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Searching for the $X(3872)$ and $Z_c^+(3900)$ on HISQ lattices

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We present preliminary simulation results for the $I = 0$ charmonium state $X(3872)(1^{++})$ and the $I = 1$ charmonium state $Z_c^+(3900)(1^{+-})$. The study is performed on gauge field configurations with 2+1+1 flavors of highly improved staggered sea quarks (HISQ) with clover (Fermilab interpretation) charm quarks and HISQ light valencee quarks. Since the $X(3872)$ lies very close to the open charm $D\bar{D}^*$ threshold, we use a combination of $\bar{c}c$ and $D\bar{D}^* + \bar{D}D^*$ interpolating operators. For the $Z_c^+(3900)$ we use a combination of $J/\psi\pi$ and $D\bar{D}^* + \bar{D}D^*$ channels. This is the first such study with HISQ sea quarks and light valence quarks. To this end, we describe a variational method for treating staggered quarks that incorporates both oscillating and non-oscillating components.

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1. Introduction

In the past decade, many excited charmonium states have been discovered that cannot be explained within the conventional quark model. Among these states, the narrow charmonium-like state $X(3872)$ and charged charmonium-like state $Z_c^+(3900)$ have attracted special attention due to the closeness of the $D\bar{D}^*$ threshold and their possible four-quark nature.

The $X(3872)$ state with $J^{\rm PC} = 1^{++}$ is one of the better established mysterious charmonium states found in B-meson decays by both Belle [1, 2] and CDF [3] and studied with more precison by CDF [4], D0 [5], BABAR [6, 7], Belle and LHCb [8, 9]. Its mass is remarkably close to the $D^0\bar{D}^{*0}$ threshold – within 1 MeV. The $Z_c^+(3900)$ is a charged, isospin one charmonium-like structure observed by the BESIII collaboration [10] as an intermediate resonance in an analysis of e^+e^- annihilation into $J/\psi\pi^+\pi^-$ at $\sqrt{s} = 4260$ MeV. This observation has been confirmed by the Belle Collaboration [11] and by Xiao *et al.* using data from the CLEO-c detector [12]. However, it has not been observed in exclusive photoproduction of $J/\psi,\pi$ on protons [13] or in conjunction with B_0 decays [14, 15]. As a charged charmonium-like structure, it must contain at least four quarks, and tetraquark and molecular interpretations have been suggested. See, for example, [16, 17] and [18].

Previous lattice studies provide theoretical support for the $X(3872)$ [19] but not the $Z_c^+(3900)$ [20, 21, 22]. Those studies were carried out on small volumes with unphysically heavy up and down quarks. Our ultimate objective is to increase the volume and work at physical values of all quarks. To this end the needed gauge field ensembles with highly improved staggered quarks (HISQ) are available [24]. We report here on a preparatory study, albeit still on a small volume with unphysically heavy up and down quarks, using the HISQ formulation for the light quarks and clover (Fermilab interpretation [23]) for the charm quark.

2. Methodology

We work with the MILC ensemble with lattice spacing approximately 0.15 fm and the lattice dimension $16^3 \times 48$, generated in the presence of highly improved staggered sea quarks (HISQ). The ensemble contains degenerate up and down sea quarks with masses approximately 1/5 the mass of the strange quark and with strange and charm sea quark masses at their physical values [24].

As mentioned above, we use clover charm quarks within the Fermilab interpretation and HISQ light valence quarks with masses matching the sea quarks. To study the $X(3872)$ with $J^{\rm PC} = 1^{++}$, we choose interpolating operators $\mathscr{O}_i$ that couple to $\bar{c}c$ as well as $D\bar{D}^* + \bar{D}D^*$ scattering states. (We use abbreviations cc and DD^* below.)

