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Gluonic Correlations around Deconfinement

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We investigate the critical behavior of electric and magnetic gluon propagators around the deconfinement temperature of SU(2) gauge theory on large lattices, employing Landau gauge. In particular, we discuss the possibility that the electric gluon propagator may signal the deconfinement transition.

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1. Introduction

Gluonic correlations of Yang-Mills theory at nonzero temperature are encoded in the chromo-electric (i.e. longitudinal) sector of the gluon propagator, which is a gauge-dependent quantity. At high temperatures, deconfinement should be felt in the electric gluon propagator as an exponential fall-off at long distances, defining a screening length and conversely a screening mass [1]. As for the magnetic (i.e. transverse) sector, the dimensional-reduction picture (based on the 3D-adjoint-Higgs model) suggests a confined magnetic gluon, associated to a nontrivial magnetic mass.

At lower temperatures, one might expect to observe some characteristic response of the electric gluon propagator to the confinement transition, allowing important physical information to be extracted from it. Of course, it is not clear if and how a screening mass would manifest itself around the critical temperature T_c. At the same time, studies of the gluon propagator at zero-temperature have shown a (dynamical) mass (see e.g. [2]). One can try to use this knowledge to define temperature-dependent masses for the region around T_c.

Lattice studies of the Landau-gauge gluon propagator around the deconfinement phase transition in pure $SU(2)$ and $SU(3)$ theory, as well as considering dynamical quarks, have been presented in [3, 4, 5, 6, 7, 8, 9, 10]. Our SU(2) study has been reported in [11, 12, 13, 14, 15, 16]. Let us discuss general findings of these studies.

In the transverse sector, one sees strong infrared suppression of the propagator, with a turning point of the curve described by the momentum-space magnetic propagator $D_T(p^2)$ for momenta p around 400 MeV. This suppression seems even more pronounced than in the zero-temperature case, reviewed in [17]. Also, $D_T(p^2)$ shows considerable finite-physical-size effects in the infrared limit, as observed for $T=0$. Furthermore, just as for $T=0$, the magnetic propagator displays a clear violation of reflection positivity in real space. Essentially these same features are seen for $D_T(p^2)$ at all nonzero temperatures considered.

The longitudinal propagator $D_L(p^2)$, on the other hand, shows significantly different behavior for different temperatures. As soon as a nonzero temperature is introduced in the system, $D_L(p^2)$ increases considerably (whereas $D_T(p^2)$ decreases monotonically). More precisely, for all fixed temperatures, the curve described by $D_L(p^2)$ seems to reach a plateau in the low-momentum region (see e.g. [12]). As the temperature is increased, this plateau increases slightly until, approaching the phase transition from below, it has been observed to rise further and then, just above the transition temperature, to drop sharply. This has been interpreted as a sign of singular behavior of the longitudinal gluon propagator around T_c and, in fact, it has been related to several proposals of a new order parameter for the deconfinement transition. (Of course, a relevant question is, then, whether this singularity survives the inclusion of dynamical quarks in the theory [8, 9].)

We note that, at all investigated temperatures, the infrared plateau just described is not long enough to justify a fit to the Yukawa form

$$D_L(p^2) \,=\, C\,\frac{1}{p^2+m^2}\,, \tag{1.1}$$

predicted at high temperatures. If this were the case, $D_L(0)^{-1/2}$ would provide a natural (temperature-dependent) mass scale. Note that this value depends also on the global constant C. On the other hand, the so-called Gribov-Stingl forms involve complex-conjugate poles, defining real and imaginary masses (independently of C).

Concerning the longitudinal propagator in real space (see e.g. [13]), positivity violation is observed unequivocally only at zero temperature and for a few cases around the critical region, in association with the severe systematic errors discussed below. For all other cases, there is no violation within errors. Also, we always observe an oscillatory behavior, indicative of a complex-mass pole.

In the present study, instead of focusing on the momentum dependence of $D_L(p^2)$, we will look at the value of $D_L(0)$ (after normalization by C) as a function of T. This quantity has the disadvantage that it does not contain information on the length of the plateau and it also has large errors, but it is very sensitive to the temperature. Let us mention that data (and preliminary fits) for $D_L(p^2)$ can be seen e.g. in [13]. In the next section, we discuss our results for the infrared values of $D_L(p^2)$.

2. Results

We have considered the pure SU(2) case, with a standard Wilson action and the largest lattice sizes to date: $N_s^3 \times N_t$ ranging from $48^3 \times 4$ to $192^3 \times 16$. For our runs we employ a cold start, performing a projection on positive-Polyakov-loop configurations. Also, gauge fixing is implemented using stochastic overrelaxation. The gluon dressing functions are normalized to 1 at 2 GeV. We considered several values of the lattice parameter β, allowing a broad range of temperatures. Our procedure for determining the physical temperature T is described in [12]. The momentum-space expressions for the transverse and longitudinal gluon propagators $D_T(p^2)$ and $D_L(p^2)$ can be found e.g. in [3].

