

Exploring the phase structure of 12-flavor $SU(3)$

Zechariah Gelzer*

Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa, USA

E-mail: zechariah-gelzer@uiowa.edu

Yuzhi Liu

Department of Physics, University of Colorado, Boulder, Colorado, USA

E-mail: yuzhi.liu@colorado.edu

Yannick Meurice

Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa, USA

E-mail: yannick-meurice@uiowa.edu

We are studying the $SU(3)$ gauge theory with 12 staggered fermions, searching for the endpoint of the line of first-order phase transitions in the mass–beta plane. This endpoint plays an important role in our understanding of the phase diagram of this model. Having found this endpoint with high statistics on a small lattice using unimproved staggered fermions, we are working to find it on larger lattices and with improved actions. For an action improved with nHYP-smearred staggered fermions, we discuss the effect of slowly turning off the improvement on the broken shift symmetry phase.

The 32nd International Symposium on Lattice Field Theory

23-28 June, 2014

Columbia University New York, NY

*Speaker.

1. Motivations

Following the recent discovery of a Higgs-like boson with an approximate mass of 126 GeV, there is a possibility that it exists as a composite particle formed by a new strongly-interacting gauge theory. To study the theory candidates, we must better understand their phase structures, such as whether a given model develops an infrared fixed point (IRFP) [1]. Models that have been receiving increasing attention include $SU(3)$ gauge theories with $N_f = 8, 12$ flavors [2, 3, 4].

We are studying $SU(3)$ gauge theory with 12 staggered fermions, adopting both unimproved and improved actions. We perform simulations by using the standard hybrid Monte Carlo (HMC) algorithm, applying code by Dr. Donald Sinclair and Dr. Yuzhi Liu for the unimproved case, as well as a modified version of the MILC code [5] for the improved case. We pay particular attention to the line of first-order phase transitions in the m - β plane, where m is our fermion mass and β is inversely proportional to the bare gauge coupling ($\beta = 6/g^2$ in $SU(3)$). This question of an endpoint for $SU(3)$ with $N_f = 12$ is also being discussed by groups using a gauge action with renormalization group (RG) improvement, known as doubly blocked Wilson (DBW2) [6].

The existence and location of the m - β transition endpoint will help us determine important features of the theory's phase diagram, primarily since all RG flows start from the m - β plane [7]. One may then analyze the endpoint from the viewpoint of Fisher zeros: zeros of the partition function Z in the complex β -plane, at which phase transitions occur. Such Fisher zeros act as indicators of an IRFP for RG flows [8] and are thus vital to understanding the continuum behavior of lattice models.

2. Numerical results

First, we employ unimproved staggered fermions and the simplest form of the Wilson gauge action, wherein we are limited to plaquette terms in the fundamental representation only (i.e., $\beta_A = 0$). Starting from the HMC algorithm, which has been made exact by its global Metropolis accept/reject step, we simulate with high statistics on small lattices to quickly yield observables with small errors. Completing simulations for a range of fermion masses from $m = 0.005$ to $m = 1$ concludes that the endpoint of the line of first-order phase transitions is in the vicinity of $m \approx 0.3$, as seen in Fig. 1. The error bars are negligible, outside of rare cases near the transitions for higher masses, and thus have been omitted from Fig. 1 to provide greater clarity. A typical example of the size of the error bars can be seen in the left panel of Fig. 2 for $m = 0.05$.

The scaling of these observables with the size of the lattice can be measured, allowing us to explore the progression of a finite temperature transition into a bulk transition. We find that the transitions in the chiral condensate $\langle \bar{\psi}\psi \rangle$ and average plaquette are bulk transitions for isotropic lattices $V \equiv L_x^3 \times L_t = L^4$ with $L \geq 12$, as seen in Fig. 3. This rapid convergence persists even with improved actions [10]. These qualities make small-lattice experiments attractive, insofar as they serve to guide the more accurate large-lattice experiments that are currently in progress.

We are working to find the transition endpoint on larger lattices; results in Fig. 4 reveal that $m = 0.15, 0.30$ are beyond this endpoint. Preliminary results for $m = 0.075, 0.125$ suggest that the endpoint lies somewhere in this range. Although the volume effects for $N_f = 12$ unimproved

