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Exploring the QCD phase diagram with conserved charge fluctuations

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We analyze cumulants of fluctuations of baryon number, electric charge and strangeness on the lattice, using highly improved staggered fermions and almost realistic quark masses. In particular we review the current stage of the sixth order fluctuations and discuss how higher order cumulants are related to the scaling behavior of QCD in the chiral limit. We argue that unlike the second and forth order cumulants, the sixth order cumulants receive a sizable contribution from the singular part of the free energy. Extracting this singular part can be helpful for the analysis of the QCD phase diagram and critical behavior.

The 32nd International Symposium on Lattice Field Theory
23-28 June, 2014
Columbia University New York, NY

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1. Introduction and formulation

The calculation of derivatives of the QCD partition function with respect to external parameters has always been a viable tool to explore the thermodynamics of quarks and gluons. However, in recent years much effort has been put into the calculation of derivatives of the logarithm of the grand canonical partition function with respect to various chemical potentials. After the Taylor expansion approach had been established as a "*solution*" to the QCD sign problem at small chemical potentials [1], it has quickly been realized that these expansion coefficients are of interest in their on right. The coefficients describe cumulants of charge fluctuations. Their temperature dependence across the QCD transition can be used to deduce the fundamental unit of charge of the effective degrees of freedom in the system [2]. In the case of the strangeness and charm charge one is evan able to separate the thermodynamic contribution of different strangeness (charm) sectors [3, 4]. Moreover, evidence for experimentally not yet observed hadrons has been found [4, 5].

Most interesting is, however, the fact that cumulants of conserved charge fluctuations can also be measured in heavy ion experiments as event-by-event fluctuations. Matching lattice data with experimental results is a model free way to extract the temperature, volume and chemical potentials of the fireball at the time of chemical freeze-out [5, 6, 7]. Event-by-event fluctuations have also been discussed as a possible experimental signature for a QCD critical point early on [8]. It has been pointed out that especially higher moments and cumulants are well suited for the critical point search [9].

Let us define the cumulants in a more precise way: assuming three independent quark flavors (up, down and strange) we have three chemical potentials associated with them, *i.e.*, $\vec{\mu} = (\mu_u, \mu_d, \mu_s)$. Phenomenological, it might be advantageous to perform a coordinate transformation in Gibbs space, and consider the baryon number (B), electric charge (Q) and strangeness (S) chemical potentials $\vec{\mu} = (\mu_B, \mu_Q, \mu_S)$. Later on, we will also chose the basis $\vec{\mu} = (\mu_q, \mu_I, \mu_S)$, where $\mu_q = (\mu_u + \mu_d)/2$ denotes the light quark chemical potential and $\mu_I = (\mu_u - \mu_d)/2$ the iso-spin chemical potential. On the lattice, the cumulants are generated as derivatives of the partition function

$$\chi_{ijk}^{BQS}(T) = \frac{1}{VT^3} \frac{\partial^{i+j+k} \ln Z(T, \mu_B, \mu_Q, \mu_S)}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} , \tag{1.1}$$

where $\hat{\mu}_X$ denotes the dimensionless expansion parameter μ_X/T with $X \in \{B, Q, S\}$ and T being the temperature. In the following we will drop indices if the corresponding oder of the derivative is zero, as *e.g.* $\chi_{200}^{BQS} \equiv \chi_2^B$. From experimental data of the detected charges for each event the cumulants are accessible by means of their original definition, here shown for the first three diagonal cumulants of even order

$$\chi_2^X = \frac{1}{VT^3} \left(\left\langle (\delta N_{X-\bar{X}})^2 \right\rangle \right), \tag{1.2}$$

$$\chi_4^X = \frac{1}{VT^3} \left(\left\langle (\delta N_{X-\bar{X}})^4 \right\rangle - 3 \left\langle (\delta N_{X-\bar{X}})^2 \right\rangle^2 \right), \tag{1.3}$$

$$\chi_6^X = \frac{1}{VT^3} \left(\left\langle (\delta N_{X-\bar{X}})^6 \right\rangle - 15 \left\langle (\delta N_{X-\bar{X}})^4 \right\rangle \left\langle (\delta N_{X-\bar{X}})^2 \right\rangle + 30 \left\langle (\delta N_{X-\bar{X}})^2 \right\rangle^3 \right) . \tag{1.4}$$

The odd order cumulants vanish at $\vec{\mu} \equiv 0$. Here $\delta N_{X-\bar{X}}$ denote the deviation of the net amount of charge from its mean, *i.e.*, $\delta N_{X-\bar{X}} = (N_X - N_{\bar{X}}) - < N_X - N_{\bar{X}} >$, for $X \in \{B, Q, S\}$. All offdiagonal cumulants, also known as correlations, can be obtained by similar formulae. Measuring the electric charges in the detector is rather easy for the experiments, a method to reconstruct net baryon number and net strangeness fluctuations from ALICE data has recently been proposed [10].