- cc interpolators ($J^{\rm PC} = 1^{++}, I = 0$)

$$\bar{c}\gamma_5\gamma_i c, \quad \bar{c}\Delta\gamma_5\gamma_i\Delta c, \quad \bar{c}\nabla_k\gamma_5\gamma_i\nabla_k c, \quad \bar{c}\varepsilon_{ijk}\gamma_j\nabla_k c, \quad \bar{c}\varepsilon_{ijk}\gamma_4\gamma_j\nabla_k c, \quad \bar{c}\left|\varepsilon_{ijk}\right|\gamma_5\gamma_j\mathscr{D}_k c\,.$$

- DD^* interpolators ($J^{\rm PC} = 1^{++}, I = 0, 1$)

$$\begin{aligned}
(DD)(t,\mathbf{p}=\mathbf{0}) \quad &: \quad [D^*(t,\mathbf{0})\bar{D}(t,\mathbf{0}) - \bar{D}^*(t,\mathbf{0})D(t,\mathbf{0})] + f_I\,\{u \leftrightarrow d\} \\
(DD)(t,\mathbf{p}=\mathbf{1}) \quad &: \quad [D^*(t,-\mathbf{1})\bar{D}(t,\mathbf{1}) - \bar{D}^*(t,\mathbf{1})D(t,-\mathbf{1})] \\
&\quad\;\; + D^*(t,\mathbf{1})\bar{D}(t,-\mathbf{1}) - \bar{D}^*(t,-\mathbf{1})D(t,\mathbf{1})] + f_I\,\{u \leftrightarrow d\}
\end{aligned}$$

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where ∇_k is a discrete covariant difference, $\mathscr{B}_k = \varepsilon_{ijk}\nabla_i\nabla_j$, $\mathscr{D}_k = |\varepsilon_{ijk}|\nabla_i\nabla_j$, $\Delta = \nabla_k \cdot \nabla_k$, $f_I = +1$ for $I = 0$ and $f_I = -1$ for $I = 1$. On the other hand, for the $Z_c^+(3900)$ we use the interpolating operators $\mathscr{O}_i$ that couple to both $cc\pi$ and DD^* scattering states with quantum number $J^{\mathrm{PC}} = 1^{+-}$ and $I = 1$

- cc interpolators ($J^{\mathrm{PC}} = 1^{--}, I = 0$)

$$\bar{c}\gamma_i c, \quad \bar{c}\gamma_4\gamma_i\Delta c, \quad \bar{c}\nabla_i c, \quad \bar{c}\varepsilon_{ijk}\gamma_5\gamma_j\nabla_k c, \quad \bar{c}\gamma_5\mathscr{B}_k c, \quad \bar{c}\gamma_4\gamma_5\mathscr{B}_k c\,.$$

- $cc\pi$ interpolators ($J^{\mathrm{PC}} = 1^{+-}, I = 1$)

$$\begin{array}{rcl} (cc,\pi)(t,\mathbf{p}=\mathbf{0}) & : & cc(t,\mathbf{0})\pi(t,\mathbf{0}) \\ (cc,\pi)(t,\mathbf{p}=\mathbf{1}) & : & cc(t,-\mathbf{1})\pi(t,\mathbf{1}) \\ & & cc(t,-\mathbf{1})\pi(t,\mathbf{1}) + cc(t,\mathbf{1})\pi(t,-\mathbf{1}) \end{array}$$

- DD^* interpolators ($J^{\mathrm{PC}} = 1^{+-}, I = 1$)

$$\begin{array}{rcl} (DD)(t,\mathbf{p}=\mathbf{0}) & : & [D^*(t,\mathbf{0})\bar{D}(t,\mathbf{0}) + \bar{D}^*(t,\mathbf{0})D(t,\mathbf{0})] - \{u \leftrightarrow d\} \\ (DD)(t,\mathbf{p}=\mathbf{1}) & : & [D^*(t,-\mathbf{1})\bar{D}(t,\mathbf{1}) + \bar{D}^*(t,\mathbf{1})D(t,-\mathbf{1})] \\ & & +D^*(t,\mathbf{1})\bar{D}(t,-\mathbf{1}) + \bar{D}^*(t,-\mathbf{1})D(t,\mathbf{1})] - \{u \leftrightarrow d\} \end{array}$$

Each charmed meson interpolating operator is given by

$$D(t,\mathbf{p}) = \sum_x e^{i\mathbf{p}\cdot\mathbf{x}}\bar{u}(\mathbf{x},t)\gamma_5 c(\mathbf{x},t)\ ,\ D^*(t,\mathbf{p}) = \sum_x e^{i\mathbf{p}\cdot\mathbf{x}}\bar{u}(\mathbf{x},t)\gamma_i c(\mathbf{x},t) \tag{2.1}$$

and stochastic and smeared-stochastic sources are used throughout.

