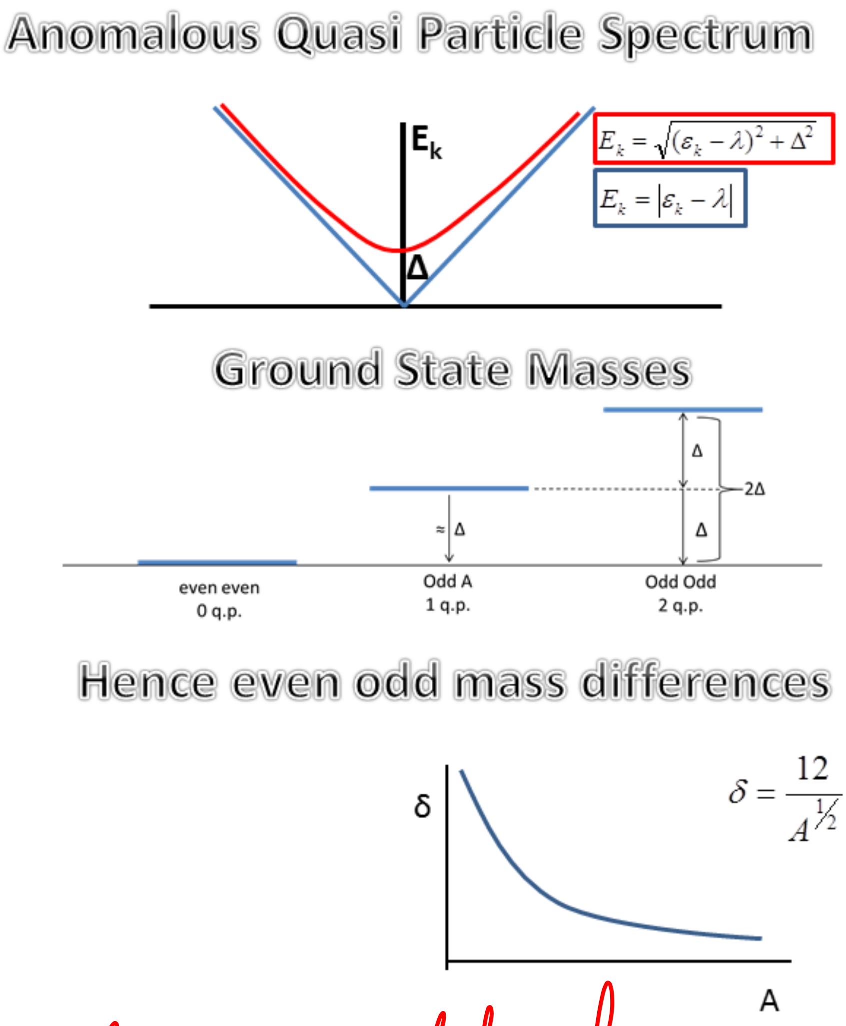

Fig.(1) Please add fig. caption

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1.1 Even-odd Mass differences

The first manifestation of this gap is the even-odd mass difference: even-even, odd-A and odd-odd nuclei in their ground state have 0,1,2 quasi-particles and differ in mass by one quasi particle energy which is approximately Δ. This is illustrated in fig.(1).

This feature is well known and reproduced in the mass formula by the term $\Delta \approx \delta \approx 12/A^{1/2}$ MeV. .

1.1.1 The Anomalous Moments of Inertia in Rotational Nuclei

Another manifestation of the pairing correlation in nuclei is the superfluid behaviour of rotational moments of inertia, which are about 60% of their expected rigid value. These moments of inertia can be related to the gap parameter Δ. An approximate closed form expression has been given by Bengtsson and Hegelsson[1]

$$\Im = \Im_{rig} \left(\frac{1}{1 + 1/x^2}\right)^{3/2}$$

where $x = \varepsilon\hbar\omega_0 / 2\Delta$, ε is the quadrupole deformation and $\hbar\omega_0$ is the single particle harmonic oscillator phonon energy.

Such an equation allows one to extract Δ from the moments of inertia, which, in turn can be extracted from the rotational spectra. This has been done by A. Macchiavelli [2] who has kindly provided fig. (2).

