

Hadron spectra and Δ_{mix} from overlap quarks on a HISQ sea

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We present results of our continuing study on mixed-action hadron spectra and decay constants using overlap valence quarks on MILC's 2+1+1 flavor HISQ gauge configurations. This study is carried out on three lattice spacings, with charm and strange masses tuned to their physical values, and with $m_l/m_s = 1/5$. We present results of an ongoing determination of the mixed-action parameter Δ_{mix} , which enters into chiral formulae for the masses and decay constants.

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1. Introduction

There has been a resurgence of interest in heavy hadron spectroscopy, with recent discoveries of numerous hadrons with one or more heavy quarks. Including heavy quarks in lattice QCD simulations remains challenging, since for present-day simulations $am_h \ll 1$ is in general not satisfied.

We have adopted a mixed-action approach using the overlap fermion action [1] on a 2+1+1 flavor HISQ sea [2]. The overlap action is $\mathcal{O}(am)$ improved; one aim of the present study is to investigate its behavior in the regime $am \lesssim 1$. Because it maintains chiral symmetry, the analysis of many lattice observables is simplified when using the overlap action. However the generation of dynamical fermions with this action is prohibitively costly. Instead we use the 2+1+1 flavor highly-improved staggered (HISQ) configurations made available by the MILC collaboration [3].

Here we present the current status of our calculation of hadrons with charm, and we also calculate the combination $a^2(\Delta_{\text{mix}} + \Delta'_{\text{mix}})$ on the coarse MILC ensemble ($a \sim 0.12$ fm), which determines the lattice-spacing dependent shift in the mass of valence-sea pions due to using a mixed action [4], similar to the quantities Δ_r that parametrize taste-breaking. We estimate the mixed-action parameter Δ_{mix} , related to the mixed-action low energy constant C_{mix} by $\Delta_{\text{mix}} = 16C_{\text{mix}}/f^2$. A similar mixed-action approach with overlap valence on 2+1 flavor dynamical domain-wall configurations has been used by the χ QCD collaboration [5, 6],

2. Simulation details

We present results from three ensembles of 2+1+1 flavor dynamical HISQ fermions, generated by the MILC collaboration. These ensembles have extents of $48^3 \times 144$, $32^3 \times 96$, and $24^3 \times 64$ with $10/g^2 = 6.72, 6.30$ and 6.00 , respectively. In all cases the charm and strange masses are tuned to near their physical values, while m_l/m_s is fixed to $1/5$. Results on the 32^3 and 48^3 ensembles were presented in [8, 9].

We have independently determined the lattice spacings by equating the lattice determined Ω_{sss} mass with its physical value. Here the valence strange mass is determined by setting the $\bar{s}s$ mass to 685 MeV [10]. We obtained lattice spacings of 0.0582(5), 0.0877(10) and 0.1192(14) fm, respectively, for finer to coarser lattices, which are consistent with the values 0.0582(5), 0.0888(8), and 0.1207(11) fm obtained by the MILC collaboration using the r_1 parameter [3].

Valence quark propagators are computed using the overlap action. The numerical implementation follows the methods used by the χ QCD collaboration [11, 12] as discussed in [8, 9]. We use periodic/antiperiodic boundary conditions in space/time. The gauge fields are fixed to Coulomb gauge and smeared with a single HYP [13] blocking transformation. We use point-point, wall-point, and wall-wall propagators to calculate the hadron correlation functions.

The valence charm mass is tuned by setting the spin-averaged 1S state mass $(m_{\eta_c} + 3m_{J/\psi})/4$ to its physical value, using the kinetic mass obtained from the 1S dispersion relation. Using the kinetic mass we find a value of c much closer to 1 as compared to the rest mass. This is discussed further in [8, 9].

3. Charmed hadron spectrum

Figure 1 shows our results for charmonia and charmed-strange mesons at three lattice spacings.