3. Staggered variational method

To extract the discrete energy spectrum E_n of the various scattering states, we use a variational approach [25, 26, 27]. The extension to staggered quarks is described in [28]. When the hadronic correlator involves staggered fermions, the multi-exponential expansion of the correlator includes terms that oscillate in time:

$$C_{ij}(t) = \langle \mathscr{O}_i(0)\mathscr{O}_j(t)\rangle = \sum_n s_n(t) Z_{in} Z_{jn}^* \frac{\exp(-E_n t)}{2E_n}\,, \tag{3.1}$$

where $s_n(t) = 1$ or $-(-)^t$ for nonoscillating and oscillating states. In terms of a pseudo-transfer matrix T with eigenvalues $\pm\exp(-E_n)$

$$C(t) = ZT^t g(2M)^{-1}Z^\dagger\,, \tag{3.2}$$

where g is diagonal with $g_{nn} = 1$ for nonoscillating and -1 for oscillating states, and M is a diagonal matrix with $M_{nn} = E_n$. We obtain the generalized eigenvalue problem:

$$C(t)V = T^{t-t_0}C(t_0)V\,, \tag{3.3}$$

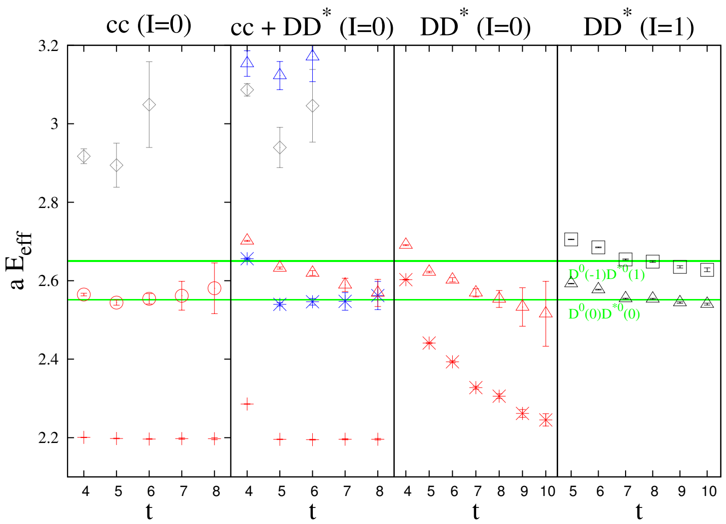

Figure 1: Effective masses in lattice units from the lowest few eigenvalues in the $X(3872)$ study. Each panel shows the result of including a different set of interpolating operators. The green lines correspond to the energies of non-interacting $\bar{D}(\mathbf{p})D(-\mathbf{p})$ scattering states. The lower one represents $\bar{D}(\mathbf{0})D(-\mathbf{0})$ and upper, $\bar{D}(\mathbf{1})D(-\mathbf{1})$. The symbols represent effective masses for different sets of interpolating operators, panel (a): cc set only, (b): combining cc and DD^*, (c): DD^* with isospin 0, and (d): DD^* with isospin 1.

where the eigenvector $V = Z^{\dagger -1}$. With a sufficiently complete interpolating operator basis and a high reference time t_0, we get the eigenvalues,

$$\lambda_n(t,t_0) = s_n(t)\exp[-E_n(t-t_0)]\,. \tag{3.4}$$

However, in practice, if the basis is not sufficiently complete and t_0 is not sufficiently high, $\lambda_n(t,t_0)$ receives contribution from higher states and often from opposite parity states, so we fit to

$$\begin{aligned}\lambda_n(t,t_0) \approx\ & [1-a_n(t_0)]s_n(t-t_0)e^{-E_n(t-t_0)} + b_n(t_0)s_n(t-t_0)e^{-\bar{E}_n(t-t_0)} + \\ & + c_n(t_0)s_n'(t-t_0)e^{-E_n'(t-t_0)} + d_n(t_0)s_n'(t-t_0)e^{-\bar{E}_n'(t-t_0)}\,.\end{aligned} \tag{3.5}$$

where s_n' oscillates if s_n does not, or vice versa.

4. Results

4.1 $X(3872)$

The resultant effective masses and spectrum in this preliminary study are shown in Fig. 1 and Fig. 2, respectively. In the isotriplet channel we do not find a candidate for the $X(3872)$. The levels observed are apparently only discrete scattering state of DD^* which inevitably appear on the lattice. Any isotriplet character for the $X(3872)$ would presumably arise after breaking the degeneracy of the up and down quarks.