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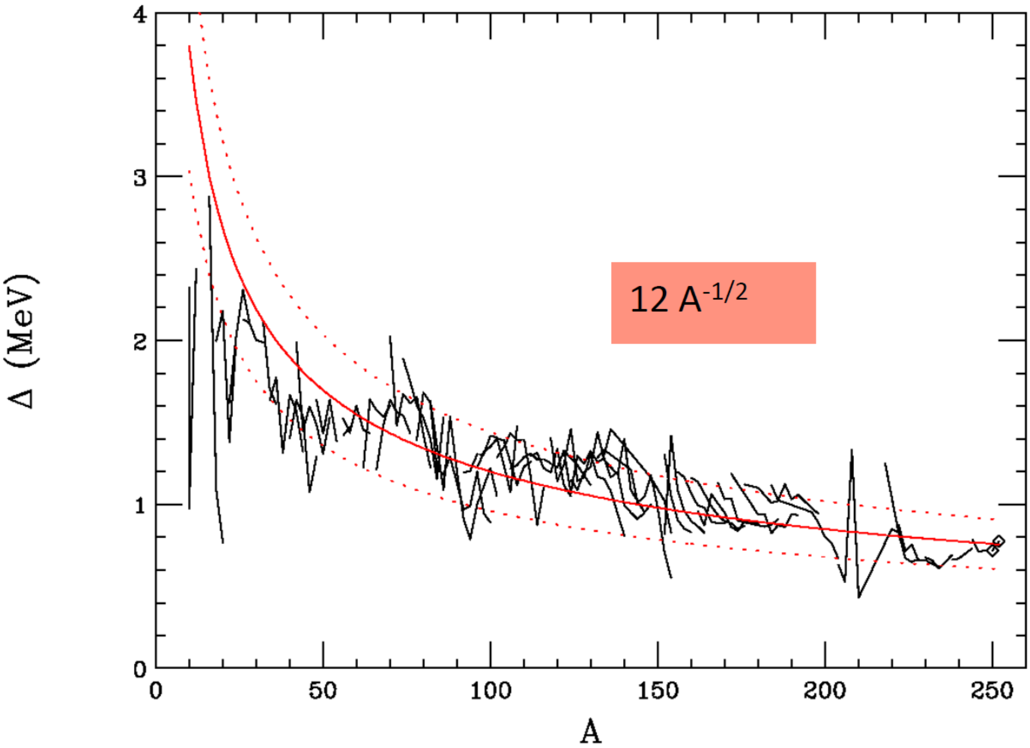

Fig.(2): Gap parameters extracted from the 2^+ states of even-even nuclei. The line represents the liquid drop values for the even-odd mass differences.

In this figure the moments of inertia, inferred for the 2^+ states of all even-even nuclei have been converted into the gap parameter Δ and compared with the even-odd mass difference $\delta \cong 12 / A^{1/2}$.

The observed remarkable agreement is most important because it shows the consistency of two very different aspects of pairing: the overall pairing correlation in the ground state, and the intrinsic quasi particle excitation of the even-odd mass differences.

1.1.2 Level Density with Pairing

The presence of pairing in nuclei away from closed shells dominates the low energy level density and its energy dependence. For the uniform model in the case of an even even nucleus, the ground state is shifted downward by an amount

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$$E_{cond.} = \frac{1}{2} g\Delta_0^2 \ ,$$

where Δ_0 is the ground state gap parameter and g is the doubly degenerate single particle level density, which is related to the single particle level density parameter a according to $a = \frac{\pi^2}{3} g$. As the temperature increases, quasi particle excitations are produced until their blocking effect leads to a decrease and eventual breakdown of the pairing correlation at the critical temperature $T_{cr} = \frac{2\Delta_0}{3.5}$.