In the following we discuss to what extent we can expect the cumulants to be governed by universal critical behavior. Close to a second order phase transition QCD can be mapped to an underling symmetry model, that exhibits the symmetries present at the phase transition point. It is generally believed that in the chiral limit of two light flavors QCD features a second order phase transition point that lies in the $O(4)$ universality class [11]. Nonetheless, the possibility of a first order transition has been discussed recently [12], which would move the second order end-point to a finite light quark mass. At physical quark masses the possibility of a QCD critical point at non-zero chemical potential is discussed widely. It shows up in various model calculations, however, due to the infamous sign problem it could not firmly been established from ab-inito QCD calculations. The RHIC low energy scan program is to a large extent motivated by the possibility of finding a QCD critical point experimentally. This critical point is believed to be an end-point of a first order line and as such belongs to the Ising ($Z(2)$) universality class. A generic QCD phase diagram is shown in Fig. 1.

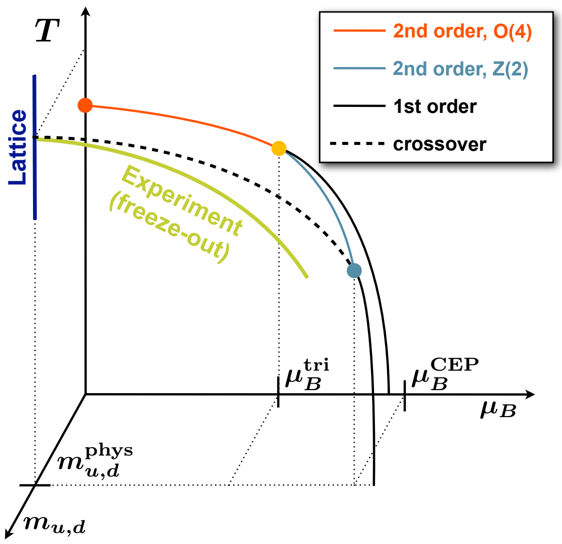

Figure 1: Generic phase diagram of QCD, based on model calculations and model independent symmetry arguments. Also indicated are the regions in the phase diagram where we are able to obtain results on fluctuation observables from lattice QCD and experiments, respectively.

2. $O(4)$ critical behavior

Independently on the universality class, RG theory predicts that the free energy has a singular part that is responsible for the power laws that thermodynamic response functions exhibits near the critical point. For degenerate light flavors $m_u = m_d = m_q$ we thus make the Ansatz

$$\frac{p}{T^4} = \frac{1}{VT^3} lnZ(T,V,m_q,m_s,\vec{\mu}) = -f_s(T,V,m_q,m_s,\vec{\mu}) - f_r(T,V,m_q,m_s,\vec{\mu}) \,, \qquad (2.1)$$

where the singular part f_s will become a generalized homogeneous function of its arguments once the correct scaling fields have been chosen as its natural variables. For all $O(N)$ and $Z(N)$ models we are left with two relevant scaling fields, the temperature-like (t) and external field-like scaling fields (h). Considering a critical point in the chiral limit ($m_q = 0$) and at zero chemical potentials ($\vec{\mu} = 0$) the leading order dependence of the scaling fields on $T, m_q, m_s, \vec{\mu}$ are determined by a symmetry argument, *i.e.* to leading order h depends only on couplings that break chiral symmetry in the light quark sector, while t depends on all other couplings. We find

$$t \equiv \frac{1}{t_0}\left(\frac{T-T_c}{T_c} + \kappa_q \hat{\mu}_q^2 + \kappa_{qs}\hat{\mu}_q\hat{\mu}_s + \kappa_s \hat{\mu}_s^2\right) \qquad \text{and} \qquad h = \frac{1}{h_0}\frac{m_q}{m_s} , \tag{2.2}$$

where T_c is the phase transition temperature and t_0, h_0 are non-universal scale parameters. Here we have adopted the $\vec{\mu} = (\mu_q, \mu_I, \mu_s)$ basis and assume $\mu_I = 0$ as a finite μ_I would break the flavor symmetry in such a way that a different symmetry breaking pattern would arise. Furthermore, we assume m_s to be fixed at the physical strange quark mass. With the above scaling fields we can exploit the property of homogeneity of the function f_s to obtain

$$f_s(t,h) = h_0 h^{1+1/\delta} f_f(z) = h_0 h^{(2-\alpha)/\beta\delta} f_f(z) \qquad \text{with} \qquad z = t/h^{1/\beta\delta} . \tag{2.3}$$

