Beta function of three-dimensional QED

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We have carried out a Schrödinger-functional calculation for the Abelian gauge theory with $N_f = 2$ four-component fermions in three dimensions. We find no fixed point in the beta function, meaning that the theory is confining rather than conformal.

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1. Introduction

It's been some time since three-dimensional QED, or QED3, has appeared at a Lattice meeting [1]. Initial interest in the theory came from its connection to finite-temperature QCD via dimensional reduction [2]. It has since acquired a number of connections to condensed-matter systems such as the quantum Hall effect [3] and high-T_c superconductors [4]. Our path to the theory came from the fact that it presents similar issues to those of borderline-conformal gauge theories in four dimensions [5, 6]. Thus we have approached it [7, 8] with the machinery that we have applied to non-Abelian theories with fermions in assorted representations of the gauge group [9, 10, 11].

For definiteness, here is the theory's action:

$$S = \int d^3x \left(\frac{1}{4e_0^2} F_{\mu\nu} F^{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}_i \slashed{D} \psi_i \right), \tag{1.1}$$

where ψ is a massless four-component Dirac field, replicated N_f times. The question we confront is whether the IR physics of the theory is that of confinement or of conformality. What makes this theory a difficult one to study is that in three dimensions one faces severe infrared problems, leading to sensitivity to the volume that makes interpretation of lattice results [12, 13] less than straightforward [14, 15].

There is nothing non-Abelian here. All we have is charged fermions, a two-dimensional ($\log r$) Coulomb potential, and a transverse photon. The complication comes from screening by the massless charges. Does the confining potential win, or do the charges screen it? Previous work [16] shows that there are two plausible regimes:

1. For small N_f, there is confinement and mass generation for the charges, with $m \sim e^2$.

2. For large N_f, screening wins.

To explain further, let us focus on the running coupling $e^2(q)$. Since the one-loop diagram involves screening, just like QED4, we have the perturbative form

$$\frac{de^2}{d\log q} = N_f b_1 e^4/q + \cdots, \tag{1.2}$$

with $b_1 > 0$. If we define a dimensionless coupling $g^2(q) = e^2/q$, this becomes

$$\frac{dg^2}{d\log q} = -g^2 + N_f b_1 g^4 + \cdots. \tag{1.3}$$

Shades of QCD! The first term, typical of a super-renormalizable theory, drives the theory towards strong coupling in the IR, inviting a condensate $\langle \bar{\psi}\psi \rangle$ and a dynamical mass for the fermions, which therefore decouple at long distances and leave us with a logarithmic, confining potential. If N_f is large, though, the coupling only runs as far as a fixed point at $g^2 = (N_f b_1)^{-1}$. At long distance we see conformal physics, with no length scale (and no particles).

If small-N_f physics differs from large N_f, there must be a critical value N_{cr} in between. Analytical calculations have converged [15] to a value in the neighborhood of $N_{\text{cr}} = 4$. Upper bounds on N_{cr}, rather larger than this, have been derived from the F-theorem governing monotonicity in

renormalization group flows [17]. I will present our study of $N_f = 2$, which falls into line with these results.[1]

2. Calculating the β function

The Schrödinger functional method [19, 20] has been widely used to define a running coupling for QCD and QCD-like theories in four dimensions. Its outstanding feature is that it uses the finite volume of the system to define the scale at which the coupling runs. Thus in QED3, plagued by infrared difficulties, the finite volume of a lattice calculation is turned from a hindrance into a tool.

We define our theory in a three-dimensional Euclidean box of dimension L. We fix simple boundary conditions on the gauge field at $t = 0$ and L, namely $A_x = A_y = \pm \phi/L$. This amounts to imposing a uniform background field $E_x = E_y = -2\phi/L^2$. Note that L is the only scale, so that the eventual running coupling will be $g^2(L)$. The latter is derived from a calculation of the free energy $\Gamma = -\log Z$ in the presence of the background field. Comparison to the classical action gives the effective coupling via

$$\Gamma = \frac{1}{e^2(L)} \int d^3x \, \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{2.1}$$

Since the integral in Eq. (2.1) is just $\frac{1}{2} L^3 E^2 = 8\phi^2/L$, a calculation of Γ gives directly[2] the running coupling $g^2(L) = e^2(L)L$ and hence the beta function.