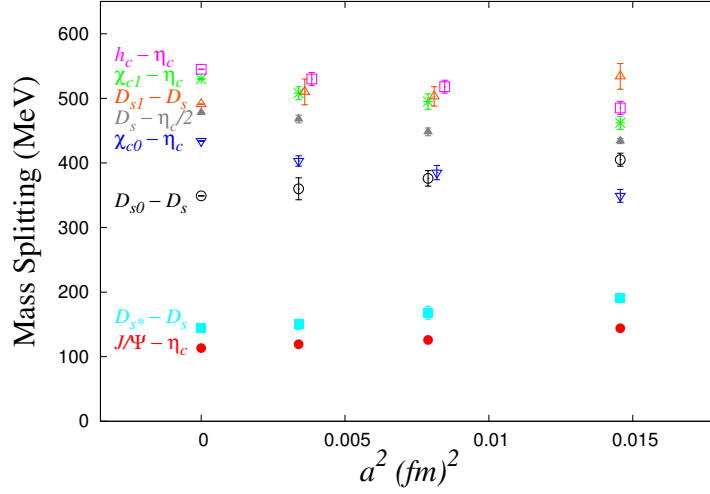


Figure 1: Meson mass splitting for charmonia and charmed-strange mesons at three lattice spacings. Experimental values are shown in the left side.

Results are presented in terms of splittings with respect to η_c and D_s mesons, obtained by fitting directly the ratios of the correlators. We have not performed any continuum or chiral extrapolation yet and the numbers at the continuum limit are taken from PDG. However, it is to be noted that there is a clear tendency of convergence of our lattice results with physical values in the continuum limit. It is expected that the discretization error will be maximum for triply-charmed baryons because of the presence of three heavy charm quarks. In Fig. 2 we plot our results for the ground state energy of the spin-3/2 triply-charmed baryon minus 3/2 times the J/Ψ mass. The 3/2 factor is included to cancel out the effect of charm quarks. Our results are shown in blue along with other lattice results. A few model results are also shown on the right. In Fig. 3(a) we show the hyperfine splitting of 1S charmonia at three lattice spacings along with its physical value. It is to be noted that the continuum limit value of this splitting for overlap fermions, utilized in this work, is approached from above. This is in contrast to the result obtained for this quantity using Wilson valence fermions where it is approached from below [14] and for HISQ fermions where it is approached from above for coarser lattices and from below for finer lattices [15]. We have not performed continuum and chiral extrapolation here though a naive fit with a form $\delta_{\text{phys}} = A + Ba^2$ gives a value 110(4) MeV which is consistent with the physical value. In Fig. 3(b) we show mass splittings $m_{\Omega_{ccc}} - \frac{3}{2}m_{J/\Psi}$ at three lattice spacings. Again a naive fit with a form $\delta_{\text{phys}} = A + Ba^2$ yields a value of this splitting as 148(10) MeV. This splitting should be comparable to the binding energy of the yet to be discovered spin-3/2 triply charmed baryon.

4. Δ_{mix} for overlap on HISQ

The low energy properties of a simulation employing different sea and valence actions can be described near the chiral limit using mixed-action chiral perturbation theory (MA χ PT) [22]. This formalism extends the usual χ PT description by terms that are proportional to new low-energy constants, and vanish in the continuum limit. For staggered simulations, χ PT is extended by

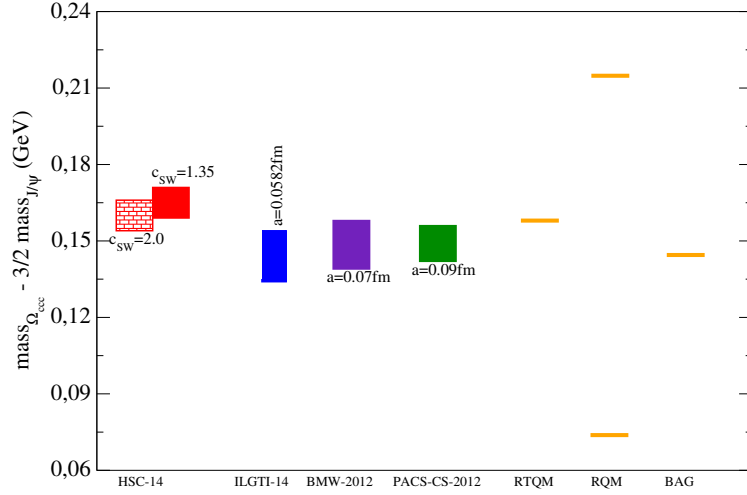


Figure 2: The mass splitting $m_{\Omega_{ccc}} - \frac{3}{2}m_{J/\Psi}$ along with other lattice and various model results. Result from this work is shown in blue color. References in the plot are (from left to right) HSC-14 [16], ILGTI-14 this work, BMW-2012 [17], PACS-CS-2012 [18], RTQM [19], RQM [20], BAG [21].