The isosinglet channel includes mixing with the $\bar{c}c$ states. Our choices of quark masses resulted in a degeneracy between the unmixed $\chi_{c1}(2P)$ and the unmixed DD^* threshold. With all

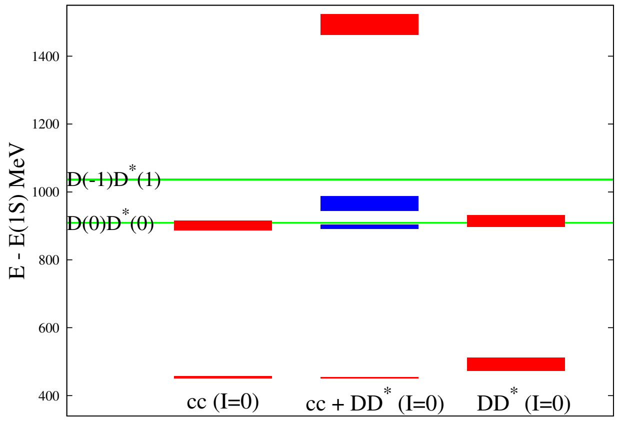

Figure 2: Energy splittings between E_n and $\overline{1S} = \frac{1}{4}(M_{\eta_c} + 3M_{J/\psi})$, the spin-averaged $1S$ charmonium masses. The towers of states are from the same operator bases as the first three panels in Fig. 1. Left: the separate $\chi_{c1}(1P)$ and $\chi_{c1}(2P)$ states from cc operators. Middle: combined cc and DD^* operators. Right: states from the DD^* $I = 0$ operators. The lower blue bar represents the $X(3872)$ candidate.

Table 1: Energy levels for the $cc + DD^*$ operator set. The level e_1 (lower blue bar in Fig. 2) corresponds to the $X(3872)$ candidate with a splitting of $13(6)$ MeV relative to the DD^* threshold with our unphysical lattice parameters.

		$E_n - \overline{1S}$ (MeV)
Non-interacting	$\bar{D}(\mathbf{0})D(\mathbf{0})$	910(2)
	$\bar{D}(\mathbf{1})D(-\mathbf{1})$	1036(3)
Interacting	e_0	452(2)
	e_1	897(6)
	e_2	966(21)
	e_3	1494(30)

interpolating operators included, level repulsion results in the weakly bound state represented by the lower blue bar, our candidate $X(3872)$. The upper blue bar can be interpreted as a scattering state shifted up due to the large negative scattering length. This shallow bound state scenario on the lattice has been confirmed in deuteron studies [29, 30]. Our results agree qualitatively with those of the pioneering lattice studies of the $X(3872)$ by Prelovsek and Leskovec [19] using clover valence and sea quarks throughout.

4.2 $Z_c^+(3900)$

Figure 3 shows the energy splittings in the various 1^{+-} channels. The mixing is evidently too weak to produce a state distinct from the noninteracting scattering states, in agreement with [20, 22].

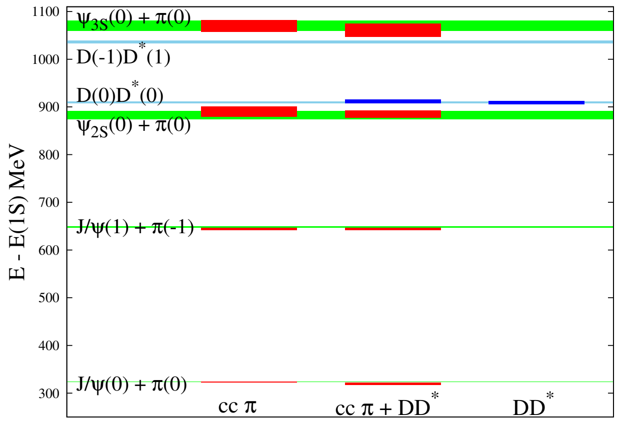

Figure 3: Same as Fig. 2, but for three choices for the 1^{+-} operator basis proposed for the $Z_c^+(3900)$. Left tower: $cc\pi$ operators only, middle tower: combined operators $cc\pi + DD^*$ and right tower: DD^* operators only. The horizontal green lines represent energy levels of the non-interacting $cc\pi$ states and the horizontal blue lines, DD^*.