As can be seen from the data in [13], the longitudinal (electric) propagator $D_L(p^2)$ displays severe systematic effects around T_c for the smaller values of N_t. These effects are strongest at temporal extent $N_t = 4$ and large values of N_s. We note that the systematic errors for small N_t come from two different sources: "pure" small-N_t effects (associated with discretization errors) and strong dependence on the spatial lattice size N_s at fixed N_t, for the cases in which the value of N_t is smaller than 16. The latter effect was observed only at temperatures slightly below T_c, whereas the former is present in a wider range of temperatures around T_c. In particular, the finite-spatial-volume effects for $D_L(p^2)$ at $N_t = 4$ are strongest at T_c, but are still very large at $T = 0.98T_c$ and are much less pronounced for $T = 1.01T_c$.

In Fig. 1 we show data for $D_L(0)$ as a function of the temperature T, for temperatures around the critical value T_c. We show such values as obtained from all our runs, grouping together (with the same symbol) the runs performed at the same temporal extent N_t. We remark that, as said above, not all curves of $D_L(p^2)$ reach a clear plateau in the infrared limit. Nevertheless, looking at the value of $D_L(0)$ gives us an indication of what this plateau might be, and is useful to expose the strong systematic effects discussed here.

We can see that the very suggestive sharp peak at T_c seen for $N_t = 4$ (corresponding to the stars in Fig. 1) turns into a finite maximum around 0.9 T_c for $N_t = 16$ (diamonds). In other words, the observed singularity at smaller values of N_t seems to disappear. The only indication of a possible

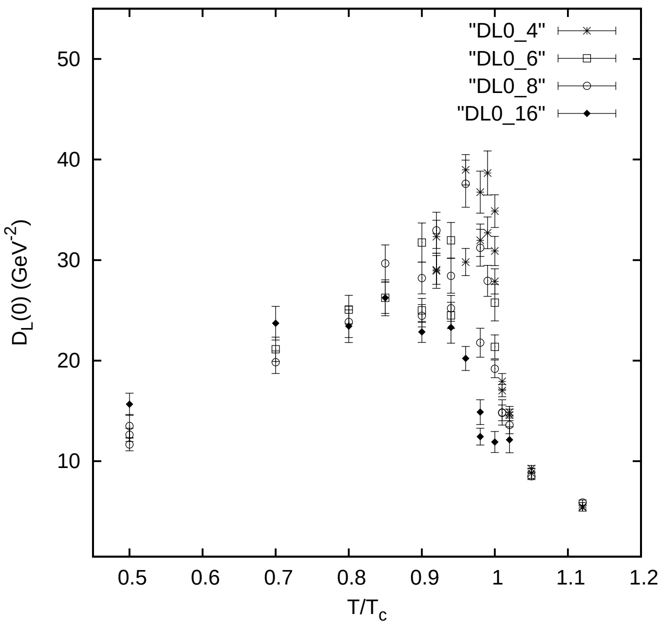

Figure 1: Infrared-plateau value for the longitudinal gluon propagator [estimated by $D_L(0)$] as a function of the temperature for the values of T/T_c around criticality. Data points from runs at the same value of N_t are grouped together and indicated by the label "DL0_N_t".

singular behavior is a finite maximum close to (but not *at*) the critical point, somewhat reminiscent of a pseudo-critical point as observed for the magnetic susceptibility of spin models in an external magnetic field (see e.g. [18, 19]).

Let us mention that, as reported in [13], good fits are obtained (in the transverse and longitudinal cases) to several generalized Gribov-Stingl forms, indicating the presence of comparable real and imaginary parts of pole masses. These masses are smooth functions of T around the transition, and the imaginary part of the electric mass seems to get smaller at higher T, as expected.

3. Conclusions

We have discussed data for the longitudinal (electric) and transverse (magnetic) Landau-gauge gluon propagator at nonzero temperature for pure SU(2) lattice gauge theory, obtained using very large lattices, especially at temperatures around the deconfinement phase transition. Fitting forms for describing the massive behavior of the propagator will be presented elsewhere [20].

As previously observed [15], results for the electric propagator $D_L(p^2)$ are plagued with unusually large systematic errors around criticality, which appear for small values of the temporal extent N_t and are seen only on the lower side of the transition temperature. These effects are not easily interpreted as a finite-size or a discretization effect, but can be avoided if we consider the data obtained with the largest value of N_t in Fig. 1. We then see that the sharp peak suggested by the stars (and even the empty symbols) turns into a smooth maximum, at around $0.9\,T_c$. Note that this feature is clearly observable even though the errors of $D_L(p^2)$ at zero momentum are quite large.

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As a result, any gluon mass extracted from the electric propagator will likely be a smooth function of the temperature, and we do not expect any specific signature of deconfinement associated with $D_L(p^2)$. Considering this behavior, and the similarity between our smaller-lattice results for the SU(2) case and existing results for SU(3), we must conclude that there is little possibility that the inverse of the zero-momentum value of the gluon propagator may provide an order parameter for the deconfinement phase transition.

Of course, as pointed out in the Introduction, there is an observed qualitative feature of the deconfined phase: the lack of violation of reflection positivity for the real-space electric propagator. However, this behavior does not signal the critical temperature, since it is present for all $T \neq 0$.

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