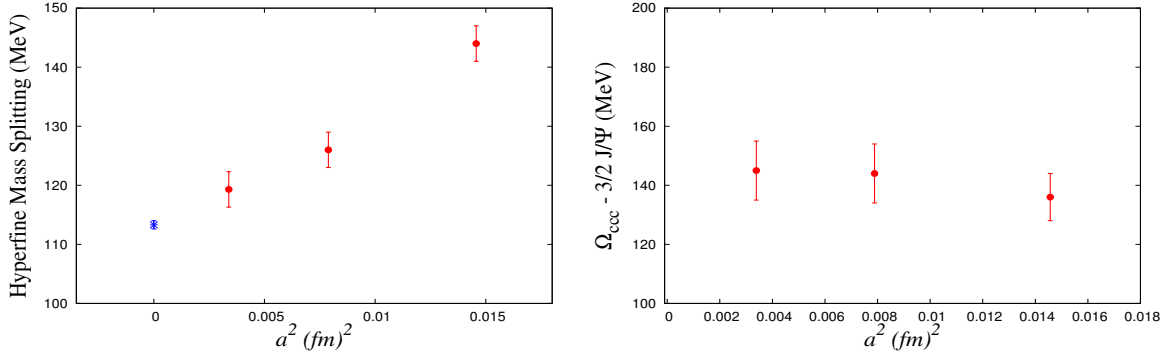


Figure 3: (a) Hyperfine mass splitting of 1S charmonia at three lattice spacings (red circles) along with its physical value (blue star). (b) The mass splitting $m_{\Omega_{ccc}} - \frac{3}{2}m_{J/\Psi}$ at three lattice spacings.

terms describing taste-breaking discretization effects, yielding staggered chiral perturbation theory ($S\chi$ PT) [23]. For chiral valence fermions on a staggered sea, it has been shown that only one new low-energy constant, C_{mix} , appears at leading order in addition to those arising in $S\chi$ PT [24]. Here we estimate the parameter $\Delta_{\text{mix}} = 16C_{\text{mix}}/f^2$ for overlap fermions on the coarse HISQ ensembles.

Several studies in recent years have studied the size of these effects in the context of staggered sea fermions. Domain-wall valence fermions on the MILC collaboration's asqtad [25] ensembles were studied in [26, 27]. In Refs. [4, 27] it was pointed out that the quantity $a^2(\Delta_{\text{mix}} + \Delta'_{\text{mix}})$ is comparable in magnitude to the size of mass splittings between pions of different tastes. One of the primary advantages of the HISQ action is the reduced taste-symmetry violations [3]. We expect a comparable reduction in this quantity when using chiral fermions on the HISQ ensembles, and find this to be the case.

At the leading order in MA(S) χ PT, the masses of pions constructed from valence (v) and sea (s) action propagators are given by

$$m_{vv'}^2 = B_{\text{ov}}(m_v + m_{v'}) \quad (4.1)$$

$$m_{ss'}^2 = B_{\text{HISQ}}(m_s + m_{s'}) + a^2 \Delta_t \quad (4.2)$$

$$m_{vs}^2 = B_{\text{ov}}m_v + B_{\text{HISQ}}m_s + a^2(\Delta_{\text{mix}} + \Delta'_{\text{mix}}). \quad (4.3)$$

Eq. (4.2) gives the well-known splitting of different taste pions in terms of Δ_t , while pions constructed from one valence-action propagator and one sea-action propagator on the sea-action ensemble are sensitive to $\Delta_{\text{mix}} + \Delta'_{\text{mix}}$. Δ_{mix} is related to the MA χ PT LEC by $\Delta_{\text{mix}} = 16C_{\text{mix}}/f^2$, while Δ'_{mix} is given in terms of staggered taste splittings [4].

Different strategies for extracting Δ_{mix} from Eqs. (4.1)–(4.3) have been proposed in the literature. Here we adopt the same technique as in [27, 7], fitting the quantity

$$\delta m^2(m_v) \equiv m_{vs}^2 - m_{ss}^2/2 = B_{\text{ov}}m_v + a^2(\Delta_{\text{mix}} + \Delta'_{\text{mix}}) \quad (4.4)$$

as a function of valence quark mass. This is convenient since it is the valence propagators that are used in e.g. spectroscopy calculations.