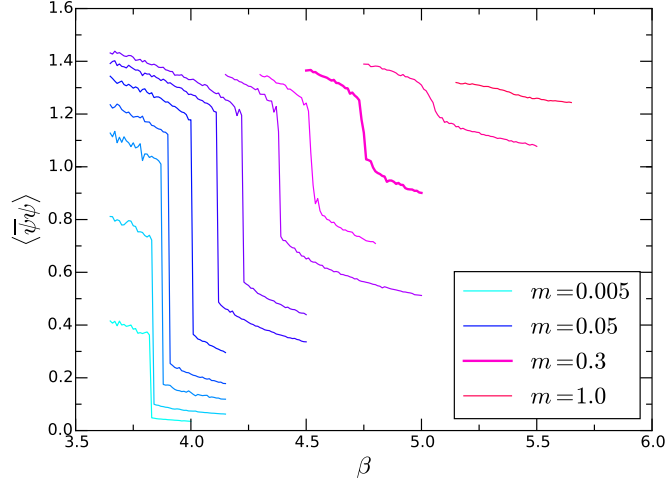


Figure 1: Unimproved HMC. Chiral condensate vs. β for increasing m , with $N_f = 12$, $V = 4^4$. The masses included (from left to right) are as follows: 0.0050, 0.0105, 0.0200, 0.0300, 0.0500, 0.0755, 0.0995, 0.1505, 0.2002, 0.3000, 0.5000, 0.9999.

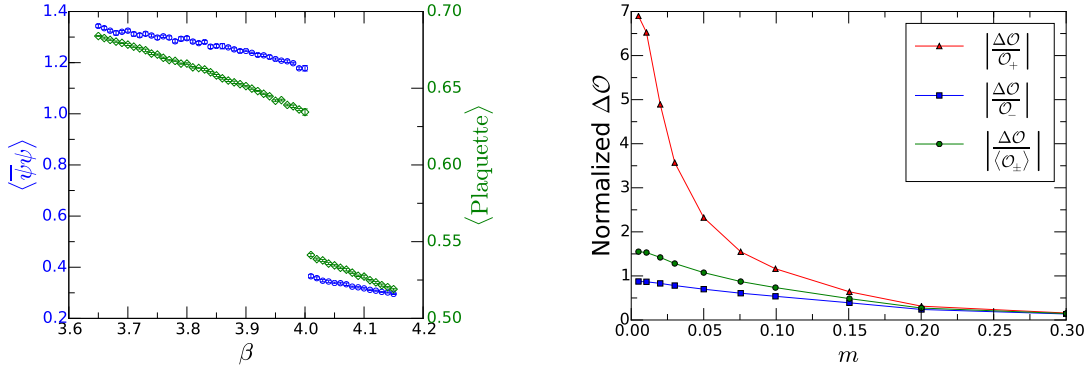


Figure 2: Unimproved HMC. Left panel: chiral condensate (blue) and average plaquette (green) vs. β , with $N_f = 12$, $V = 4^4$, $m = 0.05$. Note that the error bars are smaller than the data point markers. Right panel: normalized transitions $\Delta\mathcal{O}$ about the critical beta β_C for $\mathcal{O} = \langle \bar{\psi}\psi \rangle$ vanish with increasing m . The red curve is normalized to $\beta > \beta_C$, the blue one to $\beta < \beta_C$, and the green one to an average of the two regions.

HMC are negligible beyond $V = 12^4$, we will confirm our proximity to the continuum with select simulations on $V = 24^4$.

In addition to unimproved HMC, we make use of improved actions. We employ a gauge action with both fundamental and adjoint plaquette terms, which are tuned to $\beta_A = -0.25\beta_F$ to remove a spurious ultraviolet fixed point (UVFP), a well-known lattice artifact [10]. We also employ smeared staggered fermions. The smearing procedure [11] aims to alleviate so-called taste splitting: i.e., the symmetry breaking at nonzero lattice spacing a between the four fermion “tastes” described by each unrooted staggered fermion. We replace the “thin” links $U_{n,\mu}$ (at the lattice site n , in the direction μ) by hypercubic (HYP) links that are defined according to three smearing parameters α_i ,