At this temperature the nucleus reverts to a non interacting Fermi gas with its ground state shifted by an amount equal to the condensation energy.
Therefore, at T_{cr} the excitation energy is

$$E_{cr} = \frac{1}{2} g\Delta_0^2 + \frac{\pi^2}{3} gT_{cr}^2 = \frac{1}{2} g\Delta_0^2 (1 + \frac{8}{3}\frac{\pi^2}{(3.52)^2}) .$$

We can also evaluate the mean number of quasi particles Q_{cr} at the critical point[3]

$$Q_{cr} = 4gT_{cr} \ln 2 .$$

We can now calculate the mean energy cost per quasi particle:

$$\frac{E_{cr}}{Q_{cr}} \cong \frac{3.52\pi}{16 \ln 2} \Delta_0 \cong \Delta_0 .$$

This result is remarkable : it indicates that, if we consider the excitation energy as the indipendent variable, the energy cost per quasi particle is constant and the transition is “nearly” 1st order. This is in contrast with what one observes when the temperature is used as the independent variable, when a distinct 2nd order phase transition is visible. We can also calculate the entropy at the critical point:

$$S_{cr} = 2\frac{\pi^2}{3} gT_{cr}$$

and the entropy per quasi particle:

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$$\frac{S_{cr}}{Q_{cr}} = \frac{\pi^2}{6\ln 2} = 2.374$$

again, temperature independent.

Correcing for the BCS discontinuity in the specific heat of a factor 2.43

$$\frac{S'_{Cr}}{Q_{Cr}} = 2.374 - \frac{1}{2}\ln 2.43 = 1.92$$

Alternatively

$$\frac{S'_{Cr}}{Q_{Cr}} = \frac{E/Q}{T_{Cr}} = \frac{3.54\Delta}{2\Delta} = 1.75$$

in excellent agreement.

The number of the states associated with each quasi particle is then approximately constant and given by

$$N = e^{S'/Q} = 6.8\,.$$

Given that this transition is "nearly" 1st order, we infer that the level density should be "nearly" exponential:

$$\rho(E) \approx \exp\frac{E}{T}$$

where T should be "about" T_{cr}.

This low energy "linear" dependence of lnρ with energy has been observed widely over the nuclear chart. In fact, long ago Gilbert and Cameron[4], on the basis of much scantier experimental evidence and theoretical understanding, introduced a hybrid level density formula by matching a low energy linear dependence and a higher energy Fermi gas dependence . Despite the lack of theoretical justification, their model found great favor with astrophysicsts and reactor people in need of low energy level densities for their simulations. Using the present day available low energy temperatures determined with the Gilbert and Cameron recipe, it is possible to extract the gap parameter Δ_0 from the BCS equation $T_{cr} = \frac{2\Delta_0}{3.5}$.

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In fig.(3) the extracted Δ_0 is plotted as a function of A and compared with the "canonical" value $\Delta_0 = \frac{12}{A^{1/2}}$ that reproduces the even-odd mass differences.