The mixed-meson correlation functions are constructed using one overlap propagator and one *Wilsonized* staggered propagator. The Wilsonized propagator G_{ψ_s} is given in terms of the staggered propagator G_χ by [28]

$$G_{\psi_s}(x, y) = \Omega(x)\Omega^\dagger(y) \times G_\chi(x, y), \quad (4.5)$$

where $\Omega(x)$ is the Kawamoto-Smit transformation

$$\Omega(x) = \prod_{\mu} (\gamma_{\mu})^{x_{\mu}}. \quad (4.6)$$

Mixed-meson correlators are fit to the form

$$C_{vs}^{\Gamma}(t) \sim [A + (-1)^t B] \cosh(m_{vs}(t - T/2)). \quad (4.7)$$

Figure 4 shows δm^2 vs. m_v (in units of r_1^{-1}), for a variety of valence quark masses and at two different values of sea mass m_s , on the coarse HISQ ensemble $a = 0.121$ fm. The data is consistent with a straight line and insensitive to the HISQ sea mass m_s , indicating that Eqs. (4.1)–(4.3) are valid for the ranges of quark masses used. From the intercept of this data we find

$$r_1^2 a^2 (\Delta_{\text{mix}} + \Delta'_{\text{mix}}) = 0.104(9), \quad (4.8)$$

and combining this with $a^2 \Delta'_{\text{mix}}$ as determined from the known taste splittings [3], we find

$$a^2 \Delta_{\text{mix}} \simeq (140 \text{ MeV})^2 \quad (a = 0.121 \text{ fm}). \quad (4.9)$$

In order to convert this into a continuum determination of Δ_{mix} , the calculation needs to be repeated at finer lattice spacings and the results extrapolated to $a = 0$.

In [27] it was pointed out that the mixed-meson mass shift for domain-wall on asqtad is comparable in size to the pion taste splittings. Taste splittings for the HISQ action are reduced by a factor $\gtrsim 3$ relative to asqtad. We find that for overlap on HISQ, the shift is comparable to the HISQ taste splitting, and smaller than the asqtad taste splittings by around a factor of two.

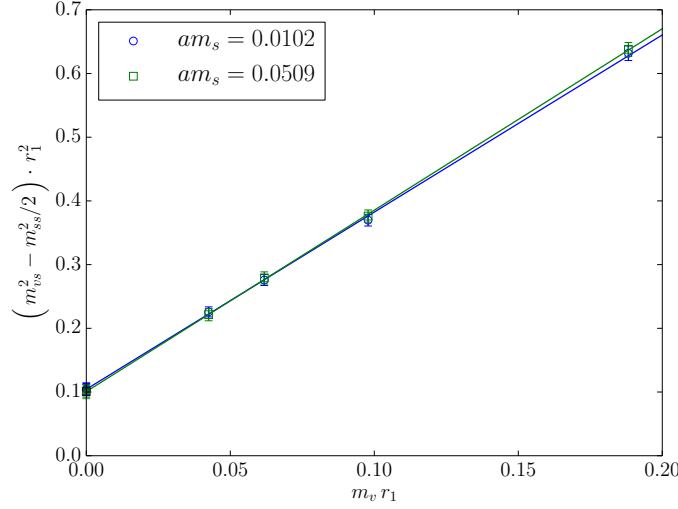


Figure 4: δm^2 vs. m_v in units of r_1^{-1} , for two different values of the HISQ sea mass.

5. Conclusions

We have presented an update of our results concerning charmed meson and baryon spectroscopy using overlap valence fermions on the HISQ ensembles made available by the MILC collaboration. Such a mixed-action approach is attractive in that one gains the advantages of the overlap Dirac operator in the valence sector while avoiding the extreme cost of ensemble generation using such an operator. One sensitive issue is whether simulating charm directly in such a setup discretization errors can be under control. Our studies of charm-meson dispersion relations employing the kinetic mass indicate that this setup is suitable for charm spectroscopy at the current lattice spacings.

The combination $a^2(\Delta_{\text{mix}} + \Delta'_{\text{mix}})$ provides a measure of mixed-action effects in the chiral regime. We determined this combination for overlap fermions on a HISQ sea at a single lattice spacing, finding it comparable in magnitude to the pion taste-splittings, and about half as large as was found for simulations of domain-wall on asqtad. This calculation needs to be repeated at finer lattice spacings to find a continuum result for Δ_{mix} .

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