A one-loop calculation shows what we might look for. From Eq. (1.3) we define the beta function for $u \equiv 1/g^2$,

$$\tilde{\beta}(u) \equiv \frac{d(1/g^2)}{d\log L} = -\frac{1}{g^2} + N_f b_1 + O(g^2), \tag{2.2}$$

a straight line that crosses zero at $u = N_f b_1$—the one-loop fixed point.

3. Lattice calculation

We use a non-compact gauge field (no instantons!) with Wilson–clover fermions and nHYP smearing,

$$S = \frac{\beta}{2} \sum_{\substack{n \\ \mu < \nu}} (\nabla \times A)_{n\mu\nu}^2 + \bar{\psi} D \psi, \tag{3.1}$$

with bare coupling $\beta = 1/(e_0^2 a)$, on a lattice of dimension $L = Na$. We fix $\kappa = \kappa_c(\beta)$ to enforce masslessness. The simulation, as described above, gives $u(L)$ directly. We can compare calculations on two lattices of size L and sL, keeping (β, κ) fixed in order to keep a fixed. This gives the "rescaled" discrete beta function,

$$R(u,s) \equiv \frac{u(sL) - u(L)}{\log s}, \tag{3.2}$$

shown in Fig. 1. (R tends to the beta function $\tilde{\beta}$ as $s \to 1$.) Two sets of data are shown in the figure,

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[1]Recently the possibility has been raised [18] of a region in N_f intermediate between mass generation at small N_f and conformality at large N_f. I have nothing to say about this, except that it's interesting.

[2]More precisely, one calculates the derivative $d\Gamma/d\phi$, which is some Green function of the theory.
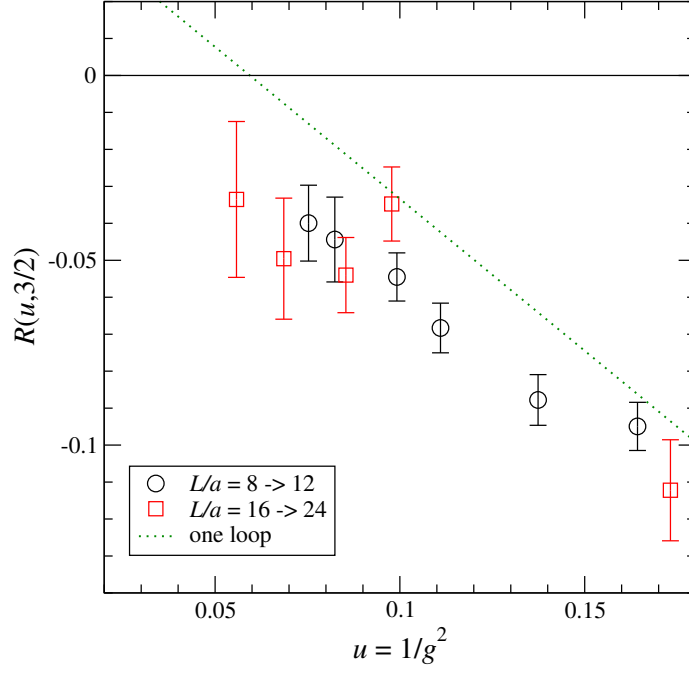
Figure 1: The discrete beta function $R(u,s)$ for scale factor $s = 3/2$.

for two different lattice sizes. Remember that $u = 1/g^2$ is the running coupling at the physical scale L. This is renormalization: Fixing u means fixing L. Increasing L/a at fixed u means that L is fixed while a is decreased. Thus the two sets of data points represent two different lattice spacings. Fig. 1 is a first look at the beta function, which apparently avoids the perturbative fixed point and levels off in strong coupling.
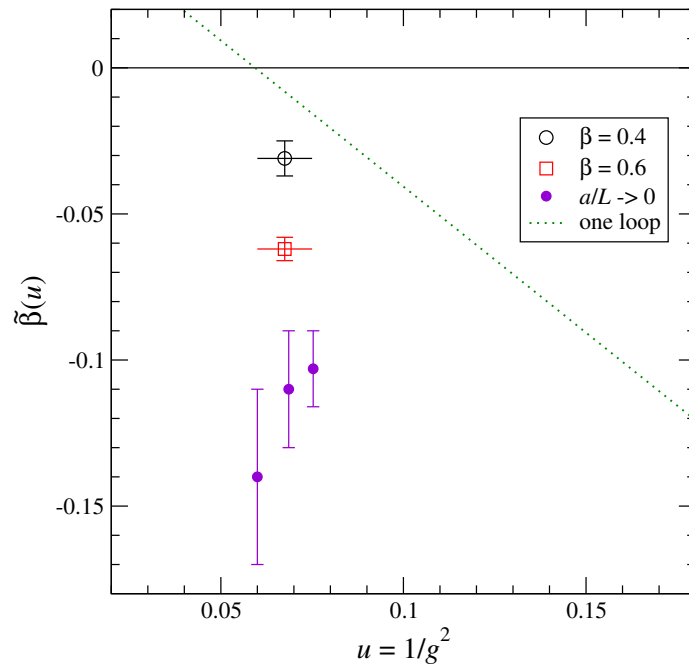
4. Continuum extrapolation