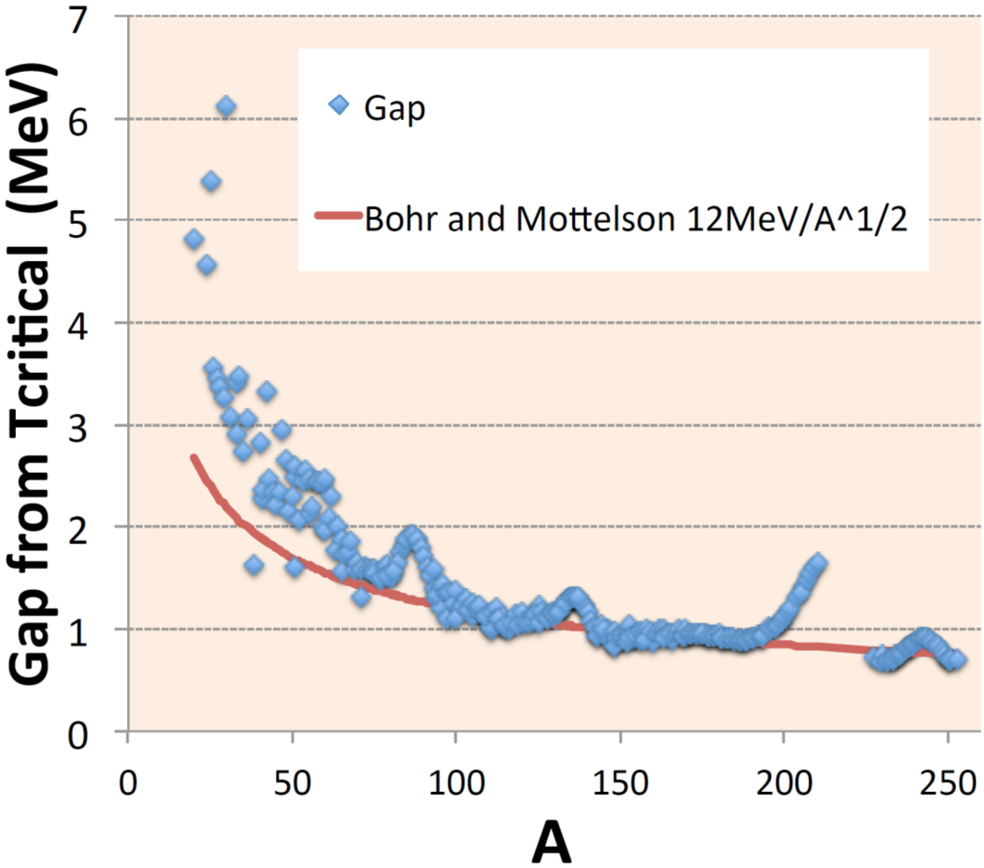

Fig.(3) Gap parameters extracted from the level density slopes compared with the even-odd mass differences.

We notice that the value of Δ extracted from the temperatures agrees in magnitude and in A dependence with the even-odd mass differences. Deviations appear, as expected, near the magic regions, where the shell effects dominate and quench the pairing correlation.
However, somewhat unexpectedly, the linearity of lnρ with E is also observed at low energy near the magic regions, so for these regions the cause of the linear behaviour must be looked for elsewhere.

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1.1.3 Spectra with any gap

As discussed above, the origin of the linear dependence is due to the constant energy cost for the production of a quasi particle and a constant entropy per quasi particle. A similar situation occurs for a magic system with a gap in the single particle spectrum. Here, the cost to promote a nucleon and thus create a quasi particle (particle, hole excitation) is constant, at least for a while. This can be illustrated with a simple model.

Let excitations (quasi particles) be created into a state of degeneracy N at the cost δ per excitation.

The excitation energy is:

$$E = n\delta$$

and the associated number of states Ω is

$$\Omega \cong N^n$$

for n<<N.

It follows that

$$S = n \ln N = \frac{E}{\delta} \ln N = \frac{E}{T}$$

where $T = \dfrac{\delta}{\ln N}$ and $\rho(E) = \exp \dfrac{E}{T}$.

Thus an exponential spectrum is expected if a gap is present irrespective of its origin.

1.1.4 Consistency between "Pairing gap" and "any gap"

The entropy per quasi particle in the pairing model is:

$$\frac{S_{cr}}{Q_{cr}} = \frac{\Delta}{T_{Cr}} = 1.75$$

For the "any gap" model we have:

$$\frac{\partial S}{\partial n} = \ln N$$

Let us put pairing Δ into δ and equate T with T_{cr}:

$$T = \frac{\Delta}{\ln N} = \frac{2\Delta}{3.5}.$$

From this we obtain: lnN=1.75 or N=5.6 in perfect agreement with the previous estimates.

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1.1.5 Even-Odd effects in level densities

In the pairing picture, an odd nucleus possesses one quasi particle in its ground state, which should control the level density at low energy. Otherwise, the odd-A nucleus should look like an even-even one except for an energy shift which should corresponds to the even-odd mass difference Δ. A simple check for this is to verify that the level densities of two adjacent nuclei overlap if an horizontal shift Δ is applied to the odd nucleus. According to the considerations made above, this *shift* Δ can be related to the level density *slope* by the expression

$$T_{\exp} \cong T_{cr} = \frac{2\Delta}{3.52}$$

The next check can be made by overlapping the two level densities by means of a vertical shift. This vertical shift ΔS should be compared with the entropy per quasi particle $\frac{S'}{Q} \cong 1.75$.

1.1.6 Conclusion

The different manifestations of pairing, the even-odd mass differences, the superfluids moments of inertia, and the low energy level densities are quantitatively consistent with the B.C.S. pairing theory.

In particular, the low energy level densities ad their energy dependence find their origin and explanation in terms of equal energy cost per quasi particle which carry a constant amount of entropy. The even-odd differences in the level densities, also find a natural explanation within this framework.

These phenomena associated with the pairing gap can be generalized to any gap arising, for instance, from shell structures.

References

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[3] L.G.Moretto, Nucl.Phys. A243 (1975) 77.
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