The function f_f is a universal scaling function that depend on a single scaling variable z. In this way we have singled out the leading order singular behavior, but have neglected sub-leading terms that are produced by irrelevant scaling variables and are known as corrections to scaling. From here we can easily obtain the magnetic equation of state [13] $M = h^{1/\delta} f_G(z)$ by taking a derivative with respect to $H = h_0 h$, where the scaling function f_G is connected to f_f by

$$f_G(z) = -\left(1+\delta^{-1}\right) f_f(z) + z\,(\beta\delta)^{-1} f_f'(z) . \tag{2.4}$$

Note that this scaling form is not the famous famous Widom-Griffiths from, as the scaling variables do not depend on the Magnetization. It is however well suited for a comparison with QCD where the calculated magnetization, which can, *e.g.*, be defined as $M = m_s \langle \bar{\psi}\psi \rangle / T^4$ has statistical and systematical errors. Such an enterprise has been undertaken and the non-universal constants t_0, h_0 have been determined [13]. More importantly it has been found that for an external field $H = m_q/m_s = 1/27$ that correspond to physical quark masses, the scaling Ansatz is still valid. The relative contribution of the regular part has been found to be small for H-derivatives of the QCD partition function. Given the parameters h_0, t_0, the non universal parameter κ_q has been determined in [14], by analyzing the mixed susceptibility $\chi_m = \partial^3 f_s / (\partial H \partial \mu_q^2)$. Systematic uncertainties of the normalization constants t_0, h_0 and κ_q have yet to be removed, *i.e.*, a continuum extrapolation of these quantities – eventually with two differed actions – is still to be performed.

If we now take derivatives with respect to μ_q we will be able to make a prediction on the singular part that is present in the light quark number cumulants as defined in Eq. (1.1) and Eqs. (1.2)-(1.4). At $\vec{\mu} = 0$ the general structure of the cumulants is given by

$$\chi_n^q \sim m_q^{2-\alpha-n/2} f_f^{(n/2)}(z) . \tag{2.5}$$

For the 3-dim. $O(4)$ symmetric model the exponent α is approximately $\alpha \approx -0.21$, which means that the fourth order cumulant develops a cusp in the chiral limit, while the sixth order cumulant is the first to diverge. As the singular part of the second and fourth order cumulants remain finite even in the chiral limit we expect this observables to be dominated by the regular part of the free energy. Once the normalization constants t_0, h_0, κ_q are determined precise enough the absolute strength of the singular part will also be fixed. For the here used HISQ action this is work in progress [15]. A parameterization of the scaling function f_f was determined in [16]. In Fig. 2 (left panel) we show the second and third derivative of the scaling function $f_f(z)$, which resembles up to a constant the singular of the cumulants χ_4^q and χ_6^q. For comparison we show lattice data on the middle