5. Conclusions and Outlook

In this exploratory study, we find a candidate $X(3872)$ state with an energy level 13(6) MeV below the DD^* threshold in the $cc + DD^*$, $I = 0$ operator set. Since the rms separation of the D and D^* mesons could be quite large ($\sim$ 6 fm) [31], we intend to repeat the calculation on a larger lattice with physical light quark masses. We were unable to observe a candidate $Z_c^+(3900)$ state, although future calculations with a larger interpolating operator basis may be able to resolve this state.

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References

[1] S. K. Choi *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003) [arXiv:hep-ex/0309032].

[2] I. Adachi *et al.* [Belle Collaboration], [arXiv:0809.1224 [hep-ex]].

[3] D. Acosta *et al.* [CDF Collaboration], Phys. Rev. Lett. **93**, 072001 (2004) [arXiv:hep-ex/0312021].

[4] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **103**, 152001 (2009) [arXiv:0906.5218 [hep-ex]].

[5] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **93**, 162002 (2004) [arXiv:hep-ex/0405004].

[6] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **71**, 071103 (2005) [arXiv:hep-ex/0406022].

[7] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. D **77**, 111101 (2008) [arXiv:0803.2838].

[8] R. Aaij *et al.* [LHCb Collaboration], Eur. Phys. J. C **72**, 1972 (2012) [arXiv:1112.5310 [hep-ex]].

[9] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **110**, 222001 (2013) [arXiv:1302.6269 [hep-ex]].

[10] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **110**, 252001 (2013) [arXiv:1303.5949 [hep-ex]].

[11] Z. Q. Liu *et al.* [Belle Collaboration], Phys. Rev. Lett. **110**, 252002 (2013) [arXiv:1304.0121 [hep-ex]].

[12] T. Xiao, S. Dobbs, A. Tomaradze and K. K. Seth, Phys. Lett. B **727**, 366 (2013) [arXiv:1304.3036 [hep-ex]].

[13] C. Adolph *et al.* [COMPASS Collaboration], [arXiv:1407.6186 [hep-ex]].

[14] K. Chilikin *et al.* [Belle Collaboration], [arXiv:1408.6457[hep-ex]].

[15] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. D **90**, 012003 (2014) [arXiv:1404.5673 [hep-ex]].

[16] E. Braaten, Phys. Rev. Lett. **111**, 162003 (2013) [arXiv:1305.6905 [hep-ph]].

[17] E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. D **90**, 014044 (2014) [arXiv:1402.0438 [hep-ph]].

[18] E. Wilbring, H.-W. Hammer and U.-G. Meissner, Phys. Lett. B **726**, 326 (2013) [arXiv:1304.2882 [hep-ph]].

[19] S. Prelovsek and L. Leskovec, Phys. Rev. Lett. **111**, 192001 (2013) [arXiv:1307.5172 [hep-lat]].

[20] S. Prelovsek and L. Leskovec, Phys. Lett. B **727**, 172 (2013) [arXiv:1308.2097 [hep-lat]].

[21] Y. Chen, M. Gong, Y. H. Lei, N. Li, J. Liang, C. Liu, H. Liu and J. L. Liu *et al.*, Phys. Rev. D **89**, 094506 (2014) [arXiv:1403.1318 [hep-lat]].

[22] S. Prelovsek, C. B. Lang, L. Leskovec and D. Mohler, [arXiv:1405.7623 [hep-lat]].

[23] A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, Phys. Rev. D **55**, 3933 (1997) [hep-lat/9604004].

[24] A. Bazavov *et al.* [MILC Collaboration], Phys. Rev. D **82**, 074501 (2010) [arXiv:1004.0342 [hep-lat]].

[25] C. Michael, Nucl. Phys. B **259**, 58 (1985).

[26] M. Lüscher and U. Wolff, Nucl. Phys. B **339**, 222 (1990).

[27] B. Blossier, M. Della Morte, G. von Hippel, T. Mendes and R. Sommer, JHEP **0904**, 094 (2009) [arXiv:0902.1265 [hep-lat]].

[28] Song-Haeng Lee and Carleton DeTar, in preparation (2014).

[29] T. Yamazaki *et al.* [PACS-CS Collaboration], Phys. Rev. D **84**, 054506 (2011) [arXiv:1105.1418 [hep-lat]].

[30] S. R. Beane *et al.* [NPLQCD Collaboration], Phys. Rev. C **88**, no. 2, 024003 (2013) [arXiv:1301.5790 [hep-lat]].

[31] E. Braaten, PoS EFT **09**, 065 (2009).