For more systematic analysis of the dependence on lattice spacing, we carry out an analysis that is close in spirit to that used in most Schrödinger functional calculations. We plot in Fig. 2 the coupling $1/g^2$ against $\log L$ for fixed bare coupling β. We are looking for a leveling off in the beta function at strong coupling. If the beta function is constant, the coupling will change by a fixed amount for each change in $\log L$ at fixed lattice spacing. Then each group of data points, at fixed β, will lie on a straight line whose slope is the beta function. We see that this works (approximately) only for the two strongest bare couplings, that is, for the bottom two groups of data points.

The horizontal lines in Fig. 2 show that at fixed g^2, which means fixed physical size L, we have two different slopes at two different bare couplings β—which means two different lattice spacings a. Thus we can extrapolate to $a/L = 0$, giving a continuum extrapolation of the slope, that is, the beta function. Fig. 3 shows this extrapolation at the strongest couplings we can reach. Again, the avoidance of the one-loop zero is clear. In fact, comparison to Fig. 1 shows that this behavior is enhanced by the continuum extrapolation. The conclusion is that QED3 with $N_f = 2$ confines.
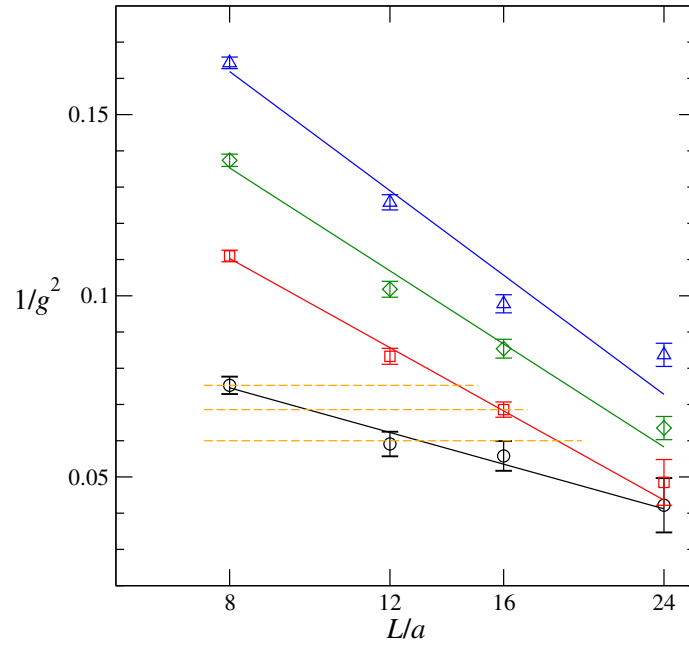
Figure 2: Running coupling vs. lattice size at fixed bare coupling β. Top to bottom: $\beta = 1.0, 0.8, 0.6, 0.4$.

Figure 3: Extrapolation of the beta function to the continuum for the three couplings marked by horizontal lines in Fig. 2.

Let me end with the comment that the one-loop beta function in this theory is very different from that of the near-conformal theories in four dimensions that we have studied in the past. Correspondingly, the results of our numerical calculations differ qualitatively as well. The slow running of the coupling in the four-dimensional theories required a rather difficult procedure of extrapolation to the continuum limit [11], and the Monte Carlo data available to us allowed only limited success. The present analysis of QED3 is more straightforward.

Acknowledgments

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