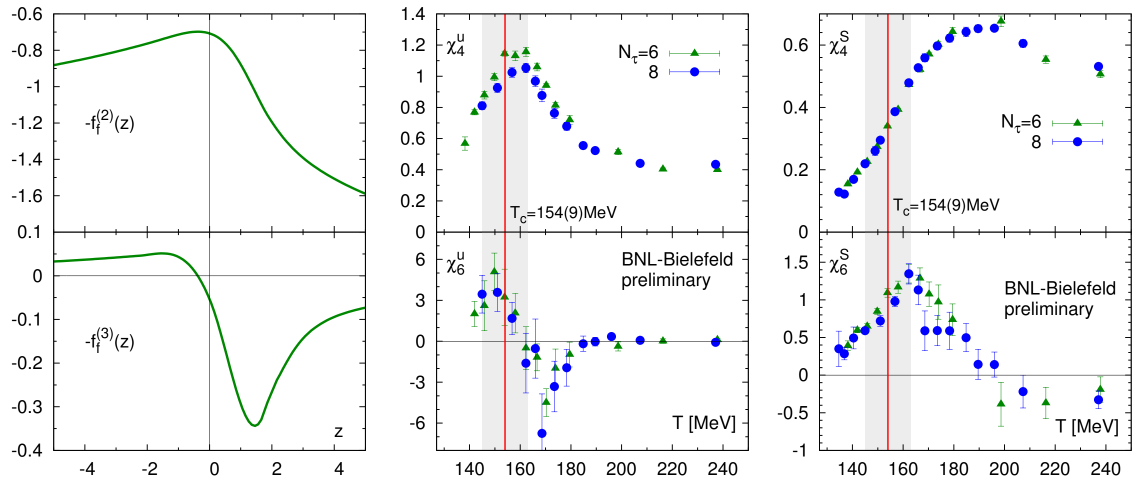

Figure 2: Second and third negative derivative of the universal scaling function $f_f(z)$ (left panel), forth and sixth oder cumulants of net up quark number fluctuations and net strangeness (middle) and (right) panels, respectively.

and right panels, obtained by using the HISQ action at quark mass values of $m_q/m_s = 1/20$. As the sixth oder light quark number fluctuations is very noisy, the middle panel shows the up quark number fluctuations χ_4^u and χ_6^u, which should serve here as a proxy for χ_4^q and χ_6^q. The singular contributions to both of these quantities should be identical, they are however likely receiving different contributions from the regular part. Indeed, if we compare the left and middle panels, the general structure of the sixth order up quark cumulant shows evidence for a typical $O(4)$ singular behavior. This is not the case for the cumulant of the net strangeness fluctuations, which are shown in the right panel.

From the structure of the third derivatives we can derive at least two distinct universal numbers, which is the ratio of the heights of the minimum above and the minimum below T_c. From the scaling function one finds a ratio of 1.734(75)[16]. Even though we lack the total normalization, and neglect the regular part, this ratio can also be determined from the χ_6^u data. At the present accuracy, we obtain a consistence result of roughly 1.5(5). If it can be shown that the regular part is small, this ratio is interesting, since the the minimum above T_c, which is much cheaper to calculate, does already determine the strength of the singular part below T_c.

The second interesting universal number is the difference between the two peaks. Given the non-universal normalization constants t_0, h_0, the universal number of $\Delta z \approx 3$ can be translated into a temperature difference. As the distance shrinks to zero in the chiral limit, one thus obtains a parameterization of the width of the crossover transition as seen by the sixth order cumulant. From

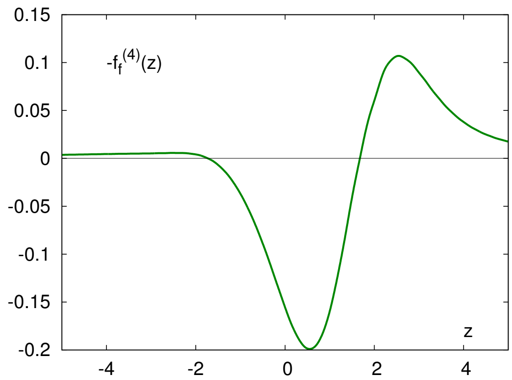

Figure 3: Negative fourth derivative the universal $O(4)$ scaling function $f_f(z)$.

χ_u^6 we estimate the width to be of the order of $(20-30)$ MeV at physical quark masses.

3. Critical behavior at nonzero chemical potential

Finally we want to mention that the structure of the singular behavior of the cumulants becomes rather different ones the critical point sits at a nonzero chemical potential. The temperature like scaling field can now be linearized around the critical chemical potential μ_q^{crit} and can be written as

$$t \equiv \frac{1}{t_0}\left(\frac{T-T_c}{T_c} + 2\kappa_q \hat{\mu}_q^{crit}(\hat{\mu}_q - \hat{\mu}_q^{crit})\right) , \tag{3.1}$$

where we assume $\mu_S^{crit} \equiv 0$ and have suppresed the μ_S dependence. Furthermore, the derivatives are supposed to be taken at (or close to) μ_q^{crit}. The general singular structure of the cumulants, which we was given in (2.5) for the case of $\vec{\mu} = 0$, now turns into

$$\chi_n^q \sim m_q^{2-\alpha-n} f_f^{(n)}(z) . \tag{3.2}$$

If we now also assume that the critical point lies in the universality class of the 3-dim. Ising model ($Z(2)$) we find $\alpha \approx 0.11$ and divergences start already at the second order.