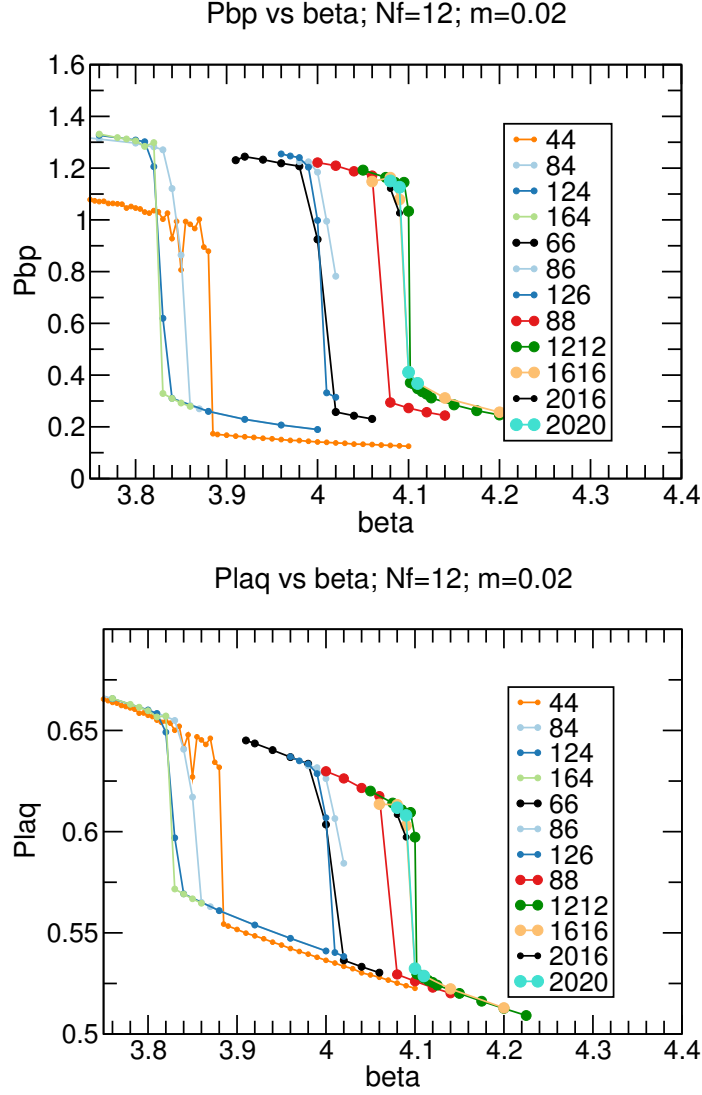


Figure 3: Unimproved HMC. Top panel: chiral condensate vs. β , with $N_f = 12$, $m = 0.02$. Bottom panel: plaquette vs. β , with $N_f = 12$, $m = 0.02$. In the legend, “124” signifies a volume of $V = 12^3 \times 4$.

which are set to $\alpha_i = (0.75, 0.6, 0.4)$ in Ref. [11]. We may then form normalized HYP (nHYP) links by projecting them to $U(3)$, rather than $SU(3)$, such that they become differentiable. In keeping with [10], we adjust to $\alpha_i = (0.5, 0.5, 0.4)$, which improves the $U(3)$ projection at strong coupling while marginally increasing the taste splitting.

Applying the smeared links with $N_f = 8, 12$ leads to a novel phase: the broken shift symmetry S^4 phase [10]. The single-site shift symmetry S^4 is an exact symmetry of the staggered fermion action, which ensures that $\langle \bar{\psi}\psi \rangle$ measured on even lattice sites is identical to that measured on odd ones. Order parameters sensitive to the spontaneous breaking of S^4 include the difference between neighboring plaquettes \square_n ,

$$\Delta P_\mu = \langle \text{Re Tr } \square_n - \text{Re Tr } \square_{n+\mu} \rangle_{n_\mu \text{ even}}, \quad (2.1)$$

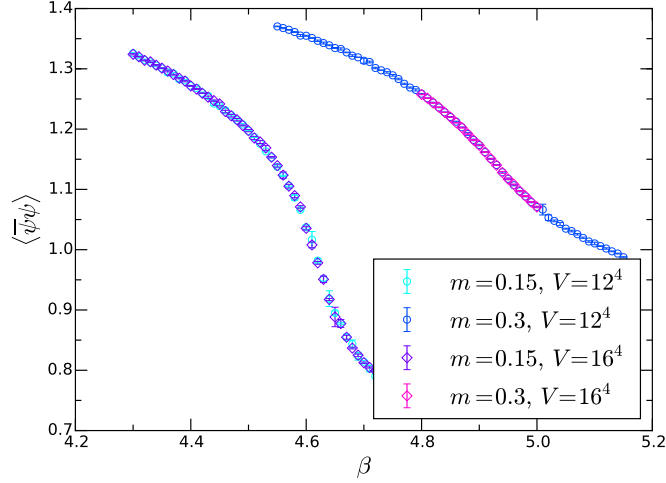


Figure 4: Unimproved HMC. Chiral condensate vs. β for different m and V , with $N_f = 12$. This shows nearly exact agreement between the two volumes.

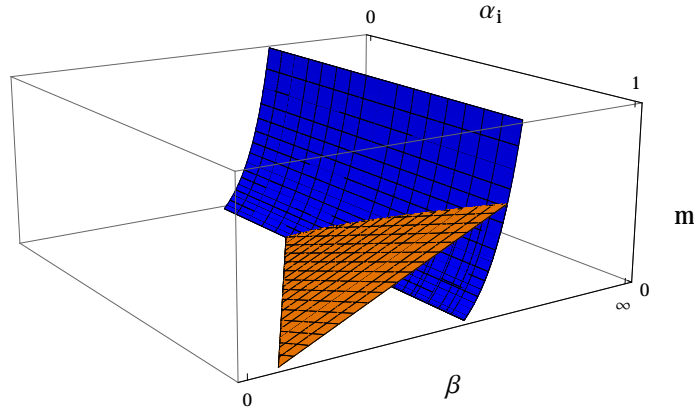


Figure 5: Sketch for nHYP-improved MILC. Expected phase diagram of transitions in the m - β - α_i space. The orange plane highlights the \mathcal{S}^4 phase, while the blue plane depicts the bulk transition between χ SB and deconfinement.

which becomes nonzero in direction(s) μ during the \mathcal{S}^4 phase [10]. A confirmation of the trend of the \mathcal{S}^4 phase has been completed for $N_f = 8$; an in-depth look at $N_f = 12$ is ongoing.