The forth order cumulant is dominated by the term $h^{-(2+\alpha)/(\beta\delta)} f^{(4)}(z)$, which we have plotted in Fig. 3 for the $O(4)$ case. We immediately see the different structure of the singular contribution if we compare with the second derivative of $f_f(z)$ in Fig. 2. The ratio between the peak below T_c an just above T_c is of the order of 25. This structure is the reason why the kurtosis (χ_4^B/χ_2^B) might have a negative dip on the freeze-out line, depending on how close the freeze-out line passes by the critical point [17].

References

[1] C. R. Allton, S. Ejiri, S. J. Hands, O. Kaczmarek, F. Karsch, E. Laermann, C. Schmidt and L. Scorzato, Phys. Rev. D **66** (2002) 074507 [hep-lat/0204010].

[2] S. Ejiri, F. Karsch and K. Redlich, Phys. Lett. B **633** (2006) 275 [hep-ph/0509051].

[3] A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Maezawa and S. Mukherjee *et al.*, Phys. Rev. Lett. **111** (2013) 082301 [arXiv:1304.7220 [hep-lat]].

[4] A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Maezawa and S. Mukherjee *et al.*, Phys. Lett. B **737** (2014) 210 [arXiv:1404.4043 [hep-lat]].

[5] A. Bazavov, H.-T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, Y. Maezawa and S. Mukherjee *et al.*, Phys. Rev. Lett. **113** (2014) 7, 072001 [arXiv:1404.6511 [hep-lat]].

[6] A. Bazavov, H. T. Ding, P. Hegde, O. Kaczmarek, F. Karsch, E. Laermann, S. Mukherjee and P. Petreczky *et al.*, Phys. Rev. Lett. **109** (2012) 192302 [arXiv:1208.1220 [hep-lat]].

[7] S. Borsanyi, Z. Fodor, S. D. Katz, S. Krieg, C. Ratti and K. K. Szabo, Phys. Rev. Lett. **111** (2013) 062005 [arXiv:1305.5161 [hep-lat]].

[8] M. A. Stephanov, K. Rajagopal and E. V. Shuryak, Phys. Rev. D **60** (1999) 114028 [hep-ph/9903292].

[9] C. Athanasiou, K. Rajagopal and M. Stephanov, Phys. Rev. D **82** (2010) 074008 [arXiv:1006.4636 [hep-ph]].

[10] P. Braun-Munzinger, A. Kalweit, K. Redlich and J. Stachel, arXiv:1412.8614 [hep-ph].

[11] R. D. Pisarski and F. Wilczek, Phys. Rev. D **29** (1984) 338.

[12] M. D'Elia, A. Di Giacomo and C. Pica, Phys. Rev. D **72** (2005) 114510 [hep-lat/0503030]; A. Pelissetto and E. Vicari, Phys. Rev. D **88** (2013) 10, 105018 [arXiv:1309.5446 [hep-lat]]; C. Bonati, P. de Forcrand, M. D'Elia, O. Philipsen and F. Sanfilippo, Phys. Rev. D **90** (2014) 7, 074030 [arXiv:1408.5086 [hep-lat]].

[13] S. Ejiri, F. Karsch, E. Laermann, C. Miao, S. Mukherjee, P. Petreczky, C. Schmidt and W. Soeldner *et al.*, Phys. Rev. D **80** (2009) 094505 [arXiv:0909.5122 [hep-lat]].

[14] O. Kaczmarek, F. Karsch, E. Laermann, C. Miao, S. Mukherjee, P. Petreczky, C. Schmidt and W. Soeldner *et al.*, Phys. Rev. D **83** (2011) 014504 [arXiv:1011.3130 [hep-lat]].

[15] H.-T. Ding, A. Bazavov, F. Karsch, Y. Maezawa, S. Mukherjee and P. Petreczky, PoS LATTICE **2013** (2014) 157 [arXiv:1312.0119 [hep-lat]].

[16] J. Engels and F. Karsch, Phys. Rev. D **85** (2012) 094506 [arXiv:1105.0584 [hep-lat]]; J. Engels and F. Karsch, Phys. Rev. D **90** (2014) 1, 014501 [arXiv:1402.5302 [hep-lat]].

[17] B. Friman, F. Karsch, K. Redlich and V. Skokov, Eur. Phys. J. C **71** (2011) 1694 [arXiv:1103.3511 [hep-ph]].

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