Interpolating between the improved and unimproved classes of actions, we expect that the \mathcal{S}^4 phase should vanish as we slowly lessen the improvement parameters. Simulations are currently in progress to ascertain this effect; a qualitative shape of the expected phase diagram is included in Fig. 5.

3. Conclusions

For the unimproved class of simulations, we will continue to seek the endpoint of the line of first-order phase transitions, which we currently believe exists between $m = 0.075$ and $m = 0.125$. Once at this endpoint, we will conduct an analysis with Fisher zeros around the critical beta β_C , as discussed by Y. Liu et al. in Ref. [7].

For the improved class of simulations, we will seek to confirm the \mathcal{S}^4 phase for $N_f = 12$. Given that \mathcal{S}^4 is expected to be reinstated in some region of the improvement parameter space α_i, β_A , interesting phenomena may appear at the boundary of this region. A tricritical point may exist between the boundaries of the phases of χ SB (chiral symmetry breaking), \mathcal{S}^4 , and deconfinement (chiral symmetry restoring).

Acknowledgements

We thank Dr. Donald Sinclair of Argonne National Laboratory for the use of his HMC code.

We also thank Dr. Anna Hasenfratz of the University of Colorado, Boulder, for the assistance she has provided in the tuning of the improvement parameters of the nHYP-improved MILC code.

Finally, we thank MILC for continuing to make its code readily available under the GNU General Public License.

This research was supported in part by the Department of Energy under Award Numbers DE-SC0010114 and FG02-91ER40664. We used the National Energy Research Scientific Computing Center (NERSC), which is supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231. Z. G. is supported by the URA Visiting Scholars program and uses the Fermilab Lattice Gauge Theory Computational Facility. Fermilab is operated by Fermi Research Alliance, LLC, under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy.

References

- [1] T. DeGrand, “Lattice studies of quantum chromodynamics-like theories with many fermionic degrees of freedom”, *Phil. Trans. R. Soc. A* **369** 2701 (2011).
- [2] X.-Y. Jin, R. Mawhinney, “Lattice QCD with 8 and 12 degenerate quark flavors”, PoS LATTICE **2009** (2009) 049 [arXiv:0910.3216 [hep-lat]].
- [3] Z. Fodor, K. Holland, J. Kuti, D. N6gr6di, C. Schroeder, C. H. Wong, “Conformal finite size scaling of twelve fermion flavors”, arXiv:1211.4238 [hep-lat] (2012).
- [4] Y. Aoki, et al. (LatKMI Collaboration), “Light Composite Scalar in Twelve-Flavor QCD on the Lattice”, *Phys. Rev. Lett.* **111**, 162001 (2013).
- [5] MIMD Lattice Collaboration (MILC), <http://www.physics.utah.edu/~detar/milc/>.
- [6] X.-Y. Jin, R. Mawhinney, “Lattice QCD with 12 Quark Flavors: A Careful Scrutiny”, arXiv:1304.0312 [hep-lat] (2013).
- [7] Z. Gelzer, Y. Liu, Y. Meurice, and D. Sinclair, “Fisher zeros and RG flows for $SU(3)$ with N_f flavors”, PoS LATTICE **2013** (2013) 078 [arXiv:1312.3906 [hep-lat]].
- [8] A. Denbleyker, D. Du, Y. Liu, Y. Meurice and H. Zou, “Fisher’s zeros as boundary of renormalization group flows in complex coupling spaces”, *Phys. Rev. Lett.* **104**, 251601 (2010).

- [9] J. Kogut, D. Sinclair, “The RHMC algorithm for theories with unknown spectral bounds”, *Phys. Rev. D* **74**, 114505 (2006).
- [10] A. Cheng, A. Hasenfratz, and D. Schaich, “Novel phase in $SU(3)$ lattice gauge theory with 12 light fermions”, *Phys. Rev. D* **85**, 094509 (2012).
- [11] A. Hasenfratz, R. Hoffmann, and S. Schaefer, “Dynamical simulations with HYP-link Wilson fermions”, *JHEP* **05**, 